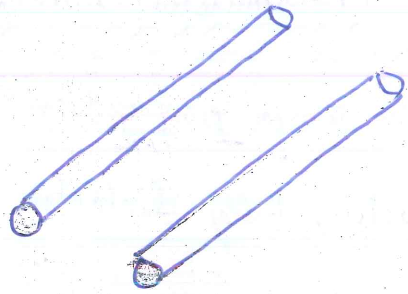
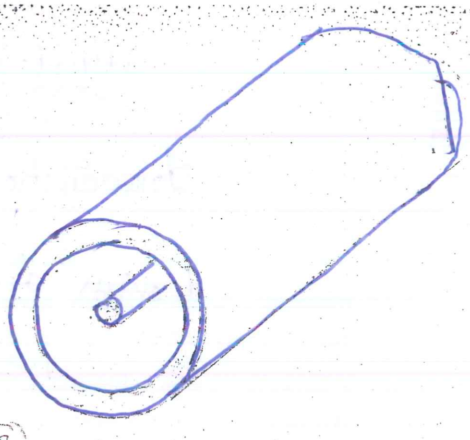
UNIT-7

Unit-7, Transmission Lines - I

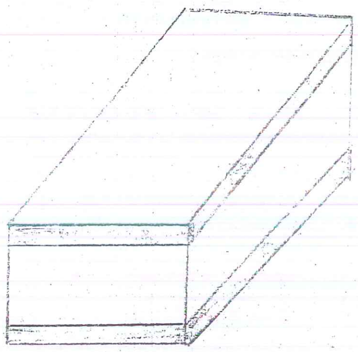
Introduction:- Wave propagation in unbounded (Free space, Lossless dielectric, Lossy dielectric and Conducting) media is used in Radio or TV broadcasting, where the infor-mation being transmitted is meant for everyone who may be interested. Such means of wave propagation will not help in a situation like telephone Conversation. where the information is received privately by one person. Another means of transmitting power or information or energy is by guided structures. Guided structures will guide or direct the propagation of energy from the source (Generator) to the Load (Receiver). Typical exam-ples of Guided structures are transmission lines and waveguides. Transmission Lines are Commonly used in power distribution and in Communications. Different types of transmission Lines used in our day to day life are listed in the following figures.



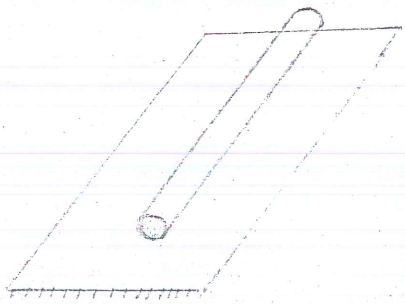
1 parallel Lines



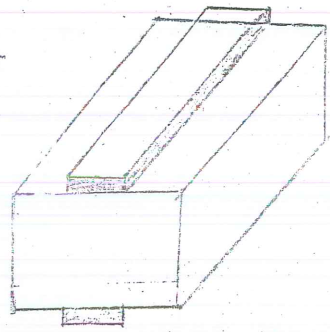
2 co-axial cable



3 parallel planes (wave Guides)



4 A Conductor on a plane



5 Microstrip line

We can see that any transmission line must consists of atleast two conductors in parallel. Any transmission line problem can be solved by using both the circuit and field theories.

2

Transmission Line parameters :- It is customary to describe a transmission line in terms of the line parameters, which are its resistance per unit length R, inductance per unit length L, Conductance per unit length, G and Capacitance per unit length C. These parameters are uniformly distributed along the entire length of the line. So these parameters can also be called as distributed or primary parameters. Any other parameters derived in terms of these primary parameters can be called as secondary parameters. Inorder to see how an EM wave propagates through a line, consider a co-axial cable connecting a generator or source to the load as shown in following figure. When switch, S is closed, the inner conductor is made positive with respect to the outer one so that the $\vec{E}$ field is radially outward as in figure ⓑ. According to Ampere's Law, the $\vec{H}$ field encircles the current carrying conductor as in figure (b)

