UNIT - IV

Estimation:

Parameters:-

Quantities appearing in distributions such as p in the Binomial distribution and μ, σ in the Normal distribution are called parameters.

Estimate:-

An estimate is a statement made to find an unknown population parameter.

Estimator:-

A procedure or rule to determine an unknown population parameter is called an estimator. For example, sample mean is an estimator of population mean because sample mean is a method of determining the population mean.

Note:-

1) The estimator must depend only on the sample and not on the parameter to be estimated.
2) The estimator must be a statistic.
3) The parameter can have one or two or many estimators.

Types of Estimation:-

1) Point estimation.
2) Interval estimation.

Point estimation:-

When estimate of the population parameter is given by a single value is called a point estimation. For example, if the height of a student is measured as 162 cms, then the measurement gives a point estimation.

Interval estimation:-

If an estimate of the population is given by two different values between which the parameter may be considered to lie, then the estimate is called an interval estimation of the parameter. For example, if the height of the student is measured as (163 ± 3.5) cms, then the height lies between 159.5 cms and 166.5 cms and the measurement gives an interval estimation.

Note:-

A point estimator is a statistic for estimating the population parameter θ and it is denoted by $\hat{\theta}$.

Properties of Estimation :-

1) An estimator is not expected to estimate the population parameter without error.
2) The estimator should be closed to the true value of the unknown parameter.

Unbiased and Biased estimates :-

A statistic is said to be an unbiased estimator of the corresponding parameter, if the mean of the sampling distribution of the statistic is equal to the mean of the corresponding population parameter.
otherwise, the statistic is called a biased estimator of the corresponding parameter. The values of the statistic in the above two cases are called unbiased and biased estimates respectively.

Problems based on unbiased and biased estimates :-

1) Prove that sample mean $\bar{x}$ is an unbiased estimator of the population mean μ.

Proof:-

Let $x_1, x_2, \ldots, x_n$ be a random sample drawn from a given population with mean μ and variance σ^2.

W.K.T. Sample mean, $\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$

$$\Rightarrow E(\bar{x}) = \frac{1}{n} E\left(\sum_{i=1}^{n} x_i\right)$$

$$= \frac{1}{n} E(x_1 + x_2 + \ldots + x_n)$$

$$= \frac{1}{n}\left[E(x_1) + E(x_2) + \ldots + E(x_n)\right]$$

$$= \frac{1}{n}\left[\mu + \mu + \ldots + \mu\right]$$

$$= \frac{1}{\cancel{n}}\left[\cancel{n}\mu\right]$$

$$\Rightarrow \boxed{E(\bar{x}) = \mu}$$

Hence the sample mean $\bar{x}$ is an unbiased estimator of the population mean μ. Hence proved. (Ans).

2) Show that the sample variance S^2 is an unbiased estimator of the population variance σ^2.

Solution :-

To prove : $E(S^2) = \sigma^2$.

W.K.T. Sample variance $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$.

Consider $\sum_{i=1}^{n}(x_i-\bar{x})^2$

$$= \sum_{i=1}^{n}\left[(x_i-\mu)-(\bar{x}-\mu)\right]^2$$

$$= \sum_{i=1}^{n}(x_i-\mu)^2 - 2(\bar{x}-\mu)\sum_{i=1}^{n}(x_i-\mu) + n(\bar{x}-\mu)^2$$

$$= \sum_{i=1}^{n}(x_i-\mu)^2 - 2(\bar{x}-\mu)\left[(x_1-\mu)+(x_2-\mu)+\dots+(x_n-\mu)\right] + n(\bar{x}-\mu)^2$$

$$= \sum_{i=1}^{n}(x_i-\mu)^2 - 2(\bar{x}-\mu)\left[\frac{n(x_1+x_2+\dots+x_n)}{n} - n\mu\right] + n(\bar{x}-\mu)^2$$

$$= \sum_{i=1}^{n}(x_i-\mu)^2 - 2(\bar{x}-\mu)n(\bar{x}-\mu) + n(\bar{x}-\mu)^2$$

$$= \sum_{i=1}^{n}(x_i-\mu)^2 - 2n(\bar{x}-\mu)^2 + n(\bar{x}-\mu)^2$$

$$\Rightarrow \boxed{\sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n}(x_i-\mu)^2 - n(\bar{x}-\mu)^2} \quad -(1)$$

Now, $E(s^2) = E\left[\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}\right]$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n}E(x_i-\bar{x})^2\right]$$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n}E(x_i-\mu)^2 - n(\bar{x}-\mu)^2\right]$$

$$= \frac{1}{n-1}\left[\sum_{i=1}^{n}\sigma_{x_i}^2 - n\sigma_{\bar{x}}^2\right] \quad -(2)$$

However, $\boxed{\sigma_{x_i}^2 = \sigma^2 \text{ for } i=1,2,\dots,n \text{ and } \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}} \quad -(3)$

