Unit-8, Transmission Lines – II

We Consider Special Cases when the line is Connected to a load $Z_L=0$, $Z_L=\infty$ and $Z_L=Z_0$. These special Cases can easily be derived from the general Case.

Shorted Line ($Z_L=0$):

We Know that for a lossless line,

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$Z_{sc} = Z_{in}\Big|_{Z_L=0} = Z_0\left[\frac{jZ_0 \tan\beta l}{Z_0}\right]$$

$$\boxed{\therefore Z_{sc} = jZ_0 \tan\beta l.}$$

and $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

Γ_L for SC line $= \dfrac{-Z_0}{Z_0} = -1$, $\therefore S = \dfrac{1+|\Gamma_L|}{1-|\Gamma_L|} = \infty$

i.e. $\boxed{\Gamma_L = -1}$ and $\boxed{S=\infty}$

Open Circuited Line:- ($Z_L=\infty$)

We Know that for a lossless line,

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

(1) $$Z_{oc} = Z_{in}\Big|_{Z_L=\infty} = Z_0\left[\frac{\cancel{Z_L}\left(1 + j\frac{Z_0}{Z_L}\tan\beta l\right)}{\cancel{Z_L}\left(\frac{Z_0}{Z_L} + j\tan\beta l\right)}\right]$$

$$= Z_0\left[\frac{(1+0)}{j\tan\beta l}\right] = \frac{Z_0}{j\tan\beta l} = -jZ_0\cot\beta l$$

i.e. $\boxed{Z_{oc} = -jZ_0\cot\beta l}$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\cancel{Z_L}\left(1 - \frac{Z_0}{Z_L}\right)}{\cancel{Z_L}\left(1 + \frac{Z_0}{Z_L}\right)} = \frac{1}{1} = 1, \text{ and } S = \frac{1+1}{1-1} = \infty$$

i.e. $\boxed{\Gamma_L = 1}$ and $\boxed{S = \infty}$

$$Z_{sc}Z_{oc} = jZ_0\tan\beta l \cdot \frac{Z_0}{j\tan\beta l} = Z_0^2$$

$$\boxed{\therefore Z_0 = \sqrt{Z_{sc}Z_{oc}}}$$

Matched Line :-

We know that for a lossless line,

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$$

$$Z_{match} = Z_{in}\Big]_{Z_L = Z_0} = Z_0\left[\frac{Z_0 + jZ_0\tan\beta l}{Z_0 + jZ_0\tan\beta l}\right] = Z_0.$$

$$\therefore \boxed{Z_{in} = Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0, \quad \therefore S = \frac{1+0}{1-0} = 1$$

$$\therefore \boxed{\Gamma_L = 0} \text{ and } \boxed{S = 1}$$

This is the most desired case from the practical point of view. For a matched line there is no reflection since $\Gamma_L = 0$. and Maximum power can be transferred.

problem

(1) A certain transmission line operating at $\omega = 10^6$ rad/s, has $\alpha = 8$ dB/m, $\beta = 1$ rad/m, and $Z_0 = 60 + j40\,\Omega$ and is 2m long. If the line is connected to a source of $10\angle 0^\circ$ V, $Z_g = 40\,\Omega$ and terminated by a load of $20 + j50\,\Omega$.

Determine, (a) The input impedance

(b) The sending end current

(c) The current at the middle of the line.

Sol:- Given; $\omega = 10^6$ rad/s, $\alpha = 8$ dB/m, $\beta = 1$ rad/m, $Z_0 = 60 + j40\,\Omega$.

$l = 2m$, $V_g = 10\angle 0^\circ$, $Z_g = 40\,\Omega$, $Z_L = 20 + j50\,\Omega$ and the line is lossy or general.

Required: $Z_{in} = ?$, $I|_{at\ l=0} = ?$, $I|_{at\ l=1m} = ?$

(a) We know that $1\,Np = 8.686\,dB$,

$$\therefore \alpha = \frac{8}{8.686} = 0.921\ Np/m.$$

$$\gamma = \alpha + j\beta = 0.921 + j1/m$$

$$\gamma l = 2(0.921 + j1) = 1.84 + j2$$

using the formula $\tanh(x + jy) = \dfrac{\sinh 2x}{\cosh 2x + \cos 2y} + j\dfrac{\sin 2y}{\cosh 2x + \cos 2y}$

we obtain $\tanh \gamma l = 1.033 - j0.03929$

$$Z_{in} = Z_0\left(\frac{Z_L + Z_0\tanh\gamma l}{Z_0 + Z_L\tanh\gamma l}\right)$$

$$= (60 + j40)\left(\frac{20 + j50 + (60 + j40)(1.033 - j0.03929)}{60 + j40 + (20 + j50)(1.033 - j0.03929)}\right)$$

$$\boxed{Z_{in} = 60.25 + j38.79\,\Omega = 71.66\angle 32.77^\circ}$$

(b) The sending end current is $I(z=0) = I_0$.

