UNIT - III

Fundamental Sampling Distributions:

Population:-

Population in a statistical study is a set or collection of totality of observations. The population is also called as totality of observations about which the inferences are to be drawn. The number of observations in the population is defined to be the size of the population. It may be finite or infinite. Size of the population is denoted by N.

Examples:-

1) The number of students in a college is a finite population.
2) Goods prepared in a factory can be considered as an infinite population.

Sample:-

A finite subset of statistical individuals in a population is known as a sample. The number of objects or observations in the sample is called size of the sample. It is denoted by 'n'.

Example:-

1) Total production of items in a month is a population and total production of items in a day is a sample.

Sampling:-

The process of selection of the sample is called sampling. A portion of the population which is examined with view to determine the population characteristics is called a sample. Thus in estimating the characteristics of the population, instead of enumerating the entire population, only the individuals in the sample are examined. Then, the sample characteristics are utilised to approximately estimate the population. The error involved in such approximation is known as sampling error.

Different methods of sampling:-

1. **Random Sampling:** A random sampling is one in which each unit of population has an equal chance of being included in it. The process of drawing such a sample is called random sampling.
2. **Stratified Sampling:** In this type of sampling, the population is first subdivided into several parts of small groups called strata, according to some relevant characteristics so that each stratum is more or less homogenous. Each stratum is called a sub-population. Then a small sample called sub-sample is selected from each stratum randomly. All the sub-samples are combined together to form the stratified sample which represents the population property. The process of obtaining and examining

a stratified sample with a view to estimate the characteristics of the population is known as stratified sampling.

3. <u>Systematic Sampling</u>: In this method all the units of the population are arranged in some order. If the population size is finite, all the units of the population are arranged in some order, then from the first 'K' items one unit is selected at random. This unit and every k^{th} unit of the serially listed population combined together constitute a systematic sample. This type of sampling is known as systematic sampling.

Non-probability Sampling Methods :-

4. <u>Purposive Sampling (Judgement)</u>: When the choice of the individual item of a sample (or samples) entirely depends on the individual judgement of the investigator, it is called a purposive (judgement) sampling.

5. <u>Sequential Sampling</u>: It consists of a sequence of samples drawn one after another from the population depending on the results of previous samples.

Classification of Samples:

Samples are classified in two ways.

1. <u>Large samples</u>: If the size of the sample $n \geq 30$, the sample is said to be a large sample.
2. <u>Small samples</u>: If the size of the sample $n < 30$, the sample is said to be a small sample (or) an exact sample.

Parameter: A parameter is a statistical measure or constant obtained from the population.

Examples:- Population mean (μ), Population variance (σ^2) etc.

Statistic: A statistic is a statistical measure computed from the sample observations.

Examples:- Sample mean ($\bar{x}$), Sample Variance (s^2) etc.

Parameters refers to population while statistic refers to sample.

1) <u>Population Mean</u> $= \mu = \dfrac{\sum_{i=1}^{N} x_i}{N}$, where N is the population size.

2) <u>Population Variance</u> $= \sigma^2 = \dfrac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2$ and Population S.D. $= \sigma$.

3) <u>Sample Mean</u>: If $x_1, x_2, \ldots, x_n$ represent a random sample of size n then the sample mean is defined by the statistic, $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{}$

4) <u>Sample Variance</u>: It is defined by the statistic $s^2 = \dfrac{1}{(n-1)}\sum_{i=1}^{n}(x_i-\bar{x})^2$, where n is the sample size.

5) <u>Sample Standard deviation</u>: The sample S.D. denoted by s is the positive square root of the sample variance.

Biased: Any sampling procedure that produces inferences that consistently overestimate or underestimate some characteristics of population is set to be biased. To eliminate any possibility of bias in sampling procedure it is desirable to choose a random sample in the sense that the observations are made independently and at random.

Random sample: Let $X_1, X_2, \ldots X_n$ be n independent r.v's each having the same probability distribution $F(x)$, define $X_1, X_2, \ldots X_n$ to be a random sample of size n from the population $f(x)$ and write its joint probability distribution as $f(x_1, x_2, \ldots, x_n) = f(x_1) \cdot f(x_2) \ldots f(x_n)$.

Statistic: Any function of the random variables constituting a random sample is called a statistic.

Location measures of a sample:-

Sample mean, $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$, where n is the sample size.

Sample median, $\tilde{x} = \begin{cases} x_{(n+1)/2}, & \text{if } n \text{ is odd,} \\ \frac{1}{2}\left[x_{n/2} + x_{\frac{n}{2}+1}\right], & \text{if } n \text{ is even.} \end{cases}$

Sample median is a location measure that shows the middle value of the sample.

Sample mode is the value of the sample that occurs most often.

Probleme based on location measures of a sample:

1) Suppose the data set is the following 1.7, 2.2, 3.9, 3.11, 14.7. Calculate the sample mean and sample median.

Solution:-

Sample mean, $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$

$= \frac{1.7+2.2+3.9+3.11+14.7}{5}$

$\Rightarrow \boxed{\text{Sample mean, } \bar{x} = 5.12}$ (Ans).

Sample median, $\tilde{x} = x_{(n+1)/2}$, if n is odd

Given: $\boxed{n=5}$

$\Rightarrow$ Sample median, $\tilde{x} = x_{6/2} = x_3$

$\Rightarrow \boxed{\text{Sample median, } \tilde{x} = 3.9}$ (Ans).

2) Suppose the data set consists of the following observations: 0.32, 0.53, 0.28, 0.37, 0.47, 0.43, 0.36, 0.42, 0.38, 0.43. Calculate the sample mode.

Solution:-

$\boxed{\text{Sample mode} = 0.43}$ ($\because$ This value occurs more than any other value). (Ans).

Note:-

Population	Sample
1) Mean $= \mu$	1) Mean $= \bar{x}$
2) Variance $= \sigma^2$	2) Variance $= S^2$
3) S.D. $= \sigma$	3) S.D. $= S$
4) Size $= N$	4) Size $= n$

Variability measures of a sample:-

Sample variance, $s^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2$, where n is the sample size.

Sample standard deviation, $s = \sqrt{s^2}$

Sample Range, $R = X_{max} - X_{min}$

Problems based on variability measures of a sample:

1) A comparison of coffee prices at 4 randomly selected grocery stored in San diego showed increases from the previous month of 12, 15, 17 and 20 cents for a 1-pound bag. Find the variance of this random sample of price increases.

Solution:-

Sample Mean, $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$

$= \frac{12+15+17+20}{4}$

$\Rightarrow$ $\boxed{\text{Sample mean}, \bar{x} = 16}$ (Ans). —①

Sample variance, $s^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$= \frac{1}{(4-1)} \sum_{i=1}^{n} (x_i - 16)^2 \quad [\because \text{From } ①]$

$= \frac{1}{3}\left[(12-16)^2 + (15-16)^2 + (17-16)^2 + (20-16)^2\right]$

$\Rightarrow$ Sample variance, $s^2 = 11.33$

Sample standard deviation, $s = \sqrt{11.33} = 3.36$

Sample Range, $R = 20 - 12 = 8$

(Ans).