Sub. (3) in (2), we get,

$$E(s^2) = \frac{1}{n-1}\left[n\sigma^2 - \cancel{n}\frac{\sigma^2}{\cancel{n}}\right]$$

$$= \frac{1}{\cancel{(n-1)}}\cancel{(n-1)}\sigma^2$$

$$\Rightarrow \boxed{E(S^2) = \sigma^2}$$

Hence the sample variance s^2 is an unbiased estimator of the population variance σ^2. (Ans)

Note:

$$\boxed{S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 \neq \sigma^2}$$

3) Suppose that we observe a random variable having the Binomial distribution. Let X be the number of successes in trials.

a) Show that $\frac{X}{n}$ is an unbiased estimate of the Binomial parameter 'p'.

b) Show that $\frac{X+1}{n+2}$ is not an unbiased estimate of the Binomial parameter 'p'.

Solutions:-

Formula: $\hat{\theta}$ is an unbiased estimator of θ if $E(\hat{\theta}) = \theta$

a) **To prove:** $E\left(\frac{X}{n}\right) = p$

$$LHS = E\left(\frac{X}{n}\right)$$

$$= \frac{1}{n} E(X)$$

$$= \frac{1}{\cancel{n}} (\cancel{n} p) \quad \left[\because \text{For Binomial distribution, Mean} = E(X) = np\right] \quad — (1)$$

$$= p = RHS$$

$\Rightarrow E\left(\frac{X}{n}\right) = p$

$\Rightarrow \frac{X}{n}$ is an unbiased estimate of the Binomial parameter 'p'.
Hence proved. (Ans).

b) **To prove:** $E\left(\frac{X+1}{n+2}\right) \neq p$

$$LHS = E\left(\frac{X+1}{n+2}\right)$$

$$= \frac{1}{n+2} E(X+1)$$

$$= \frac{E(X)}{n+2} + \frac{E(1)}{n+2}$$

$$= \frac{np}{n+2} + \frac{1}{n+2} \quad \left[\because \text{From (1)}, E(1) = 1\right]$$

$$= \frac{np+1}{n+2} \neq p \neq RHS$$

$\Rightarrow E\left(\frac{X+1}{n+2}\right) \neq p$

$\Rightarrow \frac{X+1}{n+2}$ is not an unbiased estimate of the Binomial parameter 'p'.
Hence proved. (Ans).

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4) To illustrate that the mean of the random sample is an unbiased estimate of the mean of the population.
Consider 5 slips of paper numbered 3, 6, 9, 15, 27.

a) List all possible samples of size 3 that can be taken without replacement from this finite population.

b) Calculate the mean of each of the samples listed in a) and assigning each sample a probability of each sample. Verify that the mean of these $\bar{x}$ equals 12, namely the mean of the population.

Solution:-

The total number of samples without replacement is $^N C_n = {}^5C_3$ (Given)

$$= \frac{5 \cdot \cancel{4}^{2} \cdot \cancel{3}}{1 \cdot \cancel{2} \cdot \cancel{3}} = 10 \text{ samples of size } 3,$$

where N = population size,
n = sample size.

Listing all possible samples of size 3 from finite population 3, 6, 9, 15, 27 without replacement, we get 10 samples. They are,

$$\left\{ \begin{matrix} (3,6,9), (3,6,15), (3,6,27), (3,9,15), (3,9,27), (3,15,27) \\ (6,9,15), (6,9,27), (6,15,27), (9,15,27) \end{matrix} \right\}$$

b) Population Mean, $\mu = \frac{3+6+9+15+27}{5}$

$= \frac{\cancel{60}\,12}{\cancel{5}}$

$\Rightarrow \boxed{\mu = 12}$ — ①

The mean of each of the samples, $\bar{x}$ are 6, 8, 12, 9, 13, 15, 10, 14, 16, 17.

$\bar{x}$	6	8	12	9	13	15	10	14	16	17
$p(\bar{x})$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

W.K.T. $E(\bar{x}) = \sum_{i=1}^{n} \bar{x}_i \, p_i$

$= 6\left(\frac{1}{10}\right) + 8\left(\frac{1}{10}\right) + 12\left(\frac{1}{10}\right) + 9\left(\frac{1}{10}\right) + 13\left(\frac{1}{10}\right) + 15\left(\frac{1}{10}\right) + 10\left(\frac{1}{10}\right) + 14\left(\frac{1}{10}\right) + 16\left(\frac{1}{10}\right) + 17\left(\frac{1}{10}\right)$

$= \frac{1}{10}\left[6+8+12+9+13+15+10+14+16+17\right]$

$= \frac{12\cancel{0}}{1\cancel{0}}$

$\Rightarrow \boxed{E(\bar{x}) = 12}$ — ②

$\therefore$ From ①, ② $\boxed{E(\bar{x}) = \mu = 12}$

Hence the mean of a random sample $\bar{x}$, is an unbiased estimate of the population mean μ.

