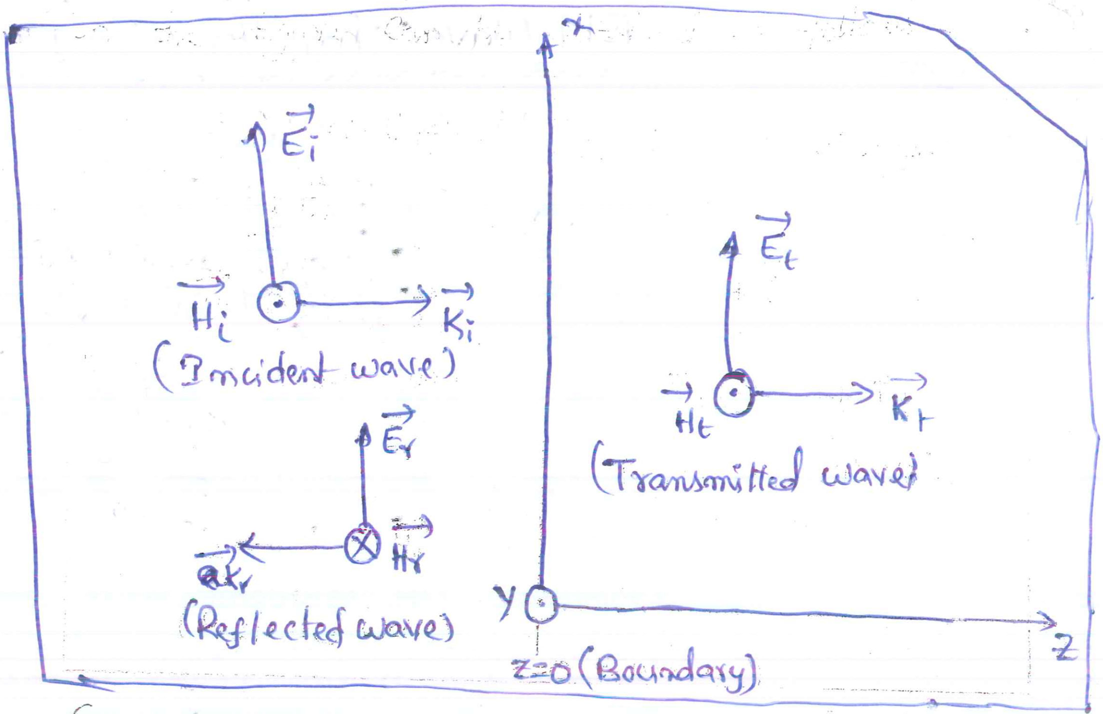
UNIT-6

EM Wave Characteristics-II

(4)

When a plane wave from one medium meets a different medium, it is partly reflected and partly transmitted. The proportion of the incident wave that is reflected or transmitted depends on the parameters ε, μ, σ of the two media involved. The incident wave can meet the boundary between two media either normally or with some angle (oblique).

Normal Incidence:- Suppose that a plane wave propagating along the $+z$ direction is incident normally on the boundary $z=0$ between medium 1 $(z<0)$ characterized by $\sigma_1, \varepsilon_1, \mu_1$ and medium 2 $(z>0)$ characterized by $\sigma_2, \varepsilon_2, \mu_2$ as shown in the below figure. In the figure, subscripts i, r and t denote incident, reflected and transmitted waves respectively.



General Case:-

Incident Wave: $(\vec{E_i}, \vec{H_i})$ is travelling along $+\vec{a_z}$ in medium 1. without Considering the time factor $e^{j\omega t}$ we can write,

$$\vec{E_i}(z) = E_{io}\, e^{-\gamma_1 z}\, \vec{a_x} \quad \text{and}$$

$$\vec{H_i}(z) = H_{io}\, e^{-\gamma_1 z}\, \vec{a_y}$$

$$= \frac{E_{io}}{\eta_1}\, e^{-\gamma_1 z}\, \vec{a_y}$$

Reflected Wave: $(\vec{E_r}, \vec{H_r})$ is travelling along $-\vec{a_z}$ in medium 1. If $\vec{E_r}(z) = E_{ro}\, e^{\gamma_1 z}\, \vec{a_x}$

then $\overrightarrow{H_r}(z) = H_{ro}\, e^{\gamma_1 z}(-\vec{a_y}) = \frac{-E_{ro}}{\eta_1} e^{\gamma_1 z} \vec{a_y}$

<u>Transmitted wave</u>: $(\overrightarrow{E_t}, \overrightarrow{H_t})$ is travelling along $+\vec{a_z}$ along medium 2. If $\overrightarrow{E_t}(z) = E_{to}\, e^{-\gamma_2 z} \vec{a_x}$ then

$$\overrightarrow{H_t}(z) = H_{to}\, e^{-\gamma_2 z} \vec{a_y}$$

$$= \frac{E_{to}}{\eta_2} e^{-\gamma_2 z} \vec{a_y}.$$

Where E_{io}, E_{ro} and E_{to} are respectively, the magnitudes of the incident, reflected and transmitted electric fields at $z=0$. The total field in medium 1 consists of both the incident and reflected fields, whereas medium 2 has only transmitted field. that is

$$\vec{E_1} = \vec{E_i} + \vec{E_r} \qquad \vec{H_1} = \vec{H_i} + \vec{H_r}$$

$$\vec{E_2} = \vec{E_t} \qquad \vec{H_2} = \vec{H_t}.$$

At the interface, $z=0$, the boundary conditions require that the tangential components of $\vec{E}$ and $\vec{H}$ fields must be continuous. Since the waves are already transverse, $\vec{E}$ and $\vec{H}$ fields both must be tangential to the interface.