2) Find the sample variance of the data 3, 4, 5, 6, 6, 7 representing the number of trout caught by a random sample of 6 fishermen on June 19, 1996 at Lake Muskoka.

Solution:-

Sample mean, $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$

$= \frac{3+4+5+6+6+7}{6}$

$= \frac{31}{6}$

$\Rightarrow$ $\boxed{\text{Sample mean}, \bar{x} = 5.1}$ (Ans). —①

Sample variance, $s^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$= \frac{1}{(6-1)} \sum_{i=1}^{n} (x_i - 5.1)^2$

$= \frac{1}{5}\left[(3-5.1)^2 + (4-5.1)^2 + (5-5.1)^2 + (6-5.1)^2 + (6-5.1)^2 + (7-5.1)^2\right]$

$= \frac{1}{5}\left[4.41 + 1.21 + 0.01 + 0.81 + 0.81 + 3.61\right]$

$= \frac{1}{5}[10.86] = 2.172$

$\Rightarrow$ Sample variance, $s^2 = 2.172$

Sample standard deviation, $s = \sqrt{2.172} = 1.47$

Sample Range, $R = 7 - 3 = 4$

(Ans).

Sampling distribution:

The sampling distribution of a statistic is the probability distribution of a statistic.

Sampling distribution of means:

The probability distribution of sample mean ($\bar{x}$) is called the sampling distribution of means.

Sampling with replacement:

If the items are selected or drawn one by one in such a way that an item drawn at a time is replaced back to the population before the next or subsequent draw is known as sampling with replacement.

Sampling without replacement:

In sampling without replacement an item of the population cannot be chosen more than once as it is not replaced.

Note:-

If N is the population size and n is the sample size, then the no. of samples with replacement is N^n.
no. of samples without replacement is $^N C_n$.

Standard Error (S.E.):

The standard deviation of the sampling distribution of a statistic is known as standard error. The standard error of some of the statistics are given below, where $\bar{x}$ is the sample mean, s^2 is the sample variance, p is the sample proportion, n is the sample size, σ^2 is the population variance, P is the population proportion.

1) S.E. of sample mean $(\bar{x}) = \dfrac{\sigma}{\sqrt{n}}$

2) S.E. of sample proportion $(p) = \sqrt{\dfrac{PQ}{n}}$

3) S.E. of sample S.D. $(s) = \dfrac{\sigma}{\sqrt{2n}}$

4) S.E. of difference of two sample means $(\bar{x}_1 - \bar{x}_2) = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

5) S.E. of difference of two sample proportions $(p_1 - p_2) = \sqrt{\dfrac{P_1 Q_1}{n_1} + \dfrac{P_2 Q_2}{n_2}}$

6) S.E. of difference of two sample S.D.'s $(s_1 - s_2) = \sqrt{\dfrac{\sigma_1^2}{2n_1} + \dfrac{\sigma_2^2}{2n_2}}$

Sampling distribution of means (σ known):

The probability distribution of sample mean $(\bar{x})$ is called the sampling distribution of means.

Case 1: Sampling with replacement or infinite population:

The mean of the sampling distribution of means is $\mu_{\bar{x}} = \mu$

(i.e.) $E(\bar{x}) = \mu$

The Variance of the sampling distribution of means is $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$

The standard deviation of the sampling distribution of means is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Case 2: Sampling without replacement or finite population:

The mean of the sampling distribution of means is $\mu_{\bar{x}} = \mu$

(i.e.) $E(\bar{x}) = \mu$

The variance the sampling distribution of means is $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)$.

where N is the population size, n is the sample size.

The standard deviation of the sampling distribution of means is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{N-n}{N-1}\right)}$

The term $\left(\frac{N-n}{N-1}\right)$ is called the finite population correction factor where N is the population size, n is the sample size.

Problems based on sampling distribution of means:

1) Find the value of the finite population correction factor if $n=10$, $N=1000$.

Solution:-

Given: $\boxed{\begin{matrix} n=10 \\ N=1000 \end{matrix}}$ —①

W.K.T. Finite population correction factor is $\left(\frac{N-n}{N-1}\right)$

where N is the population size
n is the sample size —②

Sub. ① in ②, we get, $\frac{1000-10}{1000-1} = \frac{990}{999} = 0.991$

$\Rightarrow$ $\boxed{\text{Finite population correction factor} = 0.991}$ (Ans).

HW

1) Find the value of correction factor if $n=5$ and $N=200$.

[Ans: 0.98].

2) How many different samples of size two can be chosen, from a finite population of size 25?

Solution:-

We can take NC_n samples of size n from the finite population of size N. —①

Here $\boxed{N=25, \quad n=2}$ —②

Sub. ② in ①, we get, $25C_2 = 300 \quad \left(\because \frac{25 \cdot \cancel{24}^{12}}{1 \cdot \cancel{2}} = 300\right)$

∴ We can take $25C_2 = 300$ samples of size 2 from a finite population of size 25. (Ans)

3) In a random sample of 1000 packages shipped by air freight 13 had some damage. Find the standard error of proportions.

Solution:- Given: Sample size $\boxed{n = 100}$

Proportion of damage packages, $P = \frac{13}{100}$

$\Rightarrow \boxed{P = 0.13}$ —①

W.K.T. $Q = 1 - P$

$= 1 - 0.13$

$\Rightarrow \boxed{Q = 0.87}$ —②

W.K.T. Standard error of sample proportions $= S.E.(p) = \sqrt{\frac{PQ}{n}}$ —③

Sub. ①, ② in ③, we get, $S.E.(p) = \sqrt{\frac{(0.13)(0.87)}{1000}}$

$\Rightarrow \boxed{S.E.(p) = 0.0106}$

$\Rightarrow$ $\boxed{\text{Standard error of sample proportions } S.E.(p) = 0.0106}$ (Ans).

4) A population consists of five numbers 2, 3, 6, 8 and 11. Consider all possible samples of size two which can be drawn with replacement and without replacement from this population. Find a) The mean of the population. b) The standard deviation of the population. c) The mean of the sampling distribution of means. d) The standard deviation of the sampling distribution of means [(i.e.) the standard error of means].

Solution:-

a) Mean of the population, $\mu = \frac{2+3+6+8+11}{5}$

$= \frac{30}{5}$

$\Rightarrow$ a) $\boxed{\text{Mean of the population, } \mu = 6}$ (Ans). —①

b) Variance of the population, $\sigma^2 = \sum \frac{(x_i - \mu)^2}{n}$

$= \frac{(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2}{5}$

$= \frac{16+9+0+4+25}{5}$

$= \frac{54}{5} = 10.8$

$\Rightarrow$ $\boxed{\text{Variance of the population}, \sigma^2 = 10.8}$

$\Rightarrow$ b) $\boxed{\text{Standard deviation of the population, } \sigma = \sqrt{10.8} = 3.29}$ (Ans). — (A)

Sampling with replacement (Infinite population):

c) The total number of samples with replacement is $N^n = 5^2$ (Given)
$= 25$ samples of size 2, where N = population size,
n = sample size.