Hence proved. (Ans).

Confidence Interval Estimates of Parameters:

In an interval estimation of the population parameter 'θ', if we can find two quantities t_1 and t_2 based on sample observations drawn from the population such that the unknown parameter 'θ' is included in the interval $[t_1, t_2]$ in a specified percentage of cases, then this interval is called a confidence interval for the parameter 'θ'. A confidence interval has a specified confidence or probability of correctly estimating the true value of the population parameter. In this case $P(t_1 \le \theta \le t_2) = c$ where c is a given specified probability. We know that, by central limit theorem the sampling distribution of the sample mean $\bar{x}$ of a random sample of size $n \ge 30$ drawn from a population having mean 'μ' and standard deviation 'σ' is approximately a normal distribution with mean 'μ' and

S.D. = S.E. of $\bar{x} = \frac{\sigma}{\sqrt{n}}$.

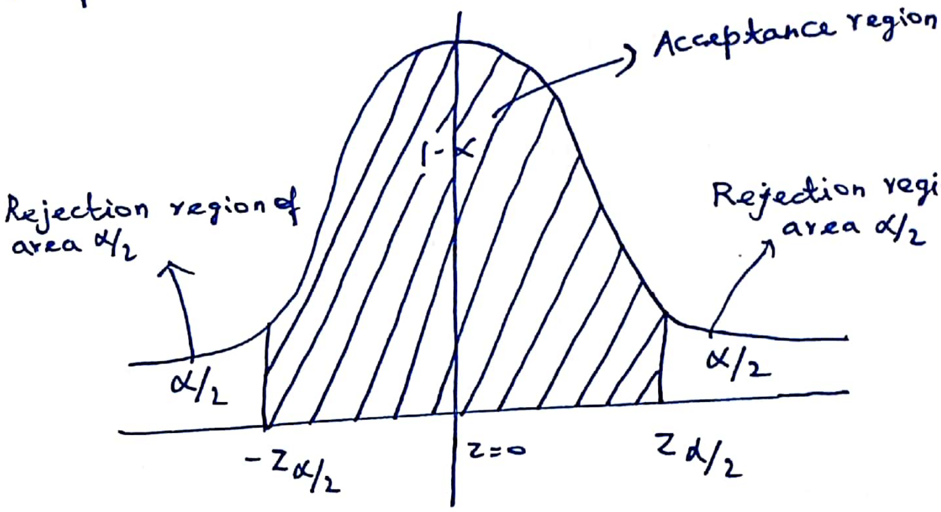
$\therefore \; Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ has a standard normal distribution with mean = 0 and S.D. = 1.

Interval Estimation:

We know that, 95.44% of area lies between the ordinates $z = \pm 2$.

$\Rightarrow P(-2 < z < 2) = 0.9544$

But $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$



$\Rightarrow -2 < \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} < 2$

$\Rightarrow \dfrac{-2\sigma}{\sqrt{n}} < \bar{x} - \mu < \dfrac{2\sigma}{\sqrt{n}}$

$\Rightarrow \dfrac{2\sigma}{\sqrt{n}} > \mu - \bar{x} > -\dfrac{2\sigma}{\sqrt{n}}$

$\Rightarrow \dfrac{-2\sigma}{\sqrt{n}} < \mu - \bar{x} < \dfrac{2\sigma}{\sqrt{n}}$

$\Rightarrow \bar{x} - \dfrac{2\sigma}{\sqrt{n}} < \mu - \bar{x} + \bar{x} < \bar{x} + \dfrac{2\sigma}{\sqrt{n}}$

$\Rightarrow \bar{x} - 2\,\text{S.E. of } \bar{x} < \mu < \bar{x} + 2\,\text{S.E. of } \bar{x}$

$\Rightarrow \bar{x} - Z_{\alpha/2}\,\text{S.E. of } \bar{x} < \mu < \bar{x} + Z_{\alpha/2}\,\text{S.E. of } \bar{x}$

Note:

1) The value of $Z\alpha/2$ for 95% confidence limits is ± 1.96
2) The value of $Z\alpha/2$ for 99% confidence limits is ± 2.58
3) The value of $Z\alpha/2$ for 99.73% confidence limits is ± 3
4) The value of $Z\alpha/2$ for 90% confidence limits is ± 1.64
5) The value of $Z\alpha/2$ for 98% confidence limits is ± 2.33

Confidence limits for population mean 'μ':

1) 95% confidence limits are $\bar{x} \pm 1.96$ S.E. of $\bar{x}$
2) 99% confidence limits are $\bar{x} \pm 2.58$ S.E. of $\bar{x}$
3) 99.73% confidence limits are $\bar{x} \pm 3$ S.E. of $\bar{x}$
4) 90% confidence limits are $\bar{x} \pm 1.64$ S.E. of $\bar{x}$

Confidence limits for population proportion 'p':