Hence at $z=0$, $\vec{E_1}\tan = \vec{E_2}\tan$ and $\vec{H_1}\tan = \vec{H_2}\tan$

or interms of amplitudes or magnitudes we can write

$$E_{io} + E_{ro} = E_{to} \quad — (1)$$

$$H_{io} - H_{ro} = H_{to} \text{ or } \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2} \quad — (2)$$

Substituting equation (2) into (1), we get

$$E_{io} + E_{ro} = \eta_2\left(\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1}\right)$$

$$\frac{E_{io}}{\eta_2} + \frac{E_{ro}}{\eta_2} = \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1}$$

$$\frac{E_{io}}{\eta_2} - \frac{E_{io}}{\eta_1} = \frac{-E_{ro}}{\eta_2} - \frac{E_{ro}}{\eta_1}$$

$$E_{io}\left(\frac{1}{\eta_2} - \frac{1}{\eta_1}\right) = E_{ro}\left(\frac{-1}{\eta_2} - \frac{1}{\eta_1}\right)$$

$$\frac{E_{ro}}{E_{io}} = \frac{-(\eta_1 - \eta_2)/\eta_1\eta_2}{(\eta_1 + \eta_2)/\eta_1\eta_2} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\boxed{E_{ro} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{io}} \quad - (3)$$

Similarly Substituting, equation (1) into (2), we get

$$\frac{E_{io}}{\eta_1} - \frac{(E_{to} - E_{io})}{\eta_1} = \frac{E_{to}}{\eta_2}$$

$$\frac{E_{io}}{\eta_1} + \frac{E_{io}}{\eta_1} = \frac{E_{to}}{\eta_2} + \frac{E_{to}}{\eta_1}$$

$$\frac{2E_{io}}{\eta_1} = E_{to}\left(\frac{1}{\eta_2} - \frac{1}{\eta_1}\right)$$

$$\frac{E_{to}}{E_{io}} = \frac{2/\cancel{\eta_1}}{(\eta_1+\eta_2)/\cancel{\eta_1}\eta_2} = \frac{2\eta_2}{\eta_2+\eta_1}$$

$$\boxed{E_{to} = E_{io}\frac{2\eta_2}{\eta_2+\eta_1}} \quad - (4)$$

Now, we define the reflection coefficient, Π and trans-~~mission~~ coefficient, γ from equations (3) and (4) as

$$\boxed{\Pi = \frac{E_{ro}}{E_{io}} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}} \quad \text{or} \quad \boxed{E_{ro} = \Pi E_{io}}$$

and $$\boxed{\gamma = \frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_2+\eta_1}} \quad \text{or} \quad \boxed{E_{to} = \gamma E_{io}}$$

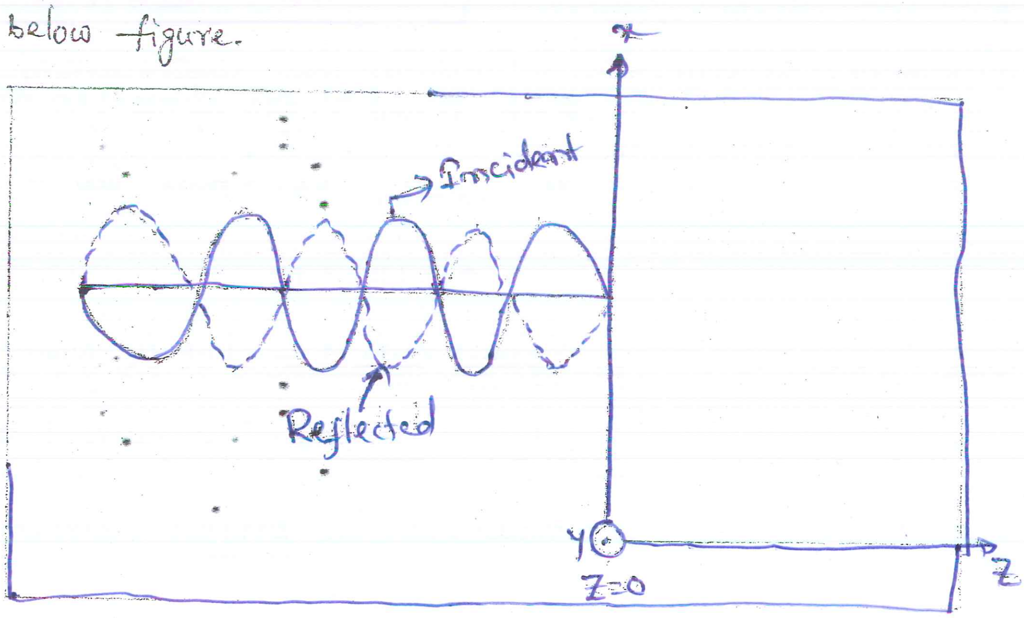
Both Π and γ are dimension less and may be Complex. the relation b/w these two is $1+\Pi=\gamma$ and Π is ranging as $0 \leq |\Pi| \leq 1$.

Special Case 1 :- Let us Now Consider a special case when medium 1 is a perfect dielectric (lossless, $\sigma_1=0$) and medium 2 is a perfect Conductor ($\sigma_2 \simeq \infty$). For this Case

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2+j\omega\epsilon_2}} = 0 \ (\because \sigma_2 \simeq \infty).$$

Hence, $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \dfrac{\eta_2\left(1 - \eta_1/\eta_2\right)}{\eta_2\left(1 + \dfrac{\eta_1}{\eta_2}\right)} = \dfrac{-\eta_1}{\eta_1} = -1 \quad (\because \eta_2 = 0)$

and $\tau = 1 + \Gamma = 1 - 1 = 0$. Showing that the wave is totally reflected. This can be expected because fields in a perfect conductor must vanish. So there can be no transmitted wave ($\vec{E}_2 = \vec{E}_t = 0$). The totally reflected wave combines with the incident wave to form a standing wave. A standing wave stands and does not travel. It consists of two travelling waves ($\vec{E}_i$ and $\vec{E}_r$) of equal amplitudes but in opposite directions as shown in below figure.



Special Case 2: when media 1 and 2 both are lossless we have another special case ($\sigma_1 = 0 = \sigma_2$). In this special case 2 again there may be two cases depending on η_1 and η_2 values.

Case A:– If $\eta_2 > \eta_1$, $\Gamma > 0$. Again there is a standing wave in medium 1 but there is also a transmitted wave in medium 2. That is the incident and reflected waves in standing wave form are not equal in magnitude.