Listing all possible samples of size 2 from infinite population 2, 3, 6, 8, 11 with replacement, we get 25 samples. They are,

$$\begin{Bmatrix} (2,2) & (2,3) & (2,6) & (2,8) & (2,11) \\ (3,2) & (3,3) & (3,6) & (3,8) & (3,11) \\ (6,2) & (6,3) & (6,6) & (6,8) & (6,11) \\ (8,2) & (8,3) & (8,6) & (8,8) & (8,11) \\ (11,2) & (11,3) & (11,6) & (11,8) & (11,11) \end{Bmatrix}$$

The corresponding sample means are
$$\begin{Bmatrix} 2 & 2.5 & 4 & 5 & 6.5 \\ 2.5 & 3 & 4.5 & 5.5 & 7 \\ 4 & 4.5 & 6 & 7 & 8.5 \\ 5 & 5.5 & 7 & 8 & 9.5 \\ 6.5 & 7 & 8.5 & 9.5 & 11 \end{Bmatrix} \quad \text{—(I)}$$

The mean of the sampling distribution of means, $\mu_{\bar{x}} = \frac{(I)}{25} = \frac{150}{25} = 6$

$\Rightarrow$ c) $\boxed{\text{The mean of the sampling distribution of means, } \mu_{\bar{x}} = 6}$ (Ans). — (2)

d) The variance of the sampling distribution of means is

$$\sigma_{\bar{x}}^2 = \frac{\begin{Bmatrix} (2-6)^2+(2.5-6)^2+(4-6)^2+(5-6)^2+(6.5-6)^2 \\ +(2.5-6)^2+(3-6)^2+(4.5-6)^2+(5.5-6)^2+(7-6)^2 \\ +(4-6)^2+(4.5-6)^2+(6-6)^2+(7-6)^2+(8.5-6)^2 \\ +(5-6)^2+(5.5-6)^2+(7-6)^2+(8-6)^2+(9.5-6)^2 \\ +(6.5-6)^2+(7-6)^2+(8.5-6)^2+(9.5-6)^2+(11-6)^2 \end{Bmatrix}}{25} \quad [\because \text{From } (2)]$$

$= \frac{135}{25}$

$= 5.40$

$\Rightarrow$ $\boxed{\text{The variance of the sampling distribution of means, } \sigma_{\bar{x}}^2 = 5.40}$

$\Rightarrow$ $\boxed{\text{The standard deviation of the sampling distribution of means, } \sigma_{\bar{x}} = \sqrt{5.40} = 2.32}$ — (3) (Ans)

Verification: For infinite population involving sampling with replacement,

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.29}{\sqrt{2}} = \frac{3.29}{1.414} = 2.32$ — (4) $\therefore$ (3) = (4)

$\Rightarrow$ $\boxed{\text{The standard deviation of the sampling distribution of means, } \sigma_{\bar{x}} = 2.32}$ (Ans)

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Sampling without replacement (Finite population):

c) The total number of samples without replacement is $^N C_n = {}^5C_2$ (Given) $= \frac{5 \cdot 4 2}{1 \cdot 2}$ $= 10$ samples of size 2, where N = population size, n = sample size.

Listing all possible samples of size 2 from finite population 2, 3, 6, 8, 11 without replacement, we get 10 samples. They are,

$$\begin{Bmatrix} (2,3) & (2,6) & (2,8) & (2,11) \\ (3,6) & (3,8) & (3,11) & \\ (6,8) & (6,11) & & \\ (8,11) & & & \end{Bmatrix}$$

The corresponding sample means are $\begin{Bmatrix} 2.5 & 4 & 5 & 6.5 \\ 4.5 & 5.5 & 7 & \\ 7 & 8.5 & & \\ 9.5 & & & \end{Bmatrix}$ — (J)

The mean of the sampling distribution of means, $\mu_{\bar{x}} = \frac{(J)}{10} = \frac{60}{10} = 6$

$\Rightarrow$ c) $\boxed{\text{The mean of the sampling distribution of means, } \mu_{\bar{x}} = 6}$ (Ans). — (5)

d) The Variance of the sampling distribution of means is

$$\sigma_{\bar{x}}^2 = \frac{\begin{Bmatrix} (2.5-6)^2 + (4-6)^2 + (5-6)^2 + (6.5-6)^2 \\ + (4.5-6)^2 + (5.5-6)^2 + (7-6)^2 \\ + (7-6)^2 + (8.5-6)^2 \\ + (9.5-6)^2 \end{Bmatrix}}{10}$$ [∵ From (5)]

$$= \frac{40.5}{10}$$

$$= 4.05$$

$\Rightarrow$ $\boxed{\text{The variance of the sampling distribution of means, } \sigma_{\bar{x}}^2 = 4.05}$

$\Rightarrow$ $\boxed{\text{The standard deviation of the sampling distribution of means, } \sigma_{\bar{x}} = \sqrt{4.05} = 2.01}$ — (6) (Ans)

Verification: For finite population involving sampling without replacement,

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\left(\sqrt{\frac{N-n}{N-1}}\right)$$

$$= 2.32\left(\sqrt{\frac{5-2}{5-1}}\right) \quad [\because \text{From } (4)]$$

$$= 2.32\left(\sqrt{\frac{3}{4}}\right)$$

$$= 2.32\left(\sqrt{0.75}\right)$$

$\Rightarrow \sigma_{\bar{x}} = 2.01$ — (7) $\quad \therefore (6) = (7)$

$\Rightarrow$ $\boxed{\text{The standard deviation of the sampling distribution of means, } \sigma_{\bar{x}} = 2.01}$ (Ans)

5) Let $u_1 = (3,7,8)$, $u_2 = (2,4)$. Find
a) μ_{u_1} b) μ_{u_2} c) Mean of the sampling distribution of the difference of means $\mu_{u_1-u_2}$
d) σ_{u_1} e) σ_{u_2} f) The standard deviation of the sampling distribution of the difference of means $\sigma_{u_1-u_2}$.