1) 95% confidence limits are $P \pm 1.96$ S.E. of P
2) 99% confidence limits are $P \pm 2.58$ S.E. of P
3) 99.73% confidence limits are $P \pm 3$ S.E. of P
4) 90% confidence limits are $P \pm 1.64$ S.E. of P

Confidence limits for the difference $\mu_1 - \mu_2$ of two population means μ_1 and μ_2:

1) 95% confidence limits are $\bar{x}_1 - \bar{x}_2 \pm 1.96$ S.E. of $\bar{x}_1 - \bar{x}_2$
2) 99% confidence limits are $\bar{x}_1 - \bar{x}_2 \pm 2.58$ S.E. of $\bar{x}_1 - \bar{x}_2$
3) 99.73% confidence limits are $\bar{x}_1 - \bar{x}_2 \pm 3$ S.E. of $\bar{x}_1 - \bar{x}_2$
4) 90% confidence limits are $\bar{x}_1 - \bar{x}_2 \pm 1.64$ S.E. of $\bar{x}_1 - \bar{x}_2$

Confidence limits for the difference $P_1 - P_2$ of two population proportions P_1 and P_2:

1) 95% confidence limits are $P_1 - P_2 \pm 1.96$ S.E. of $P_1 - P_2$
2) 99% confidence limits are $P_1 - P_2 \pm 2.58$ S.E. of $P_1 - P_2$
3) 99.73% confidence limits are $P_1 - P_2 \pm 3$ S.E. of $P_1 - P_2$
4) 90% confidence limits are $P_1 - P_2 \pm 1.64$ S.E. of $P_1 - P_2$

Maximum error estimate (E):

Maximum error estimate, $E = Z_{\alpha/2} \cdot$ S.E of $\bar{x}$

$$\Rightarrow \boxed{E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}}$$

Sample size (n):

Sample size, $n = \left(Z_{\alpha/2} \cdot \frac{\sigma}{E}\right)^2$

Note:-

If the S.D. is not given in the problem, then we have to find by using the formula $\boxed{s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$.

Problems based on Maximum error estimate, Confidence interval and Sample size:-

1) Find 95% confidence limits for the mean of the normally distributed population of which the following sample was taken from 15, 17, 10, 18, 16, 9, 7, 11, 13, 14.

Solution:- Given: Sample size, $\boxed{n = 10}$ — ①

Sample mean, $\bar{x} = \frac{15+17+10+18+16+9+7+11+13+14}{10}$

$$\Rightarrow \boxed{\bar{x} = 13} \text{ — ②}$$

Sample variance, $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

$$= \frac{1}{10-1}\left[(15-13)^2 + (17-13)^2 + (10-13)^2 + (18-13)^2 + (16-13)^2 + (9-13)^2 + (7-13)^2 + (11-13)^2 + (13-13)^2 + (14-13)^2\right]$$

$$\Rightarrow \boxed{s^2 = \frac{40}{3}}$$

Sample standard deviation, $\boxed{s = \sqrt{\frac{40}{3}}}$ — ③

W.K.T. Maximum error estimate, E

$$= Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$= 1.96 \times \frac{\sqrt{40}}{\sqrt{3}\sqrt{10}} \quad [\because \text{from ①, ③}]$$

$$\Rightarrow \boxed{E = 2.26} \text{ — ④}$$

95% confidence limits are given by, $\bar{x} \pm E$

$= (13 \pm 2.26)$ [$\because$ From ②, ④]

$\Rightarrow$ $\boxed{95\% \text{ confidence limits } = (10.74, 15.26)}$ (Ans).

2) A population random variable has mean 100 and S.D. = 16. What are the mean and S.D. of the sample mean for a the random samples of size 4 drawn with replacement.

Solution:-

Given: Population mean, $\boxed{\mu = 100}$ — ①

Population S.D. $\boxed{\sigma = 16}$ — ②

Sample size, $\boxed{n = 4}$ — ③

Since the sampling is done with replacement, the population may be considered as an infinite population.

$$\boxed{\begin{array}{l} \text{Mean of the sample mean}, \mu_{\bar{x}} = \mu = 100 \\ \text{S.D. of the sample mean}, \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{4}} = 8 \end{array}}$$ [$\because$ From ①, ②, ③] (Ans).

3) A random sample of 400 items is found to have mean 82 and S.D. 18. Find the maximum error of estimation, and the confidence limits if the mean is $\bar{x} = 82$ at 95% confidence.

Solution:-

Given: Sample size, $\boxed{n = 400}$ — ①

Sample mean, $\boxed{\bar{x} = 82}$ — ②

Sample S.D., $\boxed{S = 18}$ — ③

W.K.T. Maximum error estimate, $E = Z_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$

$= 1.96 \times \frac{18}{\sqrt{400}}$ [$\because$ From ①, ③]

$\Rightarrow \boxed{E = 1.764}$ — ④

95% confidence limits are given by, $\bar{x} \pm E$

$= (82 \pm 1.764)$ [$\because$ From ②, ④]

$\boxed{95\% \text{ confidence limits } = (80.236, 83.764)}$ (Ans).