For a standing wave we can have maxima and minima points. The maximum value of $|\vec{E}_1|$ occur at

$$-\beta_1 z_{max} = n\pi \quad (\text{integer multiples of } \pi)$$

$$\text{or} \quad z_{max} = \frac{-n\pi}{\beta_1} = \frac{-n\lambda_1}{2}, \quad n = 0, 1, 2, \cdots$$

and the minimum values of $|\vec{E}_1|$ occurs at

$$-\beta_1 z_{min} = (2n+1)\frac{\pi}{2} \quad (\text{odd multiples of } \tfrac{\pi}{2})$$

$$\text{or} \quad z_{min} = -\frac{(2n+1)\pi}{2\beta_1} = -\frac{(2n+1)}{4}\lambda_1, \quad n = 0, 1, 2, \cdots$$

Case B:- If $\eta_2 < \eta_1$, $\Gamma < 0$, In this case also there may be a standing wave in medium 1 and transmitted wave in medium2. Here also the standing wave has incident and reflected waves of unequal amplitudes and opposite sign. But only the difference is compared to case A is, the maxima and minima points will be interchanged. That is the maxima points for case B are given by $Z_{max} = -\frac{2n+1}{4}\lambda_1$, $n = 0, 1, 2, \ldots$ and the minima points for case B are given by

$$Z_{min} = \frac{-n\lambda_1}{2}, \quad n = 0, 1, 2, \ldots$$

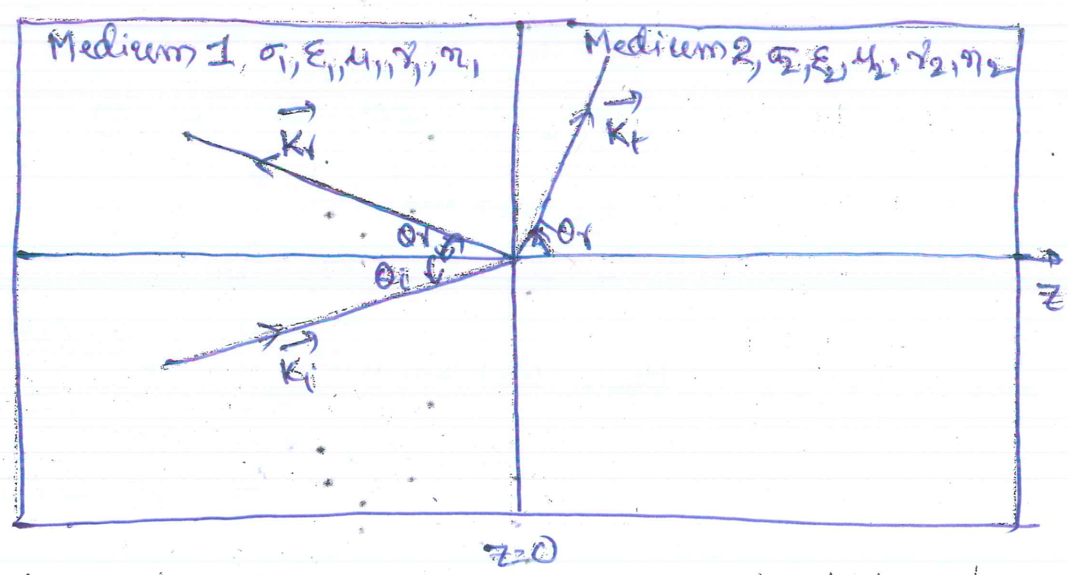
Standing wave ratio (S) :- In the special case 2 i.e. both medium 1 and medium 2 are lossless, the transmitted wave in medium 2 is purely travelling wave and there will be no maxima & minima points for travelling wave. $|\vec{H}_1|$ minimum occurs where there is $|\vec{E}_1|$ maximum and vice-versa in medium 1. The ratio of $|\vec{E}_1|_{max}$ to $|\vec{E}_1|_{min}$ or $|\vec{H}_1|_{max}$ to $|\vec{H}_1|_{min}$ is called the standing wave ratio, S

That is $S = \dfrac{|\bar{E}_1|_{max}}{|\bar{E}_1|_{min}} = \dfrac{|\bar{H}_1|_{max}}{|\bar{H}_1|_{min}} = \dfrac{1+|\Gamma|}{1-|\Gamma|}$

or $|\Gamma| = \dfrac{S-1}{S+1}$

Since $|\Gamma| \leq 1$, it follows that $1 \leq S \leq \infty$. The standing wave ratio is dimension-less and it is expressed in db's sometimes.

Oblique Incidence:- Now Consider the oblique incidence (making some angle) of a uniform plane wave at a plane boundary as shown in the below figure.



Again at oblique incidence also both incident and reflected waves are in medium 1 while transmitted or refracted wave is in medium 2. In this case the $\vec{K}$ can have all the three directions & may have 2 directions for a particular plane considered.

i.e. $\vec{K_i} = K_{ix}\vec{x} + K_{iy}\vec{y} + K_{iz}\vec{z}$ for incident wave

$\vec{K_r} = K_{rx}\vec{x} + K_{ry}\vec{y} + K_{rz}\vec{z}$ for reflected wave and

$\vec{K_t} = K_{tx}\vec{x} + K_{ty}\vec{y} + K_{tz}\vec{z}$ for transmitted or refracted wave.

Since the tangential component of $\vec{E}$ must be continuous at the boundary $z=0$

$$\vec{E_i}(z=0) + \vec{E_r}(z=0) = \vec{E_t}(z=0)$$

The only way this boundary condition will be satisfied for the xy plane ($z=0$ plane) is

1. $\omega_i = \omega_r = \omega_t = \omega$ — (1)
2. $K_{ix} = K_{rx} = K_{tx} = K_x$
3. $K_{iy} = K_{ry} = K_{ty} = K_y$. $\Big\}$ — (2)

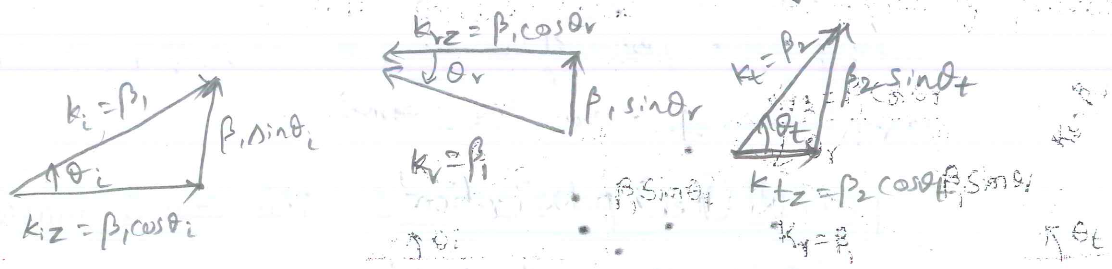
Hence, we can write $\vec{E_i} = E_{io}\cos(\omega t - K_{ix}\vec{x} - K_{iy}\vec{y} + K_{iz}\vec{z})$

$\vec{E_r} = E_{ro}\cos(\omega t - K_{rx}\vec{x} - K_{ry}\vec{y} - K_{rz}\vec{z})$.

$\vec{E_t} = E_{to}\cos(\omega t - K_{tx}\vec{x} - K_{ty}\vec{y} - K_{tz}\vec{z})$.

The tangential and normal components of $\vec{K_i}$, $\vec{K_r}$ and $\vec{K_t}$

are shown below in the form of phasors.



The above eqn (1) implies that the frequency is unchanged.

