PART-A UNIT II

PROBABILITY DISTRIBUTION

1) What is the value of correction factor if $n=5$ and $N=200$

Sol: Given N = the size of finite population = 200

n = the size of sample = 5

$\therefore$ Correction factor $= \frac{N-n}{N-1} = \frac{200-5}{200-1}$

$= \frac{195}{198} = 0.98$

2) Using Poisson's distribution, find the probability that the ace of spades will be drawn from a pack of well shuffled cards at least once in 104 consecutive trials.

Sol: Here $p = \frac{1}{52}$ and $n = 104$

$\therefore$ Mean of the distribution, $\lambda = np = \frac{104}{52} = 2$

P(atleast once) $= P(X \geq 1) = 1 - P(X=0)$

$\Rightarrow 1 - \frac{e^{-2}(2)^0}{0!} = 1 - 0.1353 = 0.8647.$

3) In eight throws of a die 5 or 6 is considered a success. Find the mean number of success and standard deviation.

Sol: p = probability of success $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

q = The probability of failure $= 1-p = 1 - \frac{1}{3} = \frac{2}{3}$

$n =$ number of throws $= 8$.

$\therefore$ Mean $= np = 8\left(\frac{1}{3}\right) = \frac{8}{3}$

Variance $= (np)q = \left(\frac{8}{3}\right)\left(\frac{2}{3}\right) = \frac{16}{9}$

Hence standard deviation $= \sqrt{\text{variance}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$

4) If X is a normal variate, find the area A.

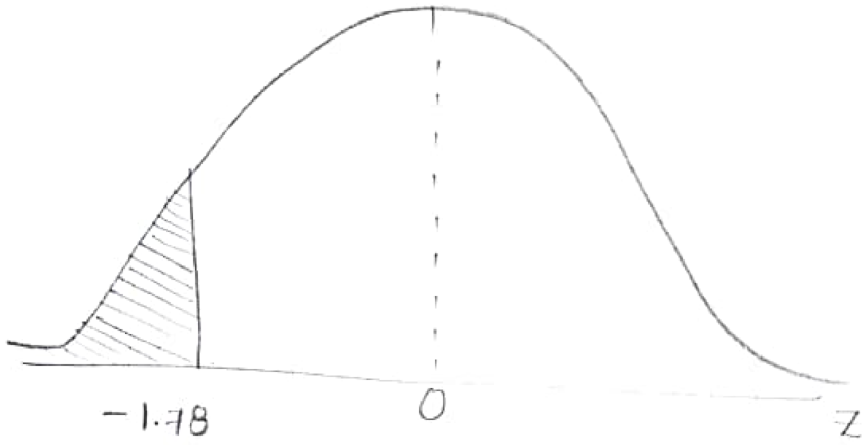
(i) to the left of $z = -1.78$

Sol:- (i) Required area, A

$= 0.5 - \text{Area}(0 \text{ to } -1.78)$

$= 0.5 - \text{Area}(0 \text{ to } 1.78)$ (By symmetry)

$= 0.5 - 0.4625$ (from tables) $= 0.0375$.



A Fair coin is tossed six times. Find the probability of getting four heads

Sol:- p = probability of getting a head $= 1/2$

q = probability of not getting head $= 1/2$

and $n = 6$, $r = 4$

We know that $P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$\therefore P(4) = 6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{6-4} = \frac{6!}{4!2!} \cdot \left(\frac{1}{2}\right)^6 = \frac{6 \times 5}{2} \cdot \frac{1}{2^6} = \frac{15}{64}$

$= 0.2344.$

PART-B.

Out of 800 families with 5 children each, how many would you expect to have

(i) 3 boys

(ii) 5 girls

(iii) Either 2 or 3 boys

(iv) atleast one boy

Assume equal probability for a boy and girl.

Given $N = 800$ families

$n = 5$

$p = \frac{1}{2}$ and $q = \frac{1}{2}$

(i) 3 boys:-

$P(X=3) = nC_3 \, p^3 q^{n-3}$

$= 5C_3 \, p^3 q^{5-3}$

$\Rightarrow 5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$

$\Rightarrow 10 \left(\frac{1}{8}\right)\left(\frac{1}{4}\right)$

$\Rightarrow 10 \left(\frac{1}{8}\right)\left(\frac{1}{4}\right)$

$P(X=3) \Rightarrow \frac{10}{32} = \frac{5}{16}$

For 800 families $= N.P(X)$

$$= 800 \times \frac{5}{16}$$

$$= 250$$

$\therefore$ 250 families are having 3 boys.