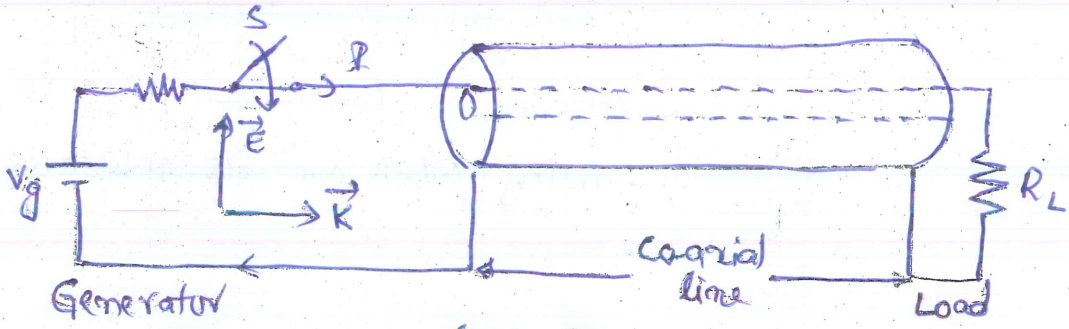


figure (a)

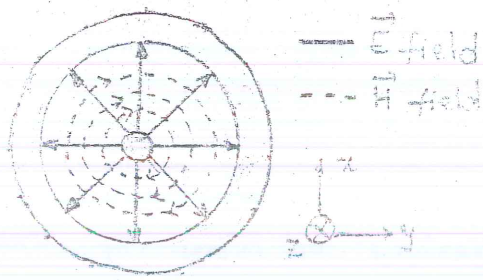


figure (b)

The poynting vector $(\vec{E} \times \vec{H})$ points along the transmission line. Thus, closing the switch simply establishes a transverse Electro Magnetic (TEM) wave propagating along the line. This wave is a non uniform plane wave and by means of it power is transmitted through the line.

<u>Transmission Line equations</u> :- Let us consider an incremental portion of the length, Δz of a two conductor transmission line. We want to find an equivalent circuit for this length, Δz, from the entire length as

shown below.

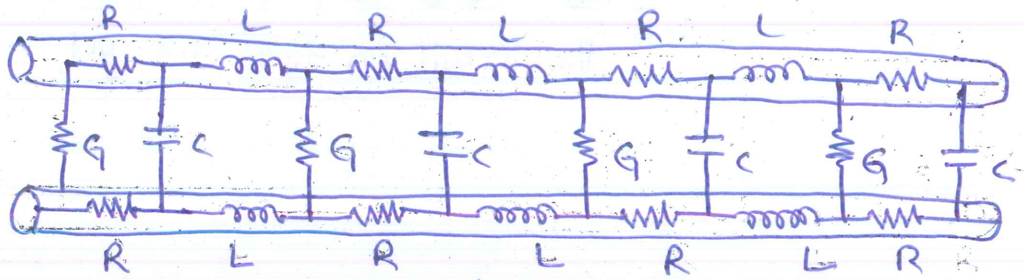


fig ⓐ

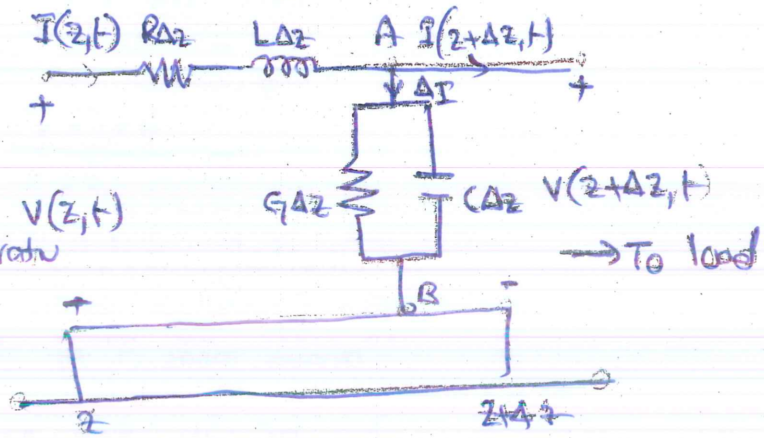


fig ⓑ

The figure ⓑ is in terms of the primary parameters R, L, G and C and may represent any of the two-conductor lines of figure ⓐ. In figure ⓑ we assume that the wave propagates along the +z direction from the generator to the load

By applying Kirchoff's voltage law to the outer loop of the circuit in fig (b), we get,

$$V(z,t) = R\Delta z \, I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z, t)$$

or $$-\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = R\,I(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

Taking the limit, as $\Delta z \to 0$. leads to

$$-\frac{\partial V(z,t)}{\partial z} = R\,I(z,t) + L\frac{\partial I(z,-)}{\partial t}$$