Equation (2) implies that the tangential components of the propagation vectors be continuous, thus

$$K_i \sin\theta_i = K_r \sin\theta_r$$

$$K_i \sin\theta_i = K_t \sin\theta_t$$

where θ_i is the angle of incidence, θ_r is the angle of reflection and θ_t is the angle of transmission or refraction. From these equations it is clear that $\boxed{\theta_r = \theta_i}$ also

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{K_i}{K_t} = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} \quad \text{where } \vec{u} = \frac{\omega}{K} \text{ is } M$$

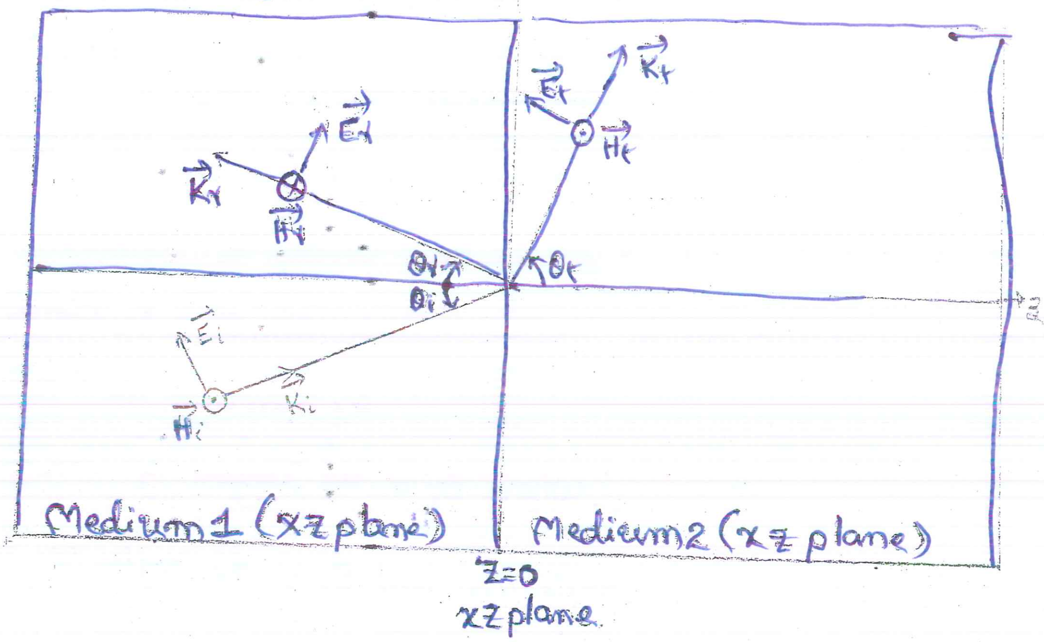
or $\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$ where $n_1 = c\sqrt{\mu_1 \epsilon_1}$ and $n_2 = c\sqrt{\mu_2 \epsilon_2}$ are refractive indices of the media and is Snell's Law.

Based on these general equations we can have two special

11 Cases, one with $\vec{E}$ parallel to the plane of incidence

and second is $\vec{E}$ ⊥lar (⊥lar polarization) to i
Any other polarization may be considered as linear combination of these two cases.

parallel (‖el) polarization :- This case is illustrated in the below figure.



The $\vec{E}$ field lies in the xz-plane, which is the plane of incidence. We have both incident and reflected fields given by

$$\vec{E}_i = E_{io}(\cos\theta_i\,\vec{a}_x - \sin\theta_i\,\vec{a}_z)\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_i = \frac{E_{io}}{\eta_1}\,e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\,\vec{a}_y$$

$$\vec{E_r} = E_{ro}(\cos\theta_r \vec{a_x} + \sin\theta_r \vec{a_z})\, e^{-j\beta(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H_r} = -\frac{E_{ro}}{\eta_1} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \vec{a_y}$$

The transmitted fields exists in medium 2 are given by

$$\vec{E_t} = E_{to}(\cos\theta_t \vec{a_x} - \sin\theta_t \vec{a_z})\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\vec{H_t} = \frac{E_{to}}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \vec{a_y}$$

where $\beta_1 = \omega\sqrt{\mu_1\epsilon_1}$ and $\beta_2 = \omega\sqrt{\mu_2\epsilon_2}$. The trick in deriving these components is to first get the polarization vector and then direction of $\vec{E}$ and $\vec{H}$ fields.

Requiring that $\theta_r = \theta_i$ and that the tangential components of $\vec{E}$ and $\vec{H}$ be continuous at the boundary $z=0$, we obtain

$$(E_{io} + E_{ro})\cos\theta_i = E_{to}\cos\theta_t$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2} E_{to}$$

Expressing E_{ro} and E_{to} in terms of E_{io}, we obtain

$$\boxed{\Gamma_{\parallel} = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}} \quad \text{or} \quad \boxed{E_{ro} = \Gamma_{\parallel} E_{io}}$$

and

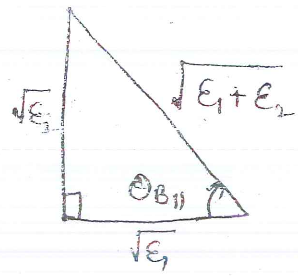
$$\boxed{\tau_{\parallel} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_r + \eta_1\cos\theta_i}} \quad \text{or} \quad \boxed{E_{to} = \tau_{\parallel} E_{io}}$$

and it is easily shown that $\boxed{1 + \Gamma_{\parallel} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)}$.

Brewster angle :- The Brewster angle is the incident angle at which $\Gamma_{\parallel} = 0$. Hence there is no reflection ($E_{ro} = 0$). And is represented as $\theta_{B\parallel}$. The Brewster angle is obtained by setting $\theta_i = \theta_{B\parallel}$ when $\Gamma_{\parallel} = 0$. That is $n_2 \cos\theta_t = n_1 \cos\theta_{B\parallel}$. The Scientist Brewster expressed this Brewster angle in terms of medium parameters also as