Solution:-

Given: $u_1 = (3,7,8)$ $u_2 = (2,4)$

$u_1 - u_2 = \{1, 5, 6, -1, 3, 4\}$

a) $\mu_{u_1} = \frac{3+7+8}{3} = \frac{18}{3} = 6$

$\Rightarrow$ a) $\boxed{\mu_{u_1} = 6}$ (Ans).

b) $\mu_{u_2} = \frac{2+4}{2} = \frac{6}{2} = 3$

$\Rightarrow$ b) $\boxed{\mu_{u_2} = 3}$ (Ans).

c) $\mu_{u_1-u_2} = \frac{1+5+6-1+3+4}{6}$
$= \frac{18}{6} = 3$

$\Rightarrow$ c) $\boxed{\mu_{u_1-u_2} = 3}$ (Ans).

d) $\sigma_{u_1} = \sqrt{\frac{(3-6)^2+(7-6)^2+(8-6)^2}{3}} = \sqrt{\frac{14}{3}} = \sqrt{4.67} = 2.16$

$\Rightarrow$ d) $\boxed{\sigma_{u_1} = 2.16}$ (Ans).

e) $\sigma_{u_2} = \sqrt{\frac{(2-3)^2+(4-3)^2}{2}} = 1$ $\Rightarrow$ e) $\boxed{\sigma_{u_2} = 1}$ (Ans).

f) $\sigma_{u_1-u_2} = \sqrt{\frac{(1-3)^2+(5-3)^2+(6-3)^2+(-1-3)^2+(3-3)^2+(4-3)^2}{6}} = \sqrt{\frac{\cancel{34}\,17}{\cancel{6}\,3}} = \sqrt{5.67} = 2.38$

$\Rightarrow$ f) $\boxed{\sigma_{u_1-u_2} = 2.38}$ (Ans).

HW

1) A population consists of 5, 10, 14, 18, 13, 24. Consider all possible samples of size two which can be drawn without replacement from the population. Find a) The mean of the population. b) The standard deviation of the population. c) The mean of the sampling distribution of means. d) The standard deviation of the sampling distribution of means.

2) Samples of size 2 and 3 are taken from the population 3, 6, 9, 15, 27 with and without replacement. Find a) The mean of the population. b) The standard deviation of the population. c) The mean of the sampling distribution of means. d) The standard deviation of the sampling distribution of means.

Central limit theorem:

If $\bar{x}$ be the mean of a random sample of size 'n' drawn from a population mean μ and finite variance σ^2 with standard deviation σ then the sampling distribution of sample mean $\bar{x}$ is approximately a normal distribution with mean μ and standard deviation equal to the standard error of $\bar{x} = \frac{\sigma}{\sqrt{n}}$ provided the sample size is large (i.e.) $n \geq 30$.

Theorem:

If $\bar{x}$ be the mean of a random sample of size n taken from a population with mean μ and finite variance σ^2 and standard deviation σ then the limiting form of the distribution is the standardized sample mean $Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ which is a random variable whose distribution function approaches that of standard normal distribution as $n \to \infty$.

Problems based on Central limit theorem:

1) The variance of a population is 2. The size of the sample collected from the population is 169. What is the standard error of mean.

Solution:- Given:

Sample size, $n = 169$ — ①

Population standard deviation, $\sigma = \sqrt{\text{Variance}} = \sqrt{2}$ — ②

W.K.T. Standard error (S.E.) of sample mean, $\boxed{\bar{x} = \frac{\sigma}{\sqrt{n}}}$ — ③

$$= \frac{\sqrt{2}}{\sqrt{169}} \quad [\because \text{Sub. ①, ② in ③}]$$

$$= \frac{1.414}{13}$$

$\Rightarrow$ $\boxed{\text{Standard error (S.E.) of sample mean, } \bar{x} = 0.109}$ (Ans).

2) What is the effect on standard error, if a sample is taken from an infinite population of sample size is increased from 400 to 900.

Solution:-

Given: $n_1 = 400$, $n_2 = 900$

W.K.T. Sample size $= n$

Standard error (S.E) of sample mean, $\bar{x} = \frac{\sigma}{\sqrt{n}}$

$$S.E._1 = \frac{\sigma}{\sqrt{n_1}} = \frac{\sigma}{\sqrt{400}} = \frac{\sigma}{20}$$

$$S.E._2 = \frac{\sigma}{\sqrt{n_2}} = \frac{\sigma}{\sqrt{900}} = \frac{\sigma}{30}$$

$$S.E._2 = \frac{\sigma}{30} = \frac{2}{3}\left(\frac{\sigma}{20}\right) = \frac{2}{3}(S.E._1)$$

$\Rightarrow$ $\boxed{S.E._2 = \frac{2}{3}(S.E._1)}$

Thus if the sample size is increased from 400 to 900 the standard error of sample mean $\bar{x}$ will be multiplied by $\frac{2}{3}$. (Ans).

3) When a sample is taken from an infinite population, what happens to the standard error of the mean if the sample size is decreased from 800 to 200.

Solution:-

Given: $n_1 = 800$
$n_2 = 200$

W.K.T. Sample size $= n$

Standard error (S.E.) of sample mean, $\bar{x} = \frac{\sigma}{\sqrt{n}}$

$$S.E_1 = \frac{\sigma}{\sqrt{n_1}} = \frac{\sigma}{\sqrt{800}} = \frac{\sigma}{20\sqrt{2}}$$

$$S.E._2 = \frac{\sigma}{\sqrt{n_2}} = \frac{\sigma}{\sqrt{200}} = \frac{\sigma}{10\sqrt{2}}$$

$$S.E._2 = \frac{\sigma}{10\sqrt{2}} = \cancel{2}\left(\frac{\sigma}{\cancel{20}_{10}\sqrt{2}}\right) = 2\,(S.E_1)$$

$\Rightarrow \boxed{S.E._2 = 2\,(S.E._1)}$

Thus if the sample size is decreased from 800 to 200, the standard error of sample mean, $\bar{x}$ will be multiplied by 2.
(Ans).

4) The mean height of students in a college is 155 cm and standard deviation is 15. What is the probability that the mean height of 36 students is less than 157 cm?

Solution:-

Given: Population Mean $= \mu =$ Mean height of students of a college $= 155$ cm

Population Standard deviation $= \sigma = 15$ cm

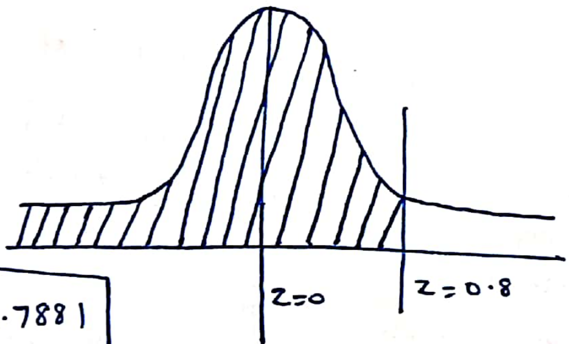
Sample size $= n = 36$

Sample mean, $\bar{x} = 157$ cm

$$\text{W.K.T.} \quad Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{157-155}{\frac{15}{\sqrt{36}}} = \frac{2}{\frac{15}{6}} = \frac{\cancel{12}\,4}{\cancel{15}\,5} = 0.8$$

$$\therefore P(\bar{x} < 157) = P(Z < 0.8)$$
$$= 0.5 + 0.2881$$

$\Rightarrow \boxed{P(\bar{x} < 157) = 0.7881}$



$\Rightarrow$ The probability that the mean height of 36 students is less than 157 cm $= 0.7881$

(Ans).