(ii) 5 girls

$$P(X=5) = nC_5\, p^5\, q^{n-5}$$

$$\Rightarrow 5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^{5-5}$$

$$\Rightarrow 1\left(\frac{1}{2^5}\right) \times 1$$

$$\Rightarrow \frac{1}{32}$$

For 800 families $\frac{1}{32} \times 800$

$$= \underline{25}$$

$\therefore$ 25 families are having 5 girls among 800 families.

(iii) Either 2 or 3 boys.

$$P(\text{Either 2 or 3 boys}) = P(X=2) + P(X=3)$$

$$\Rightarrow nC_2\, p^2\, q^{n-2} + nC_3\, p^3\, q^{n-3}$$

$$\Rightarrow 5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + 5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2$$

$\Rightarrow 10\left(\frac{1}{4}\right)\left(\frac{1}{8}\right) + 10\left(\frac{1}{8}\right)\left(\frac{1}{4}\right)$

$= \frac{5}{8}$

$\therefore$ For 800 families $N.p(x)$

$= 800 \times \frac{20}{32}$

$= 500$

$\therefore$ 500 families are having either 2 or 3 boys.

(iv) $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - n_{C_0} p^0 q^{n-0}$

$\Rightarrow 1 - 5_{C_0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{5-0}$

$\Rightarrow 1 - 1.1 . \frac{1}{2^5}$

$\Rightarrow 1 - \frac{1}{32}$

$\Rightarrow \frac{31}{32}$ //

$\therefore$ For 800 families $= N.p(x) = \frac{31}{32} \times 800$

$= 775$ families.

* Fit the binomial distribution of following data.

x	0	1	2	3	4	5	6
f	13	25	52	58	32	16	14

Sol: Let $N = \Sigma f$

$\Rightarrow 13+25+52+58+32+16+4$

$= 200$

$n = 6$

Mean $\mu = np = \frac{\Sigma f_i x_i}{N}$

$\Rightarrow \frac{13 \times 0 + 25 \times 1 + 52 \times 2 + 58 \times 3 + 32 \times 4 + 16 \times 5 + 14 \times 6}{200}$

$\mu = np = 2.67$

$np = 2.67$

$p = \frac{2.67}{6}$

$p + q = 1$

$p = 0.44$

$q = 1 - 0.44$

$q = 0.56$

$q = 0.56.$

x	f	$p(x) = {}^nc_x\, p^x q^{n-x}$	$N.p(x)$
0	13	$p(0) = 6c_0 (0.44)^0 (0.56)^{6-0}$ $= 0.03.$	$200 \times 0.03 = 6$
1	25	$p(1) = 6c_1 (0.44)^1 (0.56)^5$ $= 0.14$	$200 \times 0.14 = 28$
2	52	$p(2) = 6c_2 (0.44)^2 (0.56)^4$ $= 0.28$	$200 \times 0.28 = 56$
3	55	$p(3) = 6c_3 (0.44)^3 (0.56)^3$ $= 0.27$	$200 \times 0.27 = 60$
4	32	$p(4) = 6c_4 (0.44)^4 (0.56)^2$ $= 0.17$	$200 \times 0.17 = 35$
5	16	$p(5) = 6c_5 (0.44)^5 (0.56)^1$ $= 0.05$	$200 \times 0.05 = 11$
6	4	$p(6) = 6c_6 (0.44)^6 (0.56)^0$ $= 0.007$	200×0.007 $= 1.$

A hospital switch board receives an ananger of four emergency calls in ten minute interval. What is the probability that

(i) There are atmost two emergency calls in ten minute interval.

(ii) There are exactly is emergency calls in 10 minute interval.

(i) Given $\lambda = 4$

(ii) $P(\lambda \le 2) = P(X=0) + P(X=1) + P(X=2)$

$$= \frac{e^{-4}\lambda^0}{0!} + \frac{e^{-4}.\lambda^1}{1!} + \frac{e^{-4}!\lambda^2}{2!}$$

$$\Rightarrow e^{-4}\left[1+\lambda+\frac{\lambda^2}{2}\right]$$

$$\Rightarrow e^{-4}\left[1+4+\frac{16}{2}\right]$$

$$e^{-4}[1+4+9]$$

$$e^{-4}[13]$$

$$13e^{-4}$$

$$\boxed{P(X \le 2) = 0.238}$$

(iii) $P(X=3) = \dfrac{e^{-4}.4^3}{3!}$

$$\Rightarrow \frac{\overset{32}{\cancel{64}}.e^{-4}}{3 \times \cancel{2}}$$

$$\Rightarrow \frac{32}{3}e^{-4}$$

$P(X=3) = 0.195$

If a poisson distribution is such that $P(X=4) = \frac{1}{2} P(X=2)$

$+ P(X=0)$. Find

(i) Mean

(ii) $P(X \leq 2)$

Given $\frac{1}{2} P(X=2) + P(X=0) = 3P(X=4)$

$$\Rightarrow \frac{1}{2} \frac{e^{-\lambda} \cdot \lambda^2}{2!} + \frac{e^{-\lambda} \cdot \lambda^0}{0!} = \frac{3 \cdot e^{-\lambda} \cdot \lambda^4}{4!}$$