$$\sin\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}}$$

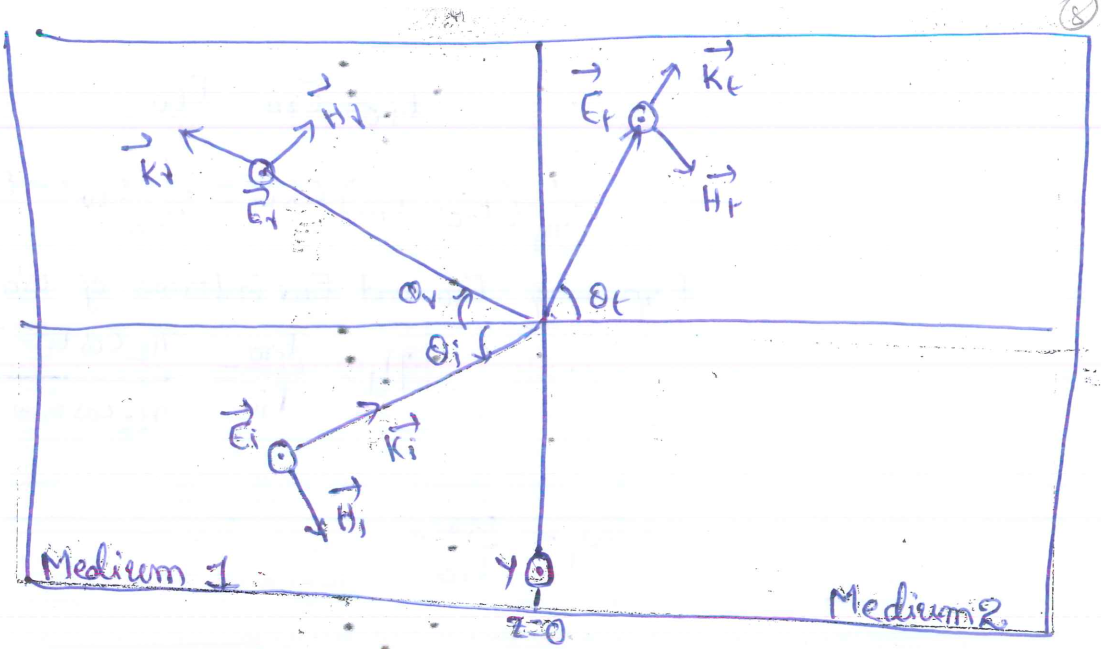
$$\cos\theta_{B\parallel} = \sqrt{\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}}$$



$$\tan\theta_{B\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1}$$

Showing that there is a Brewster angle for any combination of ε_1 and ε_2.

perpendicular polarization ($\perp$er) :- In this case, the $\vec{E}$ field is perpendicular to the plane of Incidence (xz plane) as shown in the below figure. This may also be viewed as the case where $\vec{H}$ field is parallel to the plane of incidence.



The incident and reflected fields in medium 1 are given by $\vec{E_i} = E_{io}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\,\vec{a_y}$

$$\vec{H_i} = \frac{E_{io}}{\eta_1}(\cos\theta_i\,\vec{a_x} - \sin\theta_i\,\vec{a_z})\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{E_r} = E_{ro}\, e^{-j\beta_1(x\sin\theta_r + z\cos\theta_r)}\,\vec{a_y}$$

$$\vec{H_r} = \frac{-E_{ro}}{\eta_1}(\cos\theta_r\,\vec{a_x} + \sin\theta_r\,\vec{a_z})\, e^{-j\beta_1(x\sin\theta_r + z\cos\theta_r)}$$

And the transmitted fields in medium 2 are given by

$$\vec{E_t} = E_{to}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\,\vec{a_y}$$

$$\vec{H_t} = \frac{E_{to}}{\eta_2}(\cos\theta_t\,\vec{a_x} + \sin\theta_t\,\vec{a_z})\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$$

15 The tangential Components of $\vec{E}$ and $\vec{H}$ be Continous

$$E_{io} + E_{ro} = E_{to}$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro})\cos\theta_i = \frac{1}{\eta_2} E_{to}\cos\theta_t$$

Expressing E_{ro} and E_{to} interms of E_{io} leads to

$$\boxed{\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}} \text{ or } \boxed{E_{ro} = \Gamma_\perp E_{io}}$$

and

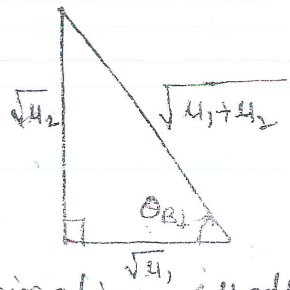
$$\boxed{\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}} \text{ or } \boxed{E_{to} = \tau_\perp E_{io}}$$

and we can show that $\boxed{1 + \Gamma_\perp = \tau_\perp}$

Brewster angle :- The Brewster angle is defined as the incident angle at which $\Gamma_\perp = 0$. that is there is no reflection ($E_{ro} = 0$). And is represented as $\theta_{B\perp}$. The Brew--ster angle is obtained by setting $\theta_i = \theta_{B\perp}$ when $\Gamma_\perp = 0$. That is $\eta_2\cos\theta_t = \eta_1\cos\theta_{B\perp}$. The Scientist Brewster expre--ssed this angle interms of medium parameters also as

$$\sin\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1+\mu_2}} \quad \text{and} \quad \tan\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1}}$$

$$\cos\theta_{B\perp} = \sqrt{\frac{\mu_1}{\mu_1+\mu_2}}$$



that is there is a Brewster angle for any combination of μ_1 and μ_2.

Poynting theorem :- States that energy can be transported from one point to another point by means of EM waves. The rate of such energy transportation can be obtained from Maxwell's equations as

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad - (1)$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad - (2)$$

Dotting both sides of equation (2) with $\vec{E}$ gives

$$\vec{E}.(\nabla \times \vec{H}) = \sigma \vec{E}^2 + \varepsilon \frac{\partial \vec{E}^2}{\partial t}$$

From the vector identity we know that

$$\nabla.(\vec{A} \times \vec{B}) = \vec{B}.(\nabla \times \vec{A}) - \vec{A}.(\nabla \times \vec{B})$$

Applying this vector identity to above equation by taking $\vec{A} = \vec{H}$ and $\vec{B} = \vec{E}$ gives

$$\nabla.(\vec{H} \times \vec{E}) = (\vec{E}. \nabla \times \vec{H}) - \vec{H}.(\nabla \times \vec{E}) \quad \text{or}$$

$$\vec{E}.(\nabla \times \vec{H}) = \nabla.(\vec{H} \times \vec{E}) + \vec{H}.(\nabla \times \vec{E}) = \sigma \vec{E}^2 + \varepsilon \frac{\partial \vec{E}^2}{\partial t}$$

$$\therefore \nabla.(\vec{H} \times \vec{E}) + \vec{H}.(\nabla \times \vec{E}) = \sigma \vec{E}^2 + \varepsilon \frac{\partial \vec{E}^2}{\partial t}$$