5) A random sample of size 100 is taken from an infinite population having the mean $\mu = 76$ and the variance $\sigma^2 = 256$. What is the probability that $\bar{x}$ will be between 75 and 78?

Solution:-

Given: Population Mean $= \mu = 76$

Population Variance $= \sigma^2 = 256$

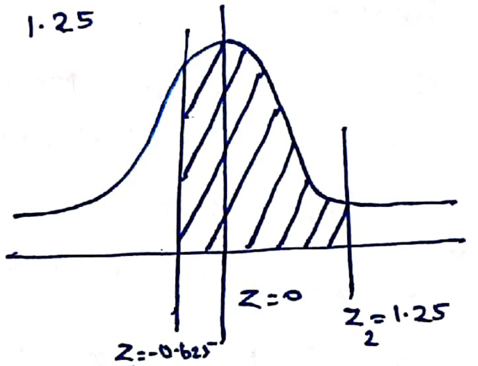
Population Standard deviation $= \sigma = \sqrt{256} = 16$

Sample size $= n = 100$

Sample means $\begin{cases} \bar{x}_1 = 75 \\ \bar{x}_2 = 78 \end{cases}$

W.K.T $z_1 = \dfrac{\bar{x}_1 - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{75-76}{\frac{16}{\sqrt{100}}} = \dfrac{-1}{\frac{16}{10}} = \dfrac{-\cancel{10}}{\cancel{16}}\dfrac{5}{8} = -0.625$

$z_2 = \dfrac{\bar{x}_2 - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{78-76}{\frac{16}{\sqrt{100}}} = \dfrac{2}{\frac{16}{10}} = \dfrac{\cancel{20}}{\cancel{16}}\dfrac{5}{4} = 1.25$



$\therefore P(75 \le \bar{x} \le 78) = P(-0.625 < z < 1.25)$

$= 0.2324 + 0.3944$

$\Rightarrow \boxed{P(75 \le \bar{x} \le 78) = 0.6268}$

$\Rightarrow$ The probability that $\bar{x}$ will be between 75 and 78 = 0.6268 (Ans).

6) A random sample of size 64 is taken from an infinite population having the mean 45 and the standard deviation 8. What is the probability that $\bar{x}$ will be between 46 and 47.5?

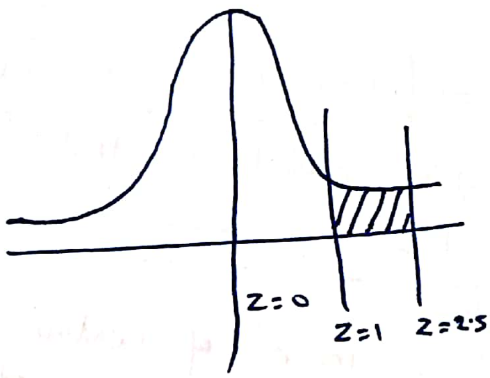
Solution:-

Given: Population Mean $= \mu = 45$

Population standard deviation $= \sigma = 8$

sample size $= n = 64$

sample means $\begin{cases} \bar{x}_1 = 46 \\ \bar{x}_2 = 47.5 \end{cases}$



W.K.T. $z_1 = \dfrac{\bar{x}_1 - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{46-45}{\frac{8}{\sqrt{64}}} = \dfrac{1}{\frac{\cancel{8}}{\cancel{8}}} = 1$

$z_2 = \dfrac{\bar{x}_2 - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{47.5-45}{\frac{8}{\sqrt{64}}} = \dfrac{2.5}{\cancel{8}/8} = 2.5$

$\therefore P(46 \le \bar{x} \le 47.5) = P(1 < z < 2.5)$

$= 0.4938 - 0.3413$

$\Rightarrow \boxed{P(46 \le \bar{x} \le 47.5) = 0.1525}$

$\Rightarrow$ The probability that $\bar{x}$ will be between 46 and 47.5 = 0.1525 (Ans).

7) A normal population has a mean of 0.1 and standard deviation of 2.1. Find the probability that mean of a sample size 900 will be negative? 14

Solution:-

Given: Population Mean $= \mu = 0.1$

Population Standard deviation $= \sigma = 2.1$

Sample size $= n = 900$

W.K.T

The standard normal variate is

$$Z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\bar{x}-0.1}{\frac{2.1}{\sqrt{900}}} = \frac{\bar{x}-0.1}{\frac{2.1}{30}} = \frac{\bar{x}-0.1}{0.07}$$

$\Rightarrow$ $0.07Z = \bar{x} - 0.1$

$\Rightarrow$ $\bar{x} = 0.07Z + 0.1$ where $Z \sim N(0,1)$

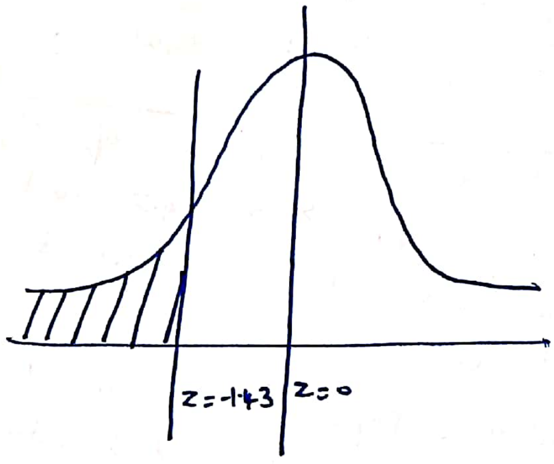
$\therefore$ $P(\bar{x}<0) = P(0.07Z + 0.1 < 0)$

$= P(0.07Z < -0.1)$

$= P\left(Z < \frac{-0.1}{0.07}\right)$

$= P(Z < -1.43)$

$= 0.5 - 0.4236$



$\Rightarrow$ $\boxed{P(\bar{x}<0) = 0.0764}$

$\Rightarrow$ The probability that mean of a sample size 900 will be negative $= 0.0764$ (Ans).

HW

1) The mean of certain normal population is equal to the standard error of the mean of the samples of 64 from that distribution. Find the probability that the mean of the sample size 36 will be negative.

2) A random sample of size 64 is taken from a normal population with $\mu = 51.4$ and $\sigma = 68$. What is the probability that the mean of the sample will a) exceed 52.9. b) fall between 50.5 and 52.3. c) less than 50.6.

Sampling distribution of means (σ unknown):

In previous problems on a population mean or the difference between two population means it was assumed that the population standard deviation σ is known. But for large sample of size $(n \geq 30)$, even if standard deviation σ of population is not known, it does not make any difference. Since we can substitute the sample S.D. 's' in the place of σ, the sample S.D. 's' is calculated using the sample mean $\bar{x}$ by the formula $s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$. For small samples of size $(n < 30)$, when σ is unknown, it can be substituted by s, provided we make the assumption that the sample is drawn from a normal population.