4) An industrial engineer intensitive use the mean of a random sample of size $n = 150$ and estimates the average mechanical attitude of assembly line works of large industry. If on the basis of experience the engineer can assume that $\sigma = 6.2$ for such data, what can he assert with probability 0.99 above the maximum size of his error.

Solution:-

Given : sample size , $\boxed{n = 150}$ — ①

Population S.D., $\boxed{\sigma = 6.2}$ — ②

W.K.T. Maximum error estimate, $E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$= 2.58 \times \frac{6.2}{\sqrt{150}}$ [∵ From ①,②]

$\Rightarrow$ $\boxed{\text{Maximum error estimate, } E = 1.30}$ (Ans).

5) In 6 determinations of the melting point of tin, a chemist obtained mean of 232.26° C with S.D. of 0.14° C. If he uses this mean to estimate the actual melting point of tin, what can the chemist assert with 98% confidence about the maximum error.

Solution:-

Given: Sample size, $\boxed{n = 6}$ — ①

Sample mean, $\boxed{\bar{x} = 232.26}$ — ②

Sample S.D., $\boxed{S = 0.14}$ — ③

W.K.T. Maximum error estimate, $E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$

$= 3.365 \times \frac{0.14}{\sqrt{6}}$ [∵ From ①,③]

$\Rightarrow$ $\boxed{\text{Maximum error estimate, } E = 0.19}$ (Ans).

6) A random sample of size n=100 is taken from a population with σ = 5.1. Given that sample mean $\bar{x} = 21.6$. Construct a 95% confidence interval for the population mean μ.

Solution:-

Given: Sample size, $\boxed{n = 100}$ — ①

Sample mean, $\boxed{\bar{x} = 21.6}$ — ②

Population S.D. $\boxed{\sigma = 5.1}$ — ③

W.K.T. Maximum error estimate, $E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$= 1.96 \times \frac{5.1}{\sqrt{100}}$ [∵ From ①,③]

$\Rightarrow$ $\boxed{E = 0.9996}$ — ④

95% confidence limits are given by, $\bar{x} \pm E$

$= (21.6, \pm 0.9996)$ [∵ From ②,④]

$\Rightarrow$ $\boxed{\text{95\% confidence limits} = (20.6004, 22.5996)}$ (Ans).

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7) With reference the sulphur oxide, $n = 80$, $\bar{x} = 18.85$, $s^2 = 30.77$, $s = 5.55$, construct a 99% confidence interval for the plans of true average daily emission of sulphur oxide.

Solution:-

Given: Sample size, $\boxed{n = 80}$ — ①

Sample mean, $\boxed{\bar{x} = 18.85}$ — ②

Sample S.D., $\boxed{s = 5.55}$ — ③

W.K.T. Maximum error estimate, $E = Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

$= 2.58 \times \frac{5.55}{\sqrt{80}}$ [∵ From ①, ③]

$\Rightarrow \boxed{E = 1.60}$ — ④

99% confidence limits are given by, $\bar{x} \pm E$

$= (18.85 \pm 1.60)$ [∵ From ②, ④]

$\Rightarrow$ $\boxed{\text{99\% confidence limits} = (17.25, 20.45)}$ (Ans).

8) The mean weight loss of $n = 16$ grinding balls after a certain length of time in mill surry is 3.42 grams with S.D. of 0.68 gram. Construct 99% confidence interval for the mean weight loss of such grinding balls under the stated condition.

Solution:-

Given: Sample size, $\boxed{n = 16}$ — ①

Sample mean, $\boxed{\bar{x} = 3.42}$ — ②

Sample S.D., $\boxed{s = 0.68}$ — ③

W.K.T. Maximum error estimate, $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

$= 2.947 \times \frac{0.68}{\sqrt{16}}$ [∵ From ①, ③]

$\Rightarrow \boxed{E = 0.4998}$ — ④

99% confidence limits are given by, $\bar{x} \pm E$

$= (3.42 \pm 0.4998)$ [∵ From ②, ④]

$\Rightarrow$ $\boxed{\text{99\% confidence limits} = (2.9202, 3.9198)}$ (Ans).

9) To estimate the mean time of continuous use until an answering machine will first require service. If it can be assumed that $\sigma = 60$ days, how large a sample is needed so that one will be able to assert with 90% confidence that the sample mean is off by atmost 10 days.

Solution :-

Given : Maximum error, $\boxed{E = 10}$ —①

Population S.D. $\boxed{\sigma = 60}$ —②

W.K.T. Sample size, $n = \left(Z_{\alpha/2} \cdot \frac{\sigma}{E}\right)^2$

$= \left(1.645 \times \frac{6\cancel{0}}{1\cancel{0}}\right)^2$ [∵ From ①, ②]

$\Rightarrow$ $\boxed{\text{Sample size, } n = 97}$ (Ans).