We know that $I(z=0) = \dfrac{V_g}{Z_{in} + Z_g} = \dfrac{10}{60.25 + j38.79 + 40}$

$$\therefore \boxed{I(z=0) = 93.03\angle -21.15^\circ \text{ mA}}$$

(c) To find the current at any point, we need V_0^+ and V_0^-. But

$$I_0 = I(z=0) = 93.03\angle -21.15^\circ \text{ mA}$$

$$V_0 = Z_{in} I_0 = (71.66\angle 32.77^\circ)(0.09303\angle -21.15^\circ) = 6.667\angle 11.62^\circ \text{ V}$$

$$\therefore V_0^+ = \frac{1}{2}(V_0 + Z_0 I_0) = \frac{1}{2}\left(6.667\angle 11.62^\circ + (60 + j40)(0.09303\angle -21.15^\circ)\right)$$

$$= 6.687\angle 12.08^\circ$$

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_0) = 0.0518\angle 260^\circ$$

At the middle of the line, $z = l/2$, $\gamma z = 0.921 + j1$. Hence the current at this point is $I_s(z = l/2) = \dfrac{V_0^+}{Z_0} e^{-\gamma z} - \dfrac{V_0^-}{Z_0} e^{\gamma z}$

$$= \frac{(6.687\, e^{j12.08^\circ})\, e^{-0.921 - j1}}{60 + j40} - \frac{(0.0518\, e^{j260^\circ})\, e^{0.921 + j1}}{60 + j40}$$

$$I_s(z = l/2) = \frac{6.687\, e^{j12.08^\circ} e^{-0.921} e^{-j57.3^\circ}}{72.1\, e^{j33.69^\circ}} - \frac{0.0518\, e^{j260^\circ} e^{0.921} e^{j57.3^\circ}}{72.1\, e^{j33.69^\circ}}$$

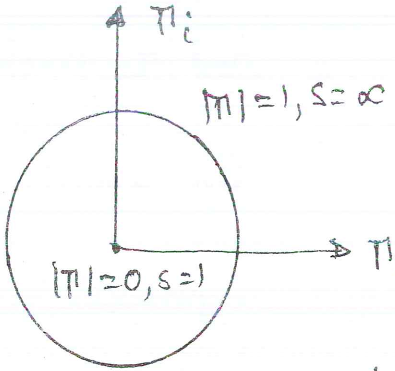
$$= 0.0369\, e^{-j78.91^\circ} - 0.001805\, e^{j283.61^\circ}$$

$$= 6.673 - j34.456 \text{ mA}$$

$$\therefore \boxed{I_s(z = l/2) = 35.10\angle 281^\circ \text{ mA}}$$

The Smith Chart :- Before the invention of digital Computers, and Calculators, engineers developped all sorts of tables, charts, graphs etc. to do Calculations for design and analysis One of them is Smith chart which can be used to solve transmission line problems.

Construction: The Smith Chart is Constructed with in a Circle of unit radius ($|\Gamma_L| \leq 1$) as shown in figure. The Construction of the Chart is based on the relation i.e.



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{or}$$

$$\Gamma_L = |\Gamma_L| \angle \theta_{\Gamma_L} = \Gamma_r + j\Gamma_i.$$ where Γ_r and Γ_i are real and imaginary parts of the reflection coefficient Γ.

Instead of having seperate Smith charts for different transmission lines, we can use only one chart if it is normalized. For example, the normalized impedance z_L is given by $z_L = \frac{Z_L}{Z_0} = r + jx$

i.e for every load impedance Z_L there are two Circles one for real part other for imaginary part. All Circles

Corresponding to real part are Complete circles where as all Circles corresponding to imaginary part are not Complete as shown in the figures (a) & (b)

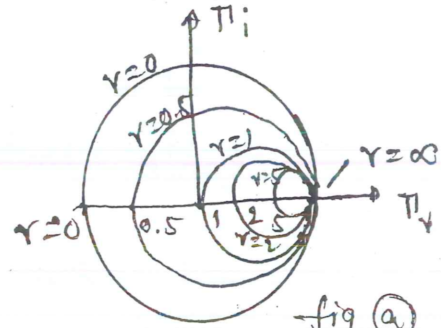


fig (a)

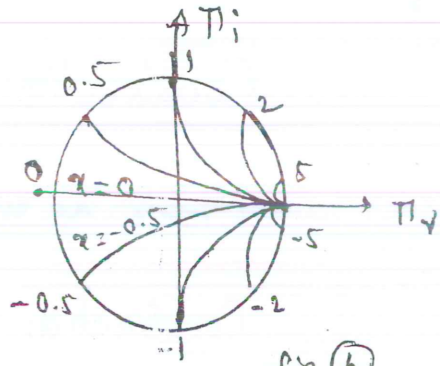


fig (b)

If we super impose the r-circles and x-circles, what we have is the smith chart as shown in figure (c). on the chart, we locate normalized impedance $z_L = 2+j1$, for example, as the point of intersection of the r=2 circle and x=1 circle. Similarly $z = 1-j0.5$ is the intersection of

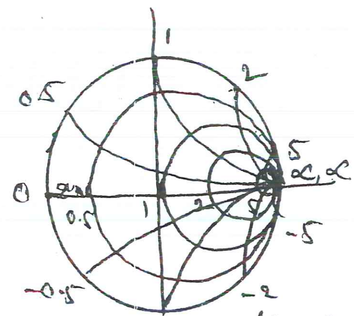


fig (c)

r=1 circle and x=-0.5 circle. Apart from the r and x-circle we can draw the s-circles or Constant standing wave ratio circles (always not shown on the chart). which are centered at the origin with s varying from 1 to ∞. The value of standing wave ratio can be determined by locating where an s-circle crosses the Γ_r axis.