Degrees of freedom:

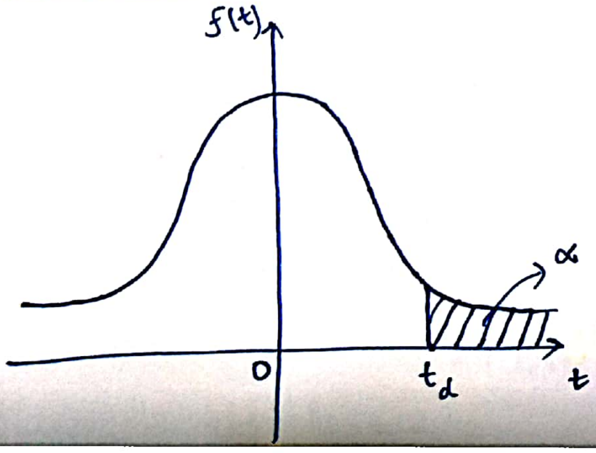
The number of independent variates which make up the statistic is known as the degrees of freedom (d.f.) and it is denoted by ν (the letter 'Nu' of the Greek alphabet).

Student's t-distribution (or) t-distribution:

It is used for testing of hypothesis when the sample size is small and population S.D. σ is not known.

If $\{x_1, x_2, \ldots, x_n\}$ be any random sample of size n drawn from a normal (or approximately normal) population with mean μ and variance σ^2, then the test statistic t is defined by $t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$ where $\bar{x}$ = Sample mean and $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})$ is an unbiased estimate of σ^2. The test statistic $t = \frac{\bar{x} - \mu}{S/\sqrt{n}}$ is a random variable having the t-distribution with $\nu = n-1$ degrees of freedom and with probability density function $f(t) = y_0 \left(1 + \frac{t^2}{\nu}\right)^{\frac{\nu+1}{2}}$ where $\nu = n-1$ and y_0 is a constant got by $\int_{-\infty}^{\infty} f(t)\, dt = 1$. This is known as Student's t distribution (or) t-distribution.

Properties of t-distribution:



(i) The shape of t-distribution is bell-shaped which is similar to that of a normal distribution and is symmetrical about the mean.

(ii) The t-distribution curve is also asymptotic to the t-axis, (i.e.) the two tails of the curve on both sides of $t=0$ extends to infinity.

(iii) It is symmetrical about the line $t=0$.
(iv) The form of the probability curve varies with degrees of freedom (i.e) with sample size.
(v) It is unimodal with Mean = Median = Mode.
(vi) The mean of standard normal distribution and as well as t-distribution is zero but the variance of t-distribution depends upon the parameter ν which is called the degrees of freedom.
(vii) The variance of t-distribution exceeds 1, but approaches 1 as $n \to \infty$. Infact the t-distribution with ν-degrees of freedom approaches standard normal distribution as $\nu = (n-1) \to \infty$.

Applications of the t-distribution :

The t-distribution has a wide number of applications in statistics, some of them are given below:

1) To test the significance of the sample mean, whose population Variance is not given.
2) To test the significance of the mean of the sample (i.e.) to test if the sample mean differs significantly from the population mean.
3) To test the significance of the difference between two sample means or to compare two samples.
4) To test the significance of an observed sample correlation coefficient and sample regression coefficient.

Problems based on t-distribution:

1) Find a) $t_{0.05}$ when $\nu = 16$
b) $t_{-0.01}$ when $\nu = 10$
c) $t_{0.995}$ when $\nu = 7$

Solutions:-

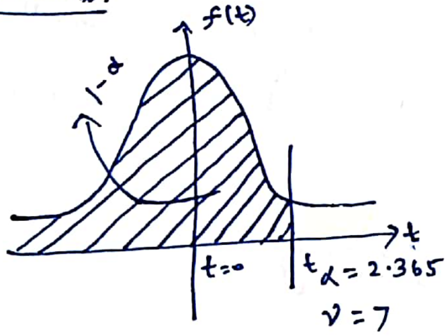
From t-tables, a) When $\nu = 16$, $t_{0.05} = 1.746$
b) When $\nu = 10$, $t_{-0.01} = -2.764$
c) When $\nu = 7$, $t_{0.995} = t_{1-0.005} = -t_{0.005} = -3.499$

(Ans).

2) Find a) $P(t < 2.365)$ when $\nu = 7$
b) $P(t > 1.318)$ when $\nu = 24$
c) $P(-1.356 < t < 2.179)$ when $\nu = 12$
d) $P(t > -2.567)$ when $\nu = 17$

Solutions:

a)

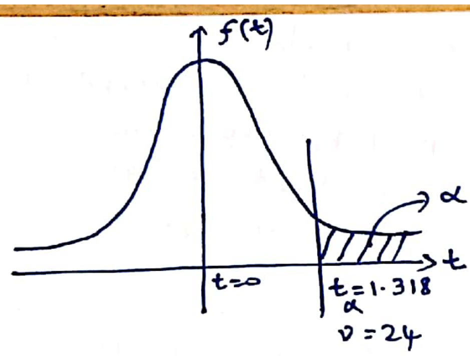


$= 1 - \alpha$

$= 1 - 0.025$ (∵ From t-tables)

$\Rightarrow \boxed{P(t < 2.365) = 0.975}$ (Ans).

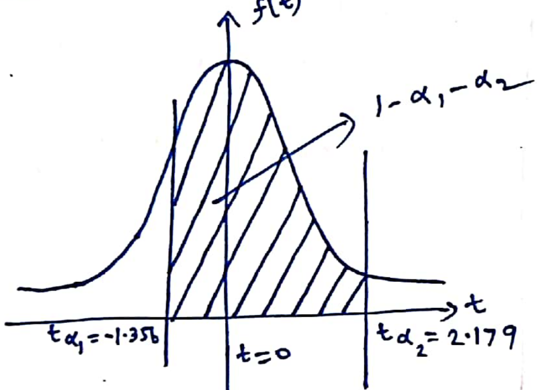
b)



$= \alpha$ (∵ From t-tables)

$\Rightarrow \boxed{P(t > 1.318) = 0.10}$ (Ans).

c)



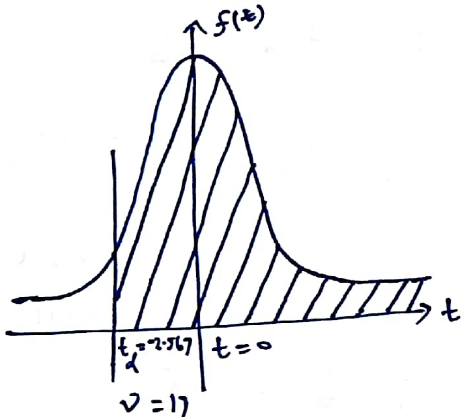
$\nu = 12$

$= 1 - \alpha_1 - \alpha_2$

$= 1 - 0.10 - 0.025$ (∵ From t-tables)

$\Rightarrow \boxed{P(-1.356 < t < 2.179) = 0.875}$ (Ans).

d)



$= 1 - \alpha$

$= 1 - 0.01$ (∵ From t-tables)

$\Rightarrow \boxed{P(t > -2.567) = 0.99}$ (Ans).