$$\Rightarrow \frac{\lambda^2 e^{-\lambda}}{4} + e^{-\lambda} = \frac{e^{-\lambda} \cdot \lambda^4}{8}$$

$$\Rightarrow \cancel{e^{-\lambda}}\left[\frac{\lambda^2}{4} + 1\right] = \cancel{e^{-\lambda}}\left[\frac{\lambda^4}{8}\right]$$

$$\Rightarrow \frac{\lambda^2}{4} + 1 = \frac{\lambda^4}{8}$$

$$\Rightarrow \frac{\lambda^2 + 4}{4} = \frac{\lambda^4}{8}$$

$$\lambda^2 + 4 = \cancel{4} \cdot \frac{\lambda^4}{\cancel{8}_{\,2}}$$

$$\Rightarrow 2\lambda^2 + 8 = \lambda^4$$

$$\Rightarrow \lambda^2 - 2\lambda^2 - 8 = 0$$

$$\lambda^4 - 4\lambda^2 + 2\lambda^2 + 8 = 0$$

$$\lambda^2(\lambda^2 - 4) + 2(\lambda^2 - 4) = 0$$

$$(\lambda^2 - 4)(\lambda^2 + 2) = 0$$

$\lambda^2 - 4 = 0$	$\lambda^2 + 2 = 0$
$\lambda^2 = 4$	$\lambda^2 = -2$
$\lambda = 2$	$\lambda = \sqrt{-2}$
✓	✗

(i) Mean $\mu = \lambda = 2$

(ii) $P(X \leq 2)$

$P(X=0) + P(X=1) + P(X=2)$

$$\Rightarrow \frac{e^{-\lambda} \cdot \lambda^0}{0!} + \frac{e^{-\lambda} \cdot \lambda^1}{1!} + \frac{e^{-\lambda} \cdot \lambda^2}{2!}$$

$$\Rightarrow e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2}\right]$$

$$\Rightarrow e^{-2}\left[1 + 2 + \frac{2^2}{2}\right]$$

$$\Rightarrow e^{-2}[1 + 2 + 2] = \underline{\underline{5e^{-2}}}$$

A sample of 1000 cases, the mean of a certain test is 14 and the standard deviation is 2.5. Assuming the distribution to be normal. Find the

(i) How many students score b/w 12 and 15.

(ii) How many students score above 18.

(iii) How many students score below 18.

Sol: Given $\mu = 14$

Standard deviation $\sigma = 2.5$

(i) $P(12 \le X \le 15) = P(X_1 \le X \le X_2)$

$= P(Z_1 \le Z \le Z_2)$

Here $X_1 = 12$ and $X_2 = 15$

$Z_1 = \frac{X_1 - \mu}{\sigma}$ $\quad Z_2 = \frac{X_2 - \mu}{\sigma}$

$\Rightarrow \frac{12 - 14}{2.5}$ $\quad \Rightarrow \frac{15 - 14}{2.5}$

$Z_1 = -0.8 < 0$ $\quad Z_2 = 0.4 > 0$

$P(Z_1 \le Z \le Z_2)$

$\Rightarrow (-0.8 \le Z \le 0.4)$

$\Rightarrow A|(0.8) + A(0.4)|$ ($\because$ From normal distibution table).