Similarly, applying Kirchoff's Current law to the main node A of fig (b), we get

$$I(z,t) = I(z+\Delta z,t) + \Delta I$$

$$= I(z+\Delta z,t) + G\Delta z\, V(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

as $\Delta z \to 0$

$$-\frac{\partial I(z,t)}{\partial z} = G\,V(z,t) + C\frac{\partial V(z,t)}{\partial t}$$

If we consider the exponential time factor for the voltage and currents

as $V(z,t) = Re[V_s(z)\, e^{j\omega t}]$

$I(z,t) = Re[I_s(z)\, e^{j\omega t}]$

where $V_s(z)$ and $I_s(z)$ are phasor forms of $V(z,t)$

and $I(z,t)$ respectively. That is

$$-\frac{dV_s}{dz} = (R+j\omega L)\, I_s \text{ and}$$

$$-\frac{dI_s}{dz} = (G+j\omega C)\, V_s.$$

The above two equations are coupled or dependent equations

To make them uncouple or independent, we take the

second derivatives as

$$\frac{d^2 V_s}{dz^2} = (R+j\omega L)(G+j\omega C) V_s \quad \text{or}$$

$$\boxed{\frac{d^2 V_s}{dz^2} - \gamma^2 V_s = 0} \quad \text{and Similarly} \quad \boxed{\frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0}$$

where $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

The above two equations are the wave equations for

Voltage and Current similar to $\vec{E}$ and $\vec{H}$ in fields. As we

are having the intrinsic impedance for unbounded mediums,

we can have the Characteristic impedance for the

 bounded structures in the similar way of fields.

That is $\boxed{Z_0 = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}}} = R_0 + jX_0$

Where Z_0 is the Characteristic impedance of the line.

The characteristic impedance Z_0 of the line is ratio of voltage to Current waves at any point on the line.

i.e. $Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R+j\omega L}{\gamma} = \frac{\gamma}{G+j\omega C}$.

from these equations we can easily write the other wave equations as $\boxed{\lambda = \frac{2\pi}{\beta}}$ and $\boxed{\vec{u} = \frac{\omega}{\beta} = f\lambda}$

The transmission line Considered here is a lossy type and it may be Considered as the general Case. We may have two special lines as lossless transmission line and distortionless transmission line.

<u>Lossless line $(R=G=0)$</u> :- A transmission line is said to be lossless if the Conductors of the line are perfect $(\sigma_c \simeq \infty)$ and the dielectric medium seperating

condition to be required is $R = G = 0$

Thus for a lossless line

$$\gamma = \alpha + j\beta$$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC}$$

i.e $\alpha = 0$ and $\beta = \omega\sqrt{LC}$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

$$\vec{u} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$$

$$X_0 = 0$$

finally for a lossless line, $\boxed{\alpha = 0}$ $\gamma = j\beta = j\omega\sqrt{LC}$ or

$$\boxed{\beta = \omega\sqrt{LC}}$$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{L}{C}}$$

$$\boxed{R_0 = Z_0 = \sqrt{\frac{L}{C}}}$$

$$\boxed{X_0 = 0}$$

$$\boxed{\vec{u} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda} \text{ and } \boxed{\lambda = \frac{2\pi}{\beta}}$$

Distortionless Line:– It is the line in which the attenuation constant α is frequency independent while the phase constant, β is linearly dependent on frequency.

Similar to the loss less line the necessary condition required for a distortionless line is

$\boxed{R/L = G/C}$. Thus for distortionless line,

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{R\left(1+\frac{j\omega L}{R}\right)G\left(1+\frac{j\omega C}{G}\right)} = \sqrt{RG\left(1+\frac{j\omega L}{R}\right)\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG\left(1+\frac{j\omega C}{G}\right)\left(1+\frac{j\omega C}{G}\right)} \quad \left(\because \frac{L}{R} = \frac{C}{G}\right)$$

$$= \sqrt{RG}\left(1+\frac{j\omega C}{G}\right) = \alpha + j\beta$$

or $\boxed{\alpha = \sqrt{RG}}$ i.e. independent of frequency

$\boxed{\beta = \omega\sqrt{LC}}$ dependent of frequency

and $$Z_0 = \sqrt{\frac{(R+j\omega L)}{(G+j\omega C)}} = \sqrt{\frac{R(1+j\omega L/R)}{G\left(1+\frac{j\omega C}{G}\right)}}$$

$$= \sqrt{\frac{R\left(1+\frac{j\omega C}{G}\right)}{G\left(1+\frac{j\omega C}{G}\right)}} \quad \left(\because \frac{L}{R} = \frac{C}{G}\right)$$

$$\boxed{Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}} = R_0 + jX_0, \ \boxed{X_0 = 0}$$

$$\boxed{\vec{u} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda}$$ and

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

$\vec{u}$ and Z_0 remains same for both lossless and distortionless lines. A lossless line can be a distortionless line. but a distortionless line is not a lossless line. Lossless lines are required in power transmission, and distortionless lines are required in telephone lines.