3) A random sample of size 25 from a normal population has the mean $\bar{x} = 47.5$ and the standard deviation $S = 8.4$. Does this information tend to support or refuse the claim that the mean of the population is $\mu = 42.5$?

Solution:-

Given:

Sample size, $n = 25$, Degrees of freedom, $\boxed{\nu = n - 1 = 24}$

Sample mean, $\bar{x} = 47.5$

Sample standard deviation, $S = 8.4$

Population mean, $\mu = 42.5$

We have for t-distribution,

Test statistic $t = \dfrac{\bar{x} - \mu}{S/\sqrt{n-1}}$

$= \dfrac{47.5 - 42.5}{8.4/\sqrt{24}}$

$\Rightarrow \boxed{t_{cal} = 2.92}$ — ①

Level of significance, $\alpha = 0.05$

$\boxed{t_{tab} = 1.797}$ — ②

∴ From ①, ② we get, $t_{cal} > t_{tab}$.

We conclude that the information given in the data tend to refuse the claim that the mean of the population is $\mu = 42.5$

(i.e.) the mean μ cannot be 42.5 (Ans).

4) 10 bearings made by a certain process have a mean diameter of 0.5060 cm with a standard deviation of 0.0040 cm. Assuming that the data may be taken as a random sample from a normal distribution, construct a 95% confidence interval for the actual average diameter of the bearings?

Solution:-

Given: Sample size, $n = 10$, Degrees of freedom, $\boxed{\nu = n-1 = 9}$

Sample mean, $\bar{x} = 0.5060$

Sample standard deviation, $s = 0.0040$

W.K.T. Maximum error estimate of 95% confidence is given by,

$$E = t_{\alpha/2} \frac{S}{\sqrt{n}}$$

$$= t_{\left(\frac{0.05}{2}\right)} \frac{0.0040}{\sqrt{9}}$$

$$= t_{0.025} (0.00216)$$

$$= 2.262 (0.00216) \quad (\because \text{From t-tables})$$

$$\Rightarrow \boxed{E = 0.00256}$$

95% confidence interval is given by $\bar{x} \pm E$

$$= 0.5060 \pm 0.00256$$

$$= (0.5031, 0.5089) \quad \text{(Ans).}$$

5) A sample of size 10 was taken from a population S.D. of sample is 0.03. Find the maximum error with 99% confidence.

Solution:-

Given: Sample size, $n = 10$, Degrees of freedom, $\boxed{\nu = n-1 = 9}$

Sample Standard deviation, $S = 0.03$

W.K.T. Maximum error estimate of 99% confidence is given by,

$$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$$

$$= t_{\left(\frac{0.01}{2}\right)} \cdot \left(\frac{0.03}{\sqrt{10}}\right)$$

$$= t_{0.005} (0.0094)$$

$$= (3.250)(0.0094) \quad (\because \text{From t-tables})$$

$$\Rightarrow \boxed{E = 0.031} \quad \text{(Ans)}$$

HW

1) A sample of 11 rats from a central population had an average blood viscosity of 3.92 with a standard deviation S.D. of 0.61. Estimate the 95% confidence limits for the mean blood viscosity of the population.

2) In 16 one hour test runs, the gasoline consumption of an engine averaged 16.4 gallons with a S.D. of 2.1 gallons. Test the claim that the average gasoline consumption of this engine is 12 gallons per hour.

Chi-square (χ^2) distribution :

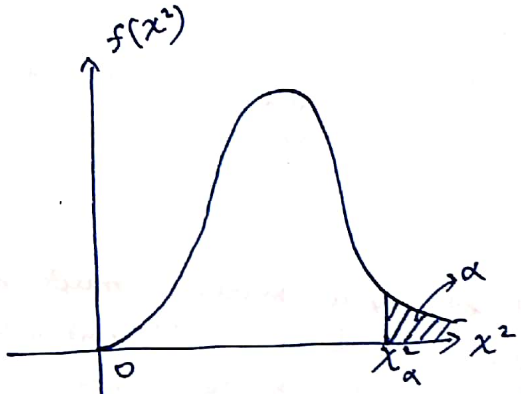
Chi-squared distribution is a probability distribution of a continuous random variable X with probability density function given by,

$$f(x) = \begin{cases} \frac{1}{2^{\nu/2}\,\Gamma(\nu/2)} x^{(\nu/2)-1} \cdot e^{-x/2}, & \text{for } x>0 \\ 0, & \text{otherwise} \end{cases}$$

where ν is a positive integer is the only single parameter of the distribution, also known as the degrees of freedom.

χ^2 - distribution is extensively used as a measure of goodness of fit and to test the independence of attributes.

Properties of χ^2 - distribution :



(i) χ^2 - distribution curve is not symmetrical, lies entirely in the first quadrant and hence not a normal curve, since χ^2 varies from 0 to ∞.

(ii) It depends only on the degrees of freedom ν.

(iii) If χ_1^2 and χ_2^2 are two independent distributions with ν_1 and ν_2 degrees of freedom, then $\chi_1^2 + \chi_2^2$ will be chi-squared distribution with $(\nu_1 + \nu_2)$ degrees of freedom.

(iv) Mean $= \nu$ and variance $= 2\nu$.

Sampling distribution of Variance s^2 :

The theoretical sampling distribution of the sample variance for random samples from normal population is related to the Chi-squared distribution as follows:

Let S^2 be the variance of a random sample of size n, taken from a normal population having the variance σ^2. Then $\chi^2 = \frac{(n-1)S^2}{\sigma^2}$ is a value of a random variable $= \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{\sigma^2}$ $\left[\because s^2 = \sum_{i=1}^{n} \frac{(x_i-\bar{x})^2}{n-1}\right]$ having the χ^2 - distribution with $\nu = n-1$ degrees of freedom.

Applications of the χ^2 - distribution :

1) To test the goodness of fit.
2) To test the independence of attributes.
3) To test the homogeneity of independent estimation of the population variance and the population correlation coefficient.

Problems based on χ^2-distribution:

1) A manufacturer claims that any of his list of items cannot have variance more than $1\,cm^2$. A sample of 25 items has a variance of $1.2\,cm^2$. Test whether the claim of the manufacturer is correct.

Solution:-

Given: Sample size, $n = 25$

Population variance, $\sigma^2 = 1$

Sample variance, $s^2 = 1.2$

W.K.T.
Test statistic, $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

$= \dfrac{24(1.2)}{1}$

$\Rightarrow \boxed{\chi^2_{cal} = 28.8}$ — ①

, Degrees of freedom, $\nu = n-1$
$= 25-1$
$\Rightarrow \boxed{\nu = 24}$

Level of significance, $\alpha = 0.05$

$\boxed{\chi^2_{tab} = 36.415}$ — ②

$\therefore$ From ①,② we get, $\chi^2 cal < \chi^2 tab$.