Substituting equation (1) into this we get

$$\nabla.(\vec{H} \times \vec{E}) + \vec{H}.\left(-\mu \frac{\partial \vec{H}}{\partial t}\right) = \sigma \vec{E}^2 + \varepsilon \frac{\partial \vec{E}^2}{\partial t} \quad \text{or}$$

$$-\nabla.(\vec{E} \times \vec{H}) - \mu \frac{\partial \vec{H}^2}{\partial t} = \sigma \vec{E}^2 + \varepsilon \frac{\partial \vec{E}^2}{\partial t}$$

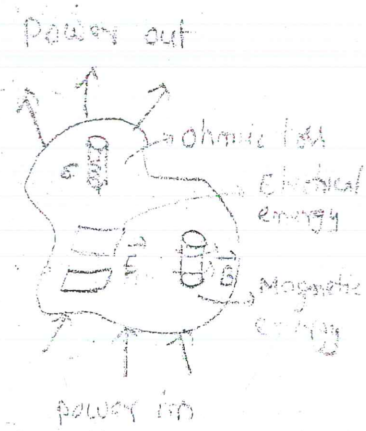
Taking Volume integral on bothsides

$$\int_V \nabla\cdot(\vec{E}\times\vec{H})\,dv = -\int_V \sigma \vec{E}^2\,dv - \frac{\partial}{\partial t}\int_V \left[\varepsilon \vec{E}^2 + \mu \vec{H}^2\right]dv$$

Applying Divergence theorem to the left hand side

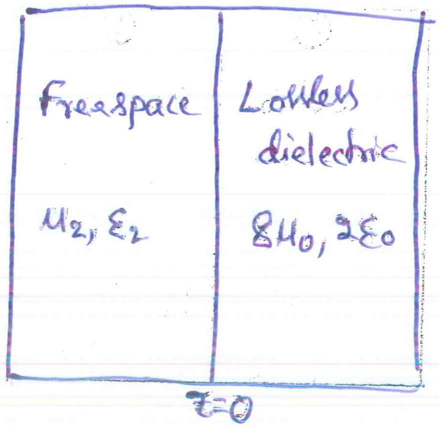
$$\boxed{\oint_S (\vec{E}\times\vec{H})\cdot d\vec{s} = -\int_V \sigma \vec{E}^2\,dv - \frac{\partial}{\partial t}\int_V (\varepsilon \vec{E}^2 + \mu \vec{H}^2)\,dv}$$

This equation is referred to as poynting's theorem. The first term on the right hand side is the ohmic power dissipated and the second term is rate of decrease in energy stored in electric and Magnetic fields. The quantity $\vec{E}\times\vec{H}$ on the left hand side is known as the poynting vector, ρ. poynting theorem states that the net power flowing out of a given volume, v is equal to the time rate of decrease in energy stored within v minus the conduction losses.



problems

① In free space ($z \leq 0$) a plane wave with magnetic field $\vec{H} = 10\cos(10^8 t - \beta z)\,\vec{a}_x$ mA/m, is incident normally on a Lossless medium ($\varepsilon = 2\varepsilon_0$, $\mu = 8\mu_0$) in region $z \geq 0$. Determine the reflected wave $\vec{H}_r$, $\vec{E}_r$ and the transmitted wave $\vec{H}_t$, $\vec{E}_t$.



Sol:- Given; Medium 1 : Free space

Medium 2 : Lossless

Incidence : Normal

$\varepsilon_2 = 2\varepsilon_0$

$\mu_2 = 8\mu_0$

$\vec{H}_i = 10\cos(10^8 t - \beta_1 z)\,\vec{a}_x$ mA/m

$\vec{a}_{ki} = \vec{a}_z$, $\vec{a}_{Hi} = \vec{a}_x$

$H_{io} = 10$, $\omega = 10^8$ (Boundary is at $z = 0$).

Required: $\vec{H}_r = ?$, $\vec{E}_r = ?$, $\vec{H}_t = ?$ and $\vec{E}_t = ?$

We know that In free space, $\beta_1 = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = \frac{1}{3}$ rad/sec

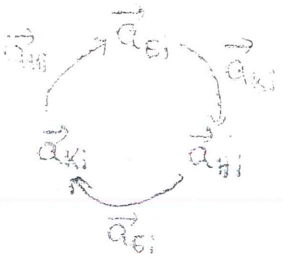
and $\eta_1 = \eta_0 = 120\pi$

In Lossless dielectric medium, $\beta_2 = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r}$

$= \frac{\omega}{c}(4) = 4\beta_1 = \frac{4}{3}$ rad/sec

and $\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 2\eta_0$

$$\vec{E}_i = E_{io}\cos(10^8 t - \beta_1 z)\,\vec{a}_{Ei}$$



$$\vec{a}_{Ei} = \vec{a}_{Hi} \times \vec{a}_{ki} = \vec{a}_x \times \vec{a}_z = -\vec{a}_y$$

and $E_{io} = \eta_1 H_{io} = 10\eta_0$

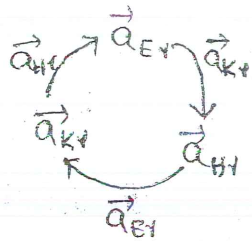
For free space there will be no change in phase

$$\therefore\ \vec{E}_i = -10\eta_0 \cos(10^8 t - \tfrac{1}{3}\cancel{\beta_1} z)\,\vec{a}_y \text{ mV/m}$$

$$\text{Now, } \Gamma = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\eta_0 - \eta_0}{2\eta_0 + \eta_0} = \frac{1}{3}$$

$$E_{ro} = \frac{1}{3}E_{io}$$

Thus $\boxed{\vec{E}_r = \frac{-10}{3}\eta_0 \cos\left(10^8 t + \frac{1}{3}z\right)\vec{a}_y \text{ mV/m}}$



from which we easily obtain $\vec{H}_r$ as

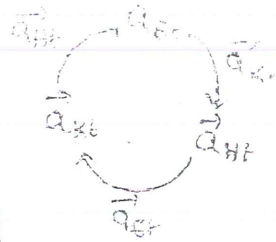
$$\boxed{\vec{H}_r = \frac{-10}{3}\cos\left(10^8 t + \frac{1}{3}z\right)\vec{a}_x \text{ mA/m}}$$

$[\vec{a}_{Hr} = \vec{a}_{kr} \times \vec{a}_{Er}$
$= -\vec{a}_z \times -\vec{a}_z$
$= -\vec{a}_x]$