10) The dean of a college wants to use the mean of a random sample to estimate the average amount of time students take to get from one class to next and she wants to be able to assert with 99% confidence that the error is atmost 0.25 minute. If it can be presumed from experience that $\sigma = 1.40$ mimutes, how large a sample will she have to take?

Solution :-

Given: Maximum error, $\boxed{E = 0.25}$ —①

Population S.D., $\boxed{\sigma = 1.40}$ —②

W.K.T. sample size, $n = \left(Z_{\alpha/2} \cdot \frac{\sigma}{E}\right)^2$

$= \left(2.58 \times \frac{1.40}{0.25}\right)^2$ [∵ From ① ②]

$\Rightarrow$ $\boxed{\text{Sample size, } n = 208}$ (Ans).

11) The mean and S.D. of population are 11795 and 14054 respectively. What can we assert with 95% confidence about the maximum error. If $\bar{x} = 11795$, $n = 50$ also construct 95% confidence interval.

Solution :-

Given: Population mean, $\boxed{\mu = 11795}$ —①

Population S.D., $\boxed{\sigma = 14054}$ —②

Sample mean, $\boxed{\bar{x} = 11795}$ —③

Sample size, $\boxed{n = 50}$ —④

W.K.T. Maximum error estimate, E

$= Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$= 1.96 \times \frac{14054}{\sqrt{50}}$

[∵ From ②④]

$\Rightarrow$ $\boxed{E = 3895.57}$ —⑤

95% confidence limits are given by, $\bar{x} \pm E$

$= (11795 \pm 3895.57)$

$\Rightarrow$ 95% confidence limits $= \boxed{(7899.43,\ 15690.57)}$ (Ans)

12) Assuming that $\sigma = 20$, how large a random sample be taken to assert with probability 0.95 so that the sample mean will not differ from the true mean by more than 3 points.

Solution:-

Given: Population S.D., $\boxed{\sigma = 20}$ — ①

Maximum error, $\boxed{E = 3}$ — ②

W.K.T. sample size, $n = \left(Z_{\alpha/2} \cdot \frac{\sigma}{E}\right)^2$

$= \left(1.96 \times \frac{20}{3}\right)^2$ [∵ From ①, ②]

$\Rightarrow$ $\boxed{\text{Sample size, } n = 171}$ (Ans).

13) A random sample of 100 teachers in a large metropolitan area revealed a mean weekly salary of Rs. 487 with S.D. Rs. 48. With what degree of confidence can he assert that area between 472 to 502 increases.

Solution :-

Given: Sample size, $\boxed{n = 100}$ — ①

Population mean, $\boxed{\mu = 487}$ — ②

Population S.D., $\boxed{\sigma = 48}$ — ③

Sample means, $\boxed{\begin{array}{l}\bar{x}_1 = 472 \\ \bar{x}_2 = 502\end{array}}$ — ④

W.K.T. $\boxed{Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}}$ — ⑤

Sub. ①,②,③, ④ in ⑤, we get,

$Z_1 = \frac{\bar{x}_1 - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{472 - 487}{\frac{48}{\sqrt{100}}}$

$\Rightarrow \boxed{Z_1 = -3.125}$

$Z_2 = \frac{\bar{x}_2 - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{502 - 487}{\frac{48}{\sqrt{100}}}$

$\Rightarrow \boxed{Z_2 = 3.125}$

Let $\bar{x}$ be the mean salary of the teacher. Then

$P(472 < \bar{x} < 502) = P(-3.125 < z < 3.125)$

$= 2P(0 < z < 3.125)$

$= 2 \int_0^{3.125} \phi(z)\, dz$

$= 2(0.4991)$

$\Rightarrow \boxed{P(472 < \bar{x} < 502) = 0.9982}$

Thus we can assert with $\boxed{99.82\%}$ confidence (Ans).

14) In a study of an automobile insurance a random sample of 80 body repair costs had a mean of Rs. 472.36 and a standard deviation of Rs 62.35. If $\bar{x} = 472.36$ is used as a point estimate to the true average repair costs, with what confidence we can assert that the maximum error does not exceed Rs. 10?

Solution:-

Given: Sample size, $\boxed{n = 80}$ —①

Sample mean, $\boxed{\bar{x} = 472.36}$ —②

Sample S.D., $\boxed{S = 62.35}$ —③

Maximum error, $\boxed{E = 10}$ —④

W.K.T. Maximum error estimate, $E = Z_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$

$$\Rightarrow 10 = Z_{\alpha/2} \cdot \frac{62.35}{\sqrt{80}} \quad [\because \text{From } ①, ③, ④]$$

$$\Rightarrow Z_{\alpha/2} = \frac{10\sqrt{80}}{62.35}$$

$$\Rightarrow \boxed{Z_{\alpha/2} = 1.43}$$

From Normal tables, $\alpha/2 = 0.4236$

$$\Rightarrow \alpha = 2(0.4236)$$

$$\Rightarrow \boxed{\alpha = 0.8472}$$

Thus, we have $\boxed{84.72\%}$ confidence so that the maximum error does not exceed Rs. 10. (Ans).