We conclude that the claim of the manufacturer is correct. (Ans).

2) Suppose that the thickness of a part used in a semiconductor is its critical dimension and that the process of manufacturing these parts is considered to be under control if the true variation among the thickness of the parts is given by a S.D. not greater than $\sigma = 0.60$ thousandth of an inch. To keep a check on the process, random samples of size $n = 20$ are taken periodically, and it is regarded to be out of control if the probability that S^2 will take on a value greater than or equal to the observed sample value is 0.01 or less (even though $\sigma = 0.60$). What can be concluded about the process if the S.D. of such a periodic random sample is $S = 0.84$ thousandth of an inch?

Solution:-

Given: Sample size, $n = 20$, Degrees of freedom, $\nu = n-1$
$= 20-1$
$\Rightarrow \boxed{\nu = 19}$

Population variance, $\sigma^2 = (0.60)^2$

Sample variance, $S^2 = (0.84)^2$ Level of significance, $\alpha = 0.01$

W.K.T.
Test statistic, $\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

$= \dfrac{(19)(0.84)^2}{(0.60)^2} = \dfrac{13.4064}{0.36}$

$\Rightarrow \boxed{\chi^2_{cal} = 37.24}$ — ①

$\boxed{\chi^2_{tab} = 36.191}$ — ②

$\therefore$ From ①,② we get, $\chi^2_{cal} > \chi^2_{tab}$.

We conclude that the process is said to be out of control. (Ans).

F- distribution :

Sampling distribution of the ratio of two sample variances :

Let S_1^2 be the sample variance of an independent sample of size n_1 drawn from a normal population $N(\mu_1, \sigma_1^2)$. Similarly, let S_2^2 be the sample variance of an independent sample of size n_2 drawn from another normal population $N(\mu_2, \sigma_2^2)$.

Thus S_1^2 and S_2^2 are the two variances of two random samples of sizes n_1 and n_2 respectively drawn from the normal population, with the variances σ_1^2 and σ_2^2. To determine whether the two samples come from two populations having equal variances.

Consider the sampling distribution of the ratio of the variances of the two independent random samples defined by $F = \dfrac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \dfrac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2}$ which follows F-distribution with $\nu_1 = n_1 - 1$, $\nu_2 = n_2 - 1$ degrees of freedom.

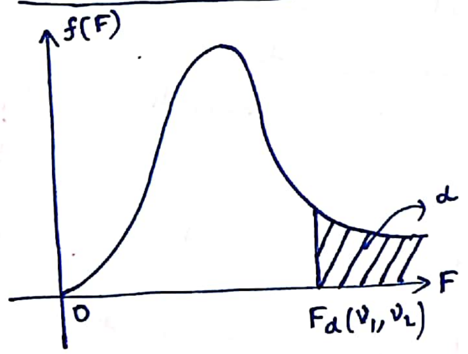
F-distribution can be used to test the equality of several population means, comparing sample variances and analysis of variance.

Under the hypothesis that two normal populations have the same variance (i.e.) $\sigma_1^2 = \sigma_2^2$, we have $F = \dfrac{S_1^2}{S_2^2}$. F determines whether the ratio of two sample variances S_1^2 and S_2^2 is too small or too large.

When F is close to 1, the two sample variances S_1 and S_2 are almost same. In practice, it is customary to take the larger sample variance as the numerator. F is always a positive number.

The sampling distribution of F is of the form $f(F) = K \dfrac{F^{(\nu_1-2)/2}}{(\nu_1 F + \nu_2)^{(\nu_1+\nu_2)/2}}$ where ν_1 and ν_2 are two degrees of freedom and K is determined by $\int_0^\infty f(F)\, dF = 1$.

Properties of F-distribution :



(i) F-distribution is free from population parameters and depends upon degrees of freedom only.

(ii) F - distribution curve lies entirely in first quadrant.

(iii) F-curve depends not only on the two parameters ν_1 and ν_2 but also on the order in which they are stated.

(iv) $F_{1-\alpha}(\nu_1, \nu_2) = \dfrac{1}{F_\alpha(\nu_2, \nu_1)}$ where $F_\alpha(\nu_1, \nu_2)$ is the value of F with ν_1 and ν_2 degrees of freedom such that the area under the F - distribution curve to the right of F_α is α.

(v) The mode of F - distribution is less than unity.

Problems based on F-distribution:

1) For an F - distribution, find
a) $F_{0.05}$ with $\nu_1 = 7$, $\nu_2 = 15$
b) $F_{0.01}$ with $\nu_1 = 24$, $\nu_2 = 19$
c) $F_{0.95}$ with $\nu_1 = 19$, $\nu_2 = 24$
d) $F_{0.99}$ with $\nu_1 = 28$, $\nu_2 = 12$

Solution:-

From F-tables, a) When $\nu_1 = 7, \nu_2 = 15$ $F_{0.05} = 2.71$

b) When $\nu_1 = 24, \nu_2 = 19$ $F_{0.01} = 2.92$

c) When $\nu_1 = 19, \nu_2 = 24$ $F_{0.95} = \dfrac{1}{F_{0.05}(24,19)} = \dfrac{1}{2.11} = 0.4739$

d) When $\nu_1 = 28, \nu_2 = 12$ $F_{0.99} = \dfrac{1}{F_{0.01}(12,28)} = \dfrac{1}{2.90} = 0.3448$ (A

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2) If two independent random samples of size $n_1 = 13$, $n_2 = 7$ are taken from a normal population, what is the probability that the variance of the first sample will be atleast four times as large as that of the second sample

Solution:-

Given: Sample sizes $n_1 = 13$, $n_2 = 7$

Degrees of freedom $\begin{cases} \nu_1 = n_1 - 1 = 13 - 1 = 12 \Rightarrow \boxed{\nu_1 = 12} \\ \nu_2 = n_2 - 1 = 7 - 1 = 6 \Rightarrow \boxed{\nu_2 = 6} \end{cases}$

Sample variances $S_1^2 = 4 S_2^2$ — ①

W.K.T. Test statistic $F = \dfrac{S_1^2}{S_2^2}$

$= \dfrac{4\cancel{S_2^2}}{\cancel{S_2^2}}$ [∵ From ①]

$\Rightarrow \boxed{F_{Cal} = 4}$ — ②

Level of significance, $\alpha = 0.05$

$\boxed{F_{tab} = 4}$ — ③

∴ From ②, ③ we get,

$F_{cal} = F_{tab}$

We conclude that,

$\boxed{\text{The required probability} = 0.05}$

(Ans)

HW

1) If two independent random samples of size $n_1 = 30$, $n_2 = 7$ are taken from a normal population, what is the probability that the variance of the first sample will be atleast 4 times as large as that of the second sample.

2) If independent random samples of size $n_1 = n_2 = 8$ come from normal population having the same variance, what is the probability that one of the sample variance will be atleast 7 times as large as the other.