Similarly,

$$\tau = \frac{E_{to}}{E_{io}} = 1 + \Gamma = \frac{4}{3} \text{ or } E_{to} = \frac{4}{3}E_{io}$$

$$\vec{a}_{Et} = \vec{a}_{Ht} \times \vec{a}_{kt} = \vec{a}_{Et} = -\vec{a}_y$$



$$\therefore \boxed{\vec{E}_t = \frac{-40}{3}\eta_0 \cos\left(10^8 t - \frac{4}{3}z\right)\vec{a}_y \text{ mV/m}}$$

$$\left[\vec{a}_{Ht} = \vec{a}_{kt} \times \vec{a}_{Et} = \vec{a}_z \times -\vec{a}_y = \vec{a}_x\right]$$

From which we obtain

$$\boxed{\vec{H}_t = \frac{20}{3}\cos\left(10^8 t - \frac{4}{3}z\right)\vec{a}_x \text{ mA/m}}$$

(2) Given a uniform plane wave in air as

$$\vec{E}_i = 40\cos(\omega t - \beta z)\vec{a}_x + 30\sin(\omega t - \beta z)\vec{a}_y \text{ V/m}$$

(a) Find $\vec{H}_i$ (b) If the wave encounters a perfectly conducting plate normal to the z-axis at $z=0$, find the reflected wave. $\vec{E}_r$ and $\vec{H}_r$ (c) What are the total $\vec{E}$ and $\vec{H}$ fields for $z \leq 0$.

Sol:- Given: Medium 1 : Air (Free space)

Medium 2 : Conducting

Incidence : Normal

Boundary is : at $z=0$

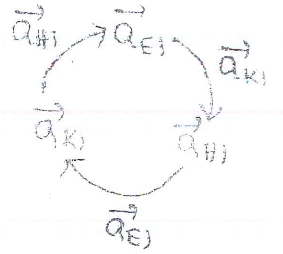
$$\vec{E}_i = 40\cos(\omega t - \beta z)\vec{a}_x + 30\sin(\omega t - \beta z)\vec{a}_y \text{ V/m.}$$

$E_{io1} = 40$, $\vec{a}_{Ei1} = \vec{a}_x$, $\vec{a}_{kEi1} = \vec{a}_z$

$E_{io2} = 30$, $\vec{a}_{Ei2} = \vec{a}_y$, $\vec{a}_{kEi2} = \vec{a}_z$

(a) Let $\vec{H_i} = \vec{H_{i1}} + \vec{H_{i2}}$

$$\vec{H_{i1}} = H_{i10} \cos(\omega t - \beta z)\, \vec{a}_{H1}$$



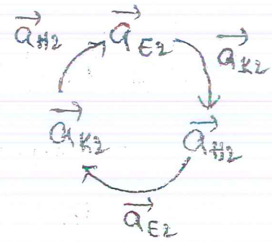
Where $H_{i10} = \dfrac{E_{i10}}{\eta_0} = \dfrac{40}{120\pi} = \dfrac{1}{3\pi}$

$$\vec{a}_{H1} = \vec{a}_{K1} \times \vec{a}_{E1} = \vec{a}_z \times \vec{a}_x = \vec{a}_y$$

$[\vec{a}_{H1} = \vec{a}_{K1} \times \vec{a}_{E1} = \vec{a}_z \times \vec{a}_x = \vec{a}_y]$

Hence $\vec{H_{i1}} = \dfrac{1}{3\pi} \cos(\omega t - \beta z)\, \vec{a}_y$

Similarly $\vec{H_{i2}} = H_{i20} \sin(\omega t - \beta z)\, \vec{a}_{H2}$



where $H_{i20} = \dfrac{E_{i20}}{\eta_0} = \dfrac{30}{120\pi} = \dfrac{1}{4\pi}$

$$\vec{a}_{H2} = \vec{a}_{K2} \times \vec{a}_{E2}$$
$$= \vec{a}_z \times \vec{a}_y = -\vec{a}_x$$

Hence, $\vec{H_{i2}} = \dfrac{-1}{4\pi} \sin(\omega t - \beta z)\, \vec{a}_x$

$$\therefore \vec{H_i} = \vec{H_{i1}} + \vec{H_{i2}}$$

$$\vec{H_i} = \frac{1}{3\pi} \cos(\omega t - \beta z)\, \vec{a}_y - \frac{1}{4\pi} \sin(\omega t - \beta z)\, \vec{a}_x \text{ A/m.}$$

(b) Since medium 2 is a perfectly Conducting, there will be a standing wave formation in medium 1.

22 i.e. $\Gamma = -1$, $\tau = 0$

showing that the incident fields are totally reflected. Since $E_{r0} = -E_{i0}$

Hence, $\vec{E}_r = -40\cos(\omega t+\beta z)\,\vec{a_x} - 30\sin(\omega t+\beta z)\,\vec{a_y}$ V/m

and $\vec{H}_r = \frac{1}{3\pi}\cos(\omega t+\beta z)\,\vec{a_y} - \frac{1}{4\pi}\sin(\omega t+\beta z)\,\vec{a_x}$ A/m

(c) The total fields in air are

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r = 40\cos(\omega t-\beta z)\,\vec{a_x} + 30\sin(\omega t-\beta z)\,\vec{a_y} - 40\cos(\omega t+\beta z)\,\vec{a_x} - 30\sin(\omega t+\beta z)\,\vec{a_y} \text{ V/m.}$$

and
$$\vec{H}_1 = \vec{H}_i + \vec{H}_r = \frac{1}{3\pi}\cos(\omega t-\beta z)\,\vec{a_y} - \frac{1}{4\pi}\sin(\omega t-\beta z)\,\vec{a_x} + \frac{1}{3\pi}\cos(\omega t+\beta z)\,\vec{a_y} - \frac{1}{4\pi}\sin(\omega t+\beta z)\,\vec{a_x} \text{ A/m.}$$

(3) A uniform plane wave in air with $\vec{E} = 8\cos(\omega t-4x-3z)\,\vec{a_y}$ V/m. is incident on a dielectric slab ($z \geq 0$) with $\mu_r = 1$, $\varepsilon_r = 2.5$, $\sigma = 0$, Find. (a) The polarization of the wave. (b) The angle of incidence (c) The reflected $\vec{E}$ field (d) The transmitted $\vec{H}$ field.

Sol:- Given; Medium 1: Air = Free Space

Medium 2: Dielectric

Incidence: oblique

Boundary is: at $z=0$

$E_{i0} = 5$, $\vec{k}_i = 4\vec{a}_x + 3\vec{a}_z$, $|\vec{k}_i| =$

$\vec{a}_{E_i} = \vec{a}_y$

$\mu_{r_2} = 1$, $\varepsilon_{r_2} = 2.5$, $\sigma = 0$.