15) A researcher wants to determine the average time it takes for a mechanic to rotate the tyres of a car an he wants to be able to assert with 95% confidence that the mean of the sample is off by atmost 0.505 min. If he can presume from past experience that $\sigma = 1.6$ minutes, how large a sample will he have to take?

Solution:-

Given: Maximum error, $\boxed{E = 0.505}$ —①

Population S.D., $\boxed{\sigma = 1.6}$ —②

W.K.T. Sample size, $n = \left(Z_{\alpha/2} \cdot \frac{\sigma}{E}\right)^2$

$$= \left(1.96 \times \frac{1.6}{0.505}\right)^2 \quad [\because \text{From } ①, ②]$$

$$\Rightarrow \boxed{\text{Sample size, } n = 39} \text{ (Ans).}$$

16) The efficiency expert of a computer company tested 40 engineers to estimate the average time it takes to assemble a certain computer component getting a mean of 12.73 minutes and S.D. of 2.06 minutes.

(i) What can we say with 99% confidence about the maximum error if $\bar{x} = 12.73$ is used as a point estimate of the actual average time required to do the job.

(ii) Use the given data to construct 98% confidence interval.

(iii) With what confidence we can accept that the sample mean does not differ from the true mean by more than 30 seconds.

Solution :-

Given: Sample mean, $\boxed{\bar{x} = 12.73}$ — ①

Sample size, $\boxed{n = 40}$ — ②

Sample S.D., $\boxed{S = 2.06}$ — ③

(i) W.K.T. Maximum error estimate, $E = Z_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$

$$= 2.58 \times \frac{2.06}{\sqrt{40}} \quad \left(\because Z_{\alpha/2} = 2.58 \text{ for } 99\%\right) \quad [\because \text{From ②, ③}]$$

$\Rightarrow$ $\boxed{\text{Maximum error estimate, } E = 0.8387}$ (Ans).

(ii) W.K.T. Maximum error estimate, $E = Z_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$

$$= 2.33 \times \frac{2.06}{\sqrt{40}} \quad \left(\begin{array}{l} \because Z_{\alpha/2} = 2.33 \text{ for } 98\% \\ \because \text{From ②, ③} \end{array}\right)$$

$\Rightarrow$ $\boxed{\text{Maximum error estimate, } E = 0.758915}$ — ④

98% confidence limits are given by, $\bar{x} \pm E$

$$= (12.73 \pm 0.758915) \quad [\because \text{From ②, ④}]$$

$\Rightarrow$ $\boxed{98\% \text{ Confidence limits} = (11.97, 13.4889)}$ (Ans).

(iii) Maximum error estimate, $E = \frac{\cancel{30}\ 1}{\cancel{60}\ 2}$

$\Rightarrow$ $\boxed{E = \frac{1}{2}}$ — ⑤

W.K.T. Maximum error estimate, $E = Z_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$

$$\Rightarrow \frac{1}{2} = Z_{\alpha/2} \cdot \frac{2.06}{\sqrt{40}} \quad [\because \text{From ②, ③, ⑤}]$$

$$\Rightarrow Z_{\alpha/2} = \frac{\sqrt{40}}{2 \times 2.06}$$

$$\Rightarrow \boxed{Z_{\alpha/2} = 1.5350}$$

From Normal tables, $\alpha/2 = 0.4370$

$$\Rightarrow \alpha = 2(0.4370)$$

$$\Rightarrow \boxed{\alpha = 0.8740}$$

Thus, we have $\boxed{87.4\%}$ confidence. (Ans).

17) What is the maximum error one can expect to make with probability 0.90 when using the mean of a random sample of size $n=64$ to estimate the population mean with $\sigma^2 = 2.56$

Solution:-

Given: Sample size, $\boxed{n=64}$ — ①

Probability $= 0.90$

$\Rightarrow$ Level of significance, $\alpha = 0.1$

W.K.T. $Z_{\alpha/2}$ for 90% $= 1.645$

Population variance, $\sigma^2 = 2.56$

Population S.D., $\sigma = \sqrt{2.56}$

$$\Rightarrow \boxed{\sigma = 1.6} \text{ — ②}$$

W.K.T. Maximum error estimate, $E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$$= 1.645 \times \frac{1.6}{\sqrt{64}} \quad [\because \text{From ①, ②}]$$

$\Rightarrow$ $\boxed{\text{Maximum error estimate}, E = 0.329}$ (Ans).

18) It is desired to estimate the mean number of hours of continuous use until a certain computer will first require repairs. If it can be assumed that $\sigma = 48$ hours, how large a sample be needed so that one will be able to assert with 90% confidence that the sample mean is off by atmost 10 hours.