$\Rightarrow 0.1881 + 0.1554$

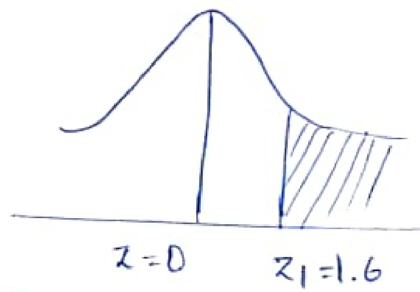
$= 0.4435$

For 1000 cases , $= 1000 \times 0.4435$

$= 443.5$

$\cong 444$

(ii) $P(X \ge 18) = P(Z \ge Z_1)$



Here $x = 18$

$$Z = \frac{X - \mu}{\sigma} = \frac{18 - 14}{2.5} = 1.6 > 0$$

$P(Z \ge 1.6) = |0.5 - A(Z_1)|$

$= |0.5 - A(1.6)|$

$= 0.5 - 0.4452$

$= 0.0548$

For 1000 cases $= 1000 \times 0.0548$

$= 54.8 \simeq 55$

$= 55$

(iii) $P(X \leq 18)$

$P(Z \leq z_1)$

Here $x = 18$

$$Z = \frac{X-\mu}{\sigma} = \frac{18-14}{2.5} = 1.6 > 0$$

$$P(Z \leq 1.6) = |0.5 + A(z_1)|$$

$$= |0.5 + A(1.6)|$$

$$= 0.5 + 0.4452$$

$$= 0.9452$$

For 1000 cases $= 1000 \times 0.9452$

$$= 945.2 \simeq 945$$

$$\Rightarrow \underline{\underline{945}}$$

* In a normal distribution, 31% of the items are under 45, 8% are over 64. Find mean and variance.

Sol: $P(X \leq 45) = 31\%$ $\qquad$ $P(X \geq 64) = 8\%$

$P(X \leq 45) = 0.31$ $\qquad$ $P(X \geq 64) = 0.08$

Here $x_1 = 45$, $x_2 = 64$

$\Rightarrow P(0 \le z \le z_1) = 0.19$ and $P(0 \le z \le z_2) = 0.42$

From the normal distribution tables,

$z_1 = 0.5$ and $z_2 = 1.41$

$z_1 = 0.\ \frac{x_1 - \mu}{\sigma}$ $\quad z_2 = \frac{x_2 - \mu}{\sigma}$

$-0.5 = \frac{45 - \mu}{\sigma}$ $\quad 1.41 = \frac{64 - \mu}{\sigma}$

$45 - \mu = (-0.5)\sigma \rightarrow (1)$ $\quad 64 - \mu = 1.41\sigma \rightarrow (2)$

Solving eq^{n}s (1) and (2)

$$
\begin{array}{rcl}
64 - \mu & = & 1.41\sigma \\
-45 - \mu & = & -0.5\sigma \\
+ & & + \\
\hline
19 & = & 1.91\sigma
\end{array}
$$

$\sigma = \frac{1.9}{1.91} = 9.94$

$\therefore \sigma = 10$

$\sigma^2 = 100$

From eq^n(1)

$64 - \mu = 1.41\sigma$

$\mu = 64 - 1.41(10)$

$\mu = 64 - 14.1$

$= 49.9 \simeq 50$

$= \boxed{\mu = 50}$

Mean

The Mean height of student in a clg is 155cm & std dv. is 15 what is the prob. that the mean height of 36 students is less than 157 cm

Given $\mu = 155$

$\sigma = 15$

sample mean $\bar{x} = 157$

sample size $n = 36$

By control limit theorem

we have

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{157 - 150}{15/\sqrt{36}}$$

$$= 7/15/6 = \frac{\cancel{42}^{14}}{\cancel{15}_{5}} = 0.8 = \cdot$$

$P(\bar{x} < 157) = P(Z < 0.8)$

$= |0.5 + A(z_1)|$

$= |0.5 + A(0.8)|$

$= |0.5 + 0.2881| = 0.7881$

$n = 100$ $\quad\quad$ $\sigma^2 = 256$

$\mu = 100$ $\quad\quad$ $\sigma = 16$

$P(76 \leq x \leq 78) = P(x_1 \leq x \leq x_2)$

$= P(z_1 \leq z \leq z_2)$

$x_2 = 78$

Here $x_1 = 75$

$z_1 = \dfrac{x_1 - \mu}{\sigma/\sqrt{n}}$

$z_1 = \dfrac{75 - 76}{16/\sqrt{100}}$

$z_1 = -0.06$

$z_1 = -0.02 < 0$

$\sigma^2 = 256$

$\sigma = 16$

$= p(x_1 \leq x \leq x_2)$

$= p(z_1 \leq z \leq z_2)$

$x_2 = 78$

$z_2 = \dfrac{x_2 - \mu}{\sigma/\sqrt{n}}$

$z_2 = \dfrac{78 - 76}{16\sqrt{100}}$

$z_2 = 1.25 > 0$

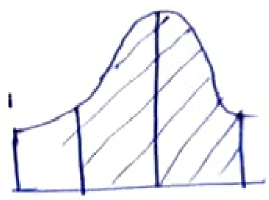
$p(z_1 \leq z \leq z_2) = p(-0.02 \leq z \leq 0.12)$