Required: polarization = ? $\theta_i = ?$ $\vec{E}_r = ?$ $\vec{H}_t = ?$

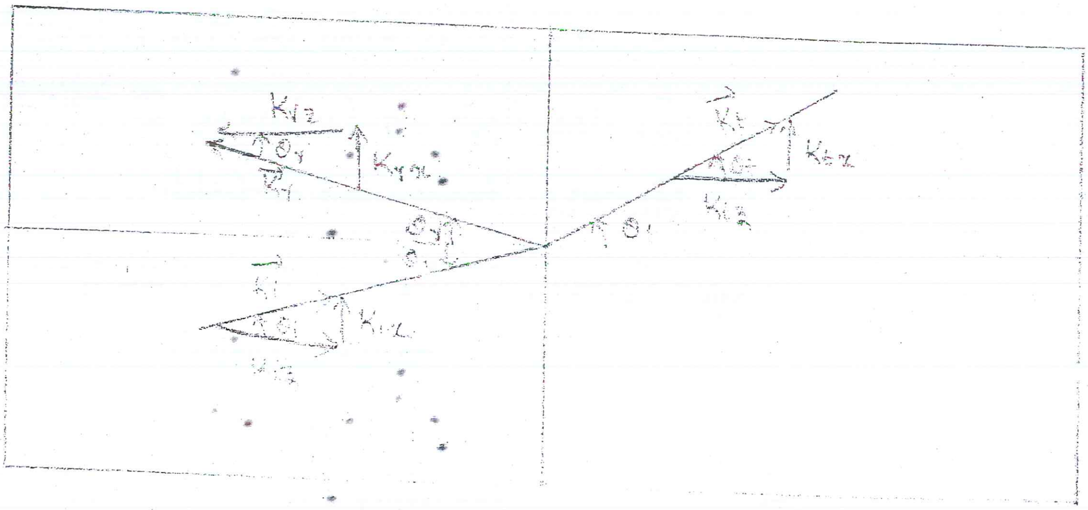
(a) From the given $\vec{E}$ field, $\vec{k}_i = 4\vec{a}_x + 3\vec{a}_z$

$$|\vec{k}_i| = \sqrt{4^2 + 3^2} = 5 = \omega\sqrt{\mu_0 \varepsilon_0} = \frac{\omega}{c}$$

or $\omega = 5c = 15 \times 10^8$ rad/sec.

A unit vector normal to the interface ($z=0$) is $\vec{a}_z$. The plane containing $\vec{k}$ and $\vec{a}_z$ is $y =$ constant, which is xz plane, the plane of incidence. Since $\vec{E}_i$ is normal to this plane, we have $\boxed{\text{perpendicular polarization}}$

(b) The propagation vectors are shown in below figure.



From the figure, $\tan\theta_i = \frac{k_{ix}}{k_{iz}} = \frac{4}{3} \rightarrow \theta_i = \tan^{-1}\left(\frac{4}{3}\right)$

$\boxed{\therefore\ \theta_i = 53.13^\circ}$

(c) In ter polarization $\vec{E}_r = E_{ro}\cos(\omega t - \vec{K}_r \cdot \vec{r})\,\vec{a}_y$

From the above figure; $\vec{K}_r = K_{rx}\vec{a}_x - K_{rz}\vec{a}_z$

$K_{rx} = K_r \sin\theta_r, \quad K_{rz} = K_r\cos\theta_r$

But $\theta_r = \theta_i$ and $K_r = K_i = 5$ because both K_r and K_i are in the same medium. Hence, $\vec{K}_r = 4\vec{a}_x - 3\vec{a}_z$

To find E_{ro} we need θ_r, From Snell's Law

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{c\sqrt{\mu_1\varepsilon_1}}{c\sqrt{\mu_2\varepsilon_2}}\sin\theta_i = \frac{\sin 53.13^\circ}{\sqrt{2.5}}$$

or $\theta_t = 30.39^\circ$

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\eta_1 = \eta_0 = 377, \quad \eta_2 = \sqrt{\frac{\mu_0\mu_{r2}}{\varepsilon_0\varepsilon_{r2}}} = \frac{377}{\sqrt{2.5}} = 238.4$$

$$\Gamma_\perp = \frac{238.4\cos 35.13^\circ - 377\cos 30.39^\circ}{238.4\cos 35.13^\circ + 377\cos 30.39^\circ} = -0.389$$

Hence, $E_{ro} = \Gamma_\perp E_{io} = -0.389(8) = -3.112$

and $\boxed{\vec{E}_r = -3.112\cos(15\times10^8 t - 4x + 3z)\,\vec{a}_y \text{ V/m}}$

(d) Similarly, the $\vec{E}_t$ will be of the form

$$\vec{E}_t = E_{to}\cos(\omega t - \vec{K}_t\cdot\vec{r})\,\vec{a}_y$$

Where $K_t =$ and $\beta_2 = \omega\sqrt{\mu_2\epsilon_2}$

$$= \frac{15\times10^8}{3\times10^8}\sqrt{1\times2.5} = 7.906$$

From the figure, $K_{tx} = K_t \sin\theta_t = 4$

$$K_{tz} = K_t \cos\theta_t = 6.819$$

or $\vec{K_t} = 4\vec{a_x} + 6.819\vec{a_z}$ (Note: $K_{ix} = K_{rx} = K_{tx}$ as expected)

$$\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} =$$

$$= \frac{2\times238.4\cos53.13^\circ}{238.4\cos53.13^\circ + 377\cos30.39^\circ}$$

$$= 0.611$$

or $\tau_\perp = 1 + \Gamma_\perp = 1 + (-0.389) = 0.611.$

$\therefore E_{to} = \tau_\perp E_{io} = 0.611\times8 = 4.888$

$$\therefore \vec{E_t} = 4.888\cos(15\times10^8 t - 4x - 6.819z)\vec{a_y}$$

From this $\vec{E_t}$, $\vec{H_t}$ will be obtained as

$$\vec{H_t} = \frac{1}{\mu_2\omega}\vec{K_t}\times\vec{E_t} = \frac{\vec{a_{kt}}\times\vec{E_t}}{\eta_2}$$

$$= \frac{4\vec{a_x} + 6.819\vec{a_z}}{7.906(238.4)}\times 4.88\vec{a_y}\cos(\omega t - \vec{K}\cdot\vec{r})$$

$$\therefore \boxed{\vec{H_t} = (-17.69\vec{a_x} + 10.37\vec{a_z})\cos(15\times10^8 t - 4x - 6.819z)\ \text{A/m}}$$