Solution:-

Given: Maximum error, $\boxed{E = 10}$ — ①

Population S.D., $\boxed{\sigma = 48}$ — ②

W.K.T. Sample size, $n = \left(Z_{\alpha/2} \cdot \frac{\sigma}{E}\right)^2$

$$= \left(1.645 \times \frac{48}{10}\right)^2 \quad [\because \text{From ①, ②}]$$

$\Rightarrow$ $\boxed{\text{Sample size}, n = 62}$ (Ans).

19) If we can assert with 95% that the maximum error is 0.05 and P = 0.2, Find the size of the sample?

Solution:-

Given: $\boxed{P = 0.2}$ — ①, $\Rightarrow Q = 1 - P = 1 - 0.2$

$\Rightarrow \boxed{Q = 0.8}$ — ②

$\boxed{\text{Maximum error, } E = 0.05}$ — ③

W.K.T. Maximum error, $E = Z_{\alpha/2} \sqrt{\frac{PQ}{n}}$

$$\Rightarrow \sqrt{n} = \frac{Z_{\alpha/2} \sqrt{PQ}}{E}$$

$$\Rightarrow \text{Sample size, } n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 (PQ)$$

$$= \left(\frac{1.96}{0.05}\right)^2 (0.2 \times 0.8) \quad [\because \text{From ①, ②, ③}]$$

$$\Rightarrow \boxed{\text{Sample size, } n = 246} \text{ (Ans).}$$

20) What is the size of the smallest sample required to estimate an unknown proportion to within a maximum error of 0.06 with atleast 95% confidence.

Solution:-

Given: $\boxed{\text{Maximum error, } E = 0.06}$ — ①

Since P is not given, we take $\boxed{P = \frac{1}{2} = 0.5}$ — ②

$\Rightarrow Q = 1 - P = 1 - 0.5$

$\Rightarrow \boxed{Q = 0.5}$ — ③

W.K.T. Sample size, $n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 (PQ)$

$$= \left(\frac{1.96}{0.06}\right)^2 (0.5 \times 0.5)$$

$$\Rightarrow \boxed{\text{Sample size, } n = 267} \text{ (Ans).}$$

21) A random sample of 500 apples was taken from a large consignment and 60 of them were found to be bad. Obtain 98% confidence limits for the percentage of bad apples in the consignment.

Solution:-

Given:

Sample size, $\boxed{n = 500}$ —①

$P = \frac{60}{500}$

$\Rightarrow \boxed{P = 0.12}$ —②

$\Rightarrow Q = 1 - P$
$= 1 - 0.12$

$\Rightarrow \boxed{Q = 0.88}$ —③

W.K.T. Maximum error estimate, $E = Z_{\alpha/2}\sqrt{\frac{PQ}{n}}$

$= 2.33 \times \sqrt{\frac{0.12 \times 0.88}{500}}$ [∵ From ①, ②, ③]

$\Rightarrow \boxed{E = 0.3386}$ —④

98% confidence limits are given by $P \pm E$

$= (0.12 \pm 0.3386)$ [∵ From ②, ④]

$\Rightarrow$ $\boxed{98\% \text{ confidence limits} = (0.0861, 0.1539)}$ (Ans).

HW

1) A random sample of size 100 has a standard deviation of 5. What can you say about the maximum error with 95% confidence.

2) A sample of 10 cam shafts intended for use in gasoline engines has an average eccentricity of 1.02 and a standard deviation of 0.044 inch. Assuming the data may be treated a random sample from a normal population, determine a 95% confidence interval for the actual mean eccentricity of the cam shaft?

3) A random sample of size 81 was taken whose variance is 20.25 and mean is 32, Construct 98% confidence interval.

4) A random sample of size 100 is taken from a population with $\sigma = 5.1$ Given that the sample mean is $\bar{x} = 21.6$. Construct a 95% confidence interval for the population mean μ.

5) Determine a 95% confidence interval for the mean of a normal distributio with variance 0.25, using a sample of $n = 100$ values with mean 212.3

6) Measurements of the weights of a random sample of 200 ball bearing made by a certain machine during one week showed a mean of 0.824 and a standard deviation of 0.042. Find the maximum error at 95% confidence interval? Find the confidence limits for the mean if $\bar{x} = 32$.

7) A sample of size 300 was taken whose variance is 225 and mean 54. Construct 95% confidence interval for the mean.

8) In a random sample of 100 packages shipped by air freight 13 had some damage. Construct 95% confidence interval for the true proportion of damage package.

9) Among 100 fishes caught in a large lake, 18 were inedible due to the pollution of the environment. With what confidence can we assert that the error of this estimate is atmost 0.065?

10) A random sample of 500 points on a heated plate resulted in an average temperature of 73.54 degrees Fahrenheit with a standard deviation of 2.79 degree Fahrenheit. Find a 99% confidence interval for the average temperature of the plate?