$= |A(z_1) + A(z_2)$

$= |A(0.62) + A(1.25)|$

$= |0.2324 + 0.39441$

$= 0.628$



$z_1 = -0.62 \quad z_2 = 1.25$

A population consists of 5 numbers 2, 3, 6, 8, 11, consider all possiable samples of size 2 which can be drawn

i) With replacement

ii) without replacement from this population, find

a) Mean of the population

b) standard deviation of population

c) the mean of the sampling distribution of means

d) The stardard deviation of sampling distribution of means

Given population = 2, 3, 6, 8, 11

$N = 5$

$n = 2$

a) Mean of the population $\mu = \frac{\Sigma x_i}{N}$

$$\mu = \frac{2+3+6+8+11}{5} = 6$$

b) Standard devition of population $\sigma^2 = \frac{\Sigma (x_i = \mu)^2}{N}$

$$= \frac{(2-6)^2 + (3-6)^2 + (0-6)^2 + (8-6)^2 + (11-6)^2}{5}$$

$$\sigma^2 = \frac{4^2 + 3^2 + 0 + 2^2 + 5^2}{5}$$

$$\sigma^2 = \frac{16+9+4+25}{5}$$

$$\sigma^2 = \frac{54}{5} = 10.8$$

variance, $\sigma^2 = 10.8$

Standard Deviation of pop $\sigma = \sqrt{10.8}$

$= 3.28$

i) with replacement $= N^n$

$= 5^2 = 25$

$$S = \begin{Bmatrix} (2,2) & (2,3) & (2,6) & (2,8) & (2,11) \\ (3,2) & (3,3) & (3,6) & (3,8) & (3,11) \\ (6,2) & (6,3) & (6,6) & (6,8) & (6,11) \\ (8,2) & (8,3) & (8,6) & (8,8) & (8,11) \\ (11,2) & (11,3) & (11,6) & (11,8) & (11,11) \end{Bmatrix}$$

sampling dist. of Mean $=$

$$\begin{Bmatrix} 2 & 2.5 & 4 & 5 & 6.5 \\ 2.5 & 3 & 4.5 & 5.5 & 7 \\ 4 & 4.5 & 6 & 7 & 8.5 \\ 5 & 5.5 & 7 & 8 & 9.5 \\ 6.5 & 7 & 8.5 & 9.5 & 11 \end{Bmatrix}$$

c) Mean of the sampling distribution of mean:

$$\mu_{\bar{x}} = \frac{\Sigma x_i}{N}$$

$$= \frac{2 + 2.5 + 4 + 6.5 + \cdots\cdots + 4}{25}$$

$$= \frac{150}{25}$$

$$\boxed{= \mu_{\bar{x}} = 6}$$

d) Standard deviation of sampling distribution of mean :-

$$\sigma^2 = \frac{\sum (x_i - \mu_{\bar{x}})^2}{}$$

$$= \frac{(2.6)^2 + (2.5-6)^2 + (4-6)^2 + \cdots\cdots (11-6)^2}{25}$$

$$= \frac{135.5}{25}$$

variances ; $\sigma^2 = 5.42$

Standard Deviation of sampling distribution of means $\sigma_{\bar{x}} = \sqrt{5.42}$

$\sigma_{\bar{x}} = 2.32$

ii) without replacement

Total no. of sampling distribution without replacement

$= N_{C_n} = 5C_2 = 10$

$$\left\{ \begin{array}{llll} (2,3 & (2,6) & (2,8) & (2,11) \\ (3,6) & (3,8) & (3,11) & \\ (6,8) & (6,11) & & \\ (8,11 & & & \end{array} \right\}$$

Sampling dist. of means = $\begin{Bmatrix} 2.5 & 4.5 & 5 & 4.5 \\ 4.5 & 5.5 & 7 & \\ 7 & 8.5 & & \\ 9.5 & & & \end{Bmatrix}$

c) Mean of the sampling dist. of means

$$u_x = \frac{\Sigma x_i}{N}$$

$$= \frac{2.5 + 4 + 5 + 6.5 + 4.5 + 5.5 + 7 + 7 + 8.5 + 9.5}{10}$$

$$= \frac{60}{10}$$

$$\boxed{u_x = 6}$$

Standard dev of sampling dist of means

$$\sigma x^2 = \frac{\Sigma (x_i - u)^2}{N}$$

$$\sigma^2 = (2.5-6)^2 + (4-6)^2 + (5-6)^2 + \cdots + (9.5-6)^2$$

variance σ^2

$x = 4.25$

S.D of sampling dist. of mean $\sigma x = \sqrt{4.05}$

$\sigma x = 2.01$