All Transmission Line equations at Glance:-

parameter.	Lossy (General)	Lossless	Distortionless.
$\gamma =$	$\sqrt{(R+j\omega L)(G+j\omega C)}$	~~$0+j\omega\sqrt{LC}$~~ $0+j\omega\sqrt{LC}$	~~$\sqrt{\frac{L}{C}}+j0$~~ $\sqrt{RG}+j\omega\sqrt{LC}$
$Z_0 =$	$\sqrt{\frac{R+j\omega L}{G+j\omega C}}$	$\sqrt{\frac{L}{C}}+j0$	$\sqrt{\frac{L}{C}}+j0.$
$\vec{u} =$	$\frac{\omega}{\beta} = f\lambda =$	$\vec{u} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$	$\vec{u} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\lambda$
$\lambda =$	$\frac{2\pi}{\beta}$	$\frac{2\pi}{\beta}$	$\frac{2\pi}{\beta}.$

Input impedance & Standing wave Ratio:

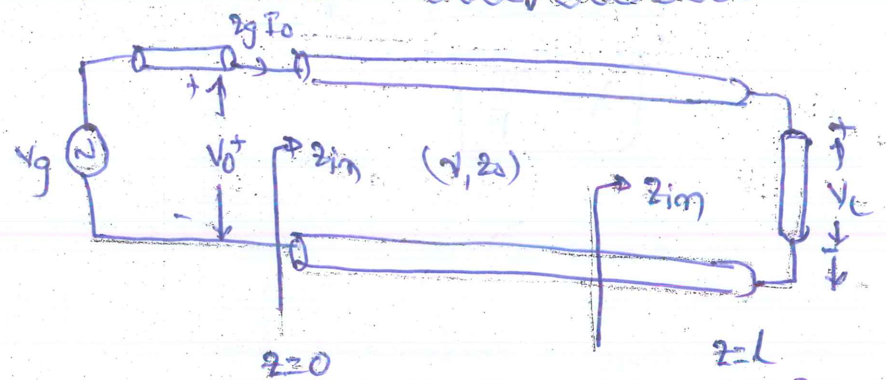


fig (a)

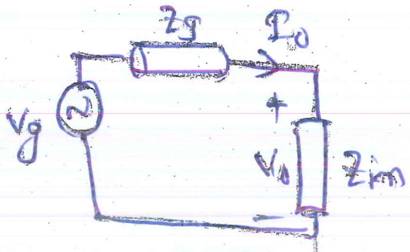


fig (b)

Consider a Transmission Line extending from $z=0$ at the generator to $z=l$ at the load. First of all, we need the voltage and current waves that is

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}.$$

To find V_0^+ and V_0^-, we take the terminal conditions as

$$V_0 = V(z=0), \quad I_0 = I(z=0).$$

Substituting these conditions in the above equation we get

$$V_0^+ = \frac{1}{2}(V_0 + Z_0 I_0)$$

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_0)$$

If the input impedance at the input terminals is Z_{in} the input voltage, V_0 and the input current I_0 are easily obtained from fig (b) as

$$V_0 = \frac{Z_{in}}{Z_{in}+Z_g} V_g, \quad I_0 = \frac{V_g}{Z_{in}+Z_g}$$

Similarly, if we take the terminal conditions at the load as $V_L = V(z=l)$, $I_L = I(z=l)$

Substituting these conditions into above equations, we get

$$V_0^+ = \frac{1}{2}(V_L + Z_0 I_L) e^{\gamma l}$$

$$V_0^- = \frac{1}{2}(V_L - Z_0 I_L) e^{-\gamma l}$$

The input impedance, Z_{in} at the generator is

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_0 (V_0^+ + V_0^-)}{V_0^+ - V_0^-}$$

Substituting V_0^+ and V_0^- values into this we get

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right]} \text{ lossy}$$

where $\tanh \gamma l = \dfrac{\sinh \gamma l}{\cosh \gamma l} = \dfrac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}}$.

It is the general expression for finding Z_{in} at any point on the line.

for a lossless line, $\gamma = j\beta$, $\tanh j\beta l = j\tan\beta l$, and $Z_0 = R_0$, it becomes $\boxed{Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]}$ (lossless line)

The quantity βl is usually referred to as the electrical length of the line and can be expressed in degrees or radians.

We now define Γ_L as the voltage reflection co-efficient at the load. Γ_L is the ratio of the voltage reflection wave to the incident wave at the load, i.e.

$$\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}}$$

Substituting the values of V_0^+ and V_0^- and taking $V_L = Z_L I_L$ gives

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}.$$

The current reflection co-efficient at ~~the load~~ any point on the line is negative of voltage reflection coefficient at that point. We define the standing wave ratio s (or SWR) as

$$\boxed{s = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}}.$$

At the maxima and minima of the voltage and current

14 standing wave,

$$|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = \frac{V_{max}}{\left(\frac{V_{min}}{Z_0}\right)} = \frac{V_{max}}{V_{min}} Z_0 = S Z_0$$

$$\therefore \boxed{|Z_{in}|_{max} = S Z_0}$$

and $$|Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{V_{min}}{\left(\frac{V_{max}}{Z_0}\right)} = \frac{V_{min}}{V_{max}} Z_0 = \frac{Z_0}{S}$$

$$\boxed{\therefore |Z_{in}|_{min} = \frac{Z_0}{S}}$$

problems

① An air has characteristic impedance of $70\,\Omega$ and phase constant of 3 rad/m. at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

Sol:- Given: $Z_0 = 70\,\Omega$

$\beta = 3$ rad/m

$f = 100$ MHz.

Line = Air = lossless line, since $\sigma \simeq 0$. for air.

Required: $L = ?$, $C = ?$

We know that for a lossless line, $R = G = 0$ and $\alpha = 0$.

$$Z_0 = R_0 = \sqrt{\frac{L}{C}} \quad \text{and} \quad \beta = \omega\sqrt{LC}$$

$$\therefore C = \frac{\beta}{\omega R_0} = \frac{3}{2\pi \times 100 \times 10^6 \times 70} = \boxed{68.2\,pF/m}$$

and from $Z_0 = R_0 = \sqrt{\frac{L}{C}}$, we can write

$$L = R_0^2 C = (70)^2 (68.2 \times 10^{-12}) = \boxed{334.2\,nH/m}$$

(2) A distortionless line has $Z_0 = 60\Omega$, $\alpha = 20\,mNp/m$, $\vec{u} = 0.6c$, where c is the velocity of light in vaccum. Find R, L, G, C, and λ at $100\,MHz$.

Sol:- Given: $Z_0 = 60\Omega$

$\alpha = 20\,mNp/m$.

$\vec{u} = 0.6c$

$f = 100\,MHz$

Required: $R = ?$, $L = ?$, $G = ?$, $C = ?$ and $\lambda = ?$

We know that for a distortionless line, $RC = GL$ or

$$G = \frac{RC}{L} \text{ and } \cancel{\text{hence}} \quad Z_0 = \sqrt{\frac{L}{C}}, \quad \alpha = \sqrt{RG}$$

$$\alpha = \sqrt{R\left(\frac{RC}{L}\right)} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$$

or $R = \alpha Z_0 = (20 \times 10^{-3})(60) = \boxed{1.2\,\Omega/m}$

and also we know that, $\vec{u} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$

$$\frac{Z_0}{\vec{u}} = \frac{\sqrt{L}}{\sqrt{C}} \times \sqrt{L}\sqrt{C} = L$$

$$\frac{Z_0}{\vec{u}} = \frac{60}{0.6 \times 3\times10^8} = \frac{600}{6\times3\times10^8} = \frac{100}{3\times10^8} = \frac{10^{-1}}{3\times10^6}$$

$0.3\,\mu H$

and from $\alpha = \sqrt{RG}$ we can write

$$\alpha^2 = RG \text{ or } G = \frac{\alpha^2}{R} = \frac{400\times10^{-6}}{1.2} = \boxed{333\,\mu S/m}$$

$$\vec{u}\, Z_0 = \frac{1}{\sqrt{L}\sqrt{c}} \frac{\sqrt{L}}{\sqrt{c}} = \frac{1}{c}$$

$$c = \frac{1}{\vec{u}\, Z_0} = \frac{1}{0.6\times3\times10^8\times60} = \boxed{92.59\ pF/m}$$

$$\lambda = \frac{\vec{u}}{f} = \frac{0.6\times3\times10^8}{100\times10^6} \boxed{= 1.8\ m}$$