While using Smith chart, the following points should be noted.

(1). The left most point on $r=0$ circle is shorted point, P_{sc} and the right most point on at $r=\infty$ circle is opend point, P_{oc}

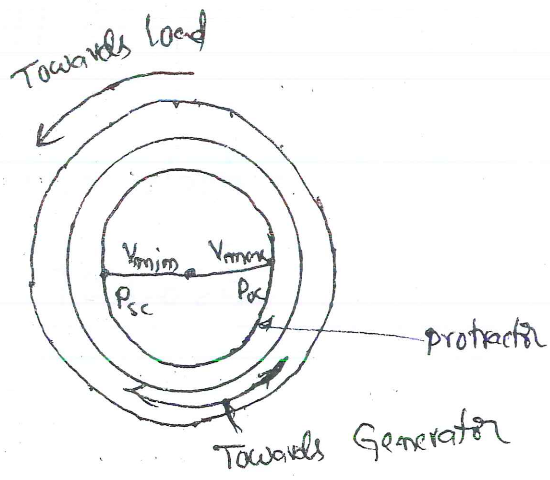
(2) A Complete revolution around the chart is equal to a distance of $\frac{\lambda}{2} = 360°$. or $\lambda = 720°$.

(3) Clock wise movement on the chart is regarded as moving towards the generator

(4) Anti clock wise movement represents the movement towards the Load.

(5) There will be 3 scales around the periphery of the chart. The outer most scale is used to determine the distance on the line from the generator end in terms of wavelengths

(6) And the next scale determines the distance from the load end interms of wavelengths.



(7) The inner most scale is a protractor (in degrees) and can be used to determine θ_{Γ_L}

(8) From origin to P_{sc} we have V_{min} point and from origin to P_{oc} we have V_{max} point

(7) (9) Smith Chart Can be used for both impedances and Admittances.

(Pb) ② A 30m-long lossless transmission line with $Z_0 = 50\Omega$ operating at 2MHz is terminated with a load $Z_L = 60+j40\Omega$. If $\vec{u} = 0.6c$ on the line find ⓐ Γ_L ⓑ S ⓒ Z_{in}.

Sol:- Given: $l = 30m$, $Z_0 = 50\Omega$, $f = 2MHz$, $Z_L = 60+j40\Omega$

$\vec{u} = 0.6 \times 3 \times 10^8$ m/s.

Required: $\Gamma_L = ?$ $S = ?$ $Z_{in} = ?$

ⓐ Inorder to use Smith chart, first we normalize the load impedance as $z_L = \frac{Z_L}{Z_0} = \frac{60+j40}{50} = 1.2+j0.8$

Locate z_L on the chart at point P where r = 1.2 circle and x = 0.8 circle meet. To get Γ_L, extend OP (O is origin) to meet r = 0 circle at Q and measure OP and OQ. Then

$$|\Gamma_L| = \frac{OP}{OQ} = \frac{3.2\,cm}{9.1\,cm} = 0.3516 \text{ (from chart)}.$$

Angle θ_{Γ_L} is read directly on the chart as the angle between OS and OP that is θ_{Γ_L} = angle POS = 56° from the chart.

Thus $\Gamma = 0.3516\angle 56^\circ$. (OS = OP on the Γ_r-axis).

ⓑ To obtain the standing wave ratio, S, draw circle with radius OP and center at O. Locate S where this circle meets Γ_r axis. The value of r at this point is S that is

$S = r = 2.1$ from the chart.

(c) To obtain Z_{in} first express l in terms of λ or in degrees.

$$\lambda = \frac{\vec{u}}{f} = \frac{0.6 \times 3\times10^8}{2\times10^6} = 90m.$$

$$l = 30m = \frac{30}{90}\lambda = \frac{\lambda}{3} \rightarrow \frac{720^\circ}{3} = 240^\circ.$$

Move toward the generator (or away from the load) in clockwise direction by 240° on the S-circle from point, P to point G. At G we obtain, $z_{in} = 0.47 + j0.035$ (from chart)

$$\therefore Z_{in} = Z_0 z_{in} = 50(0.47 + j0.035) = 23.5 + j1.75\Omega.$$

(3) A $100 + j150\Omega$ load is connected to a 75Ω lossless line. Find (a) Γ_L (b) s (c) Load Admittance, Y_L (d) Z_{in} at 0.4λ from the load (e) The location of V_{max} and V_{min} with respect to the load if the line is 0.6λ long (f) Z_{in} at the generator

Sol: Given: $Z_L = 100 + j150\Omega$, $Z_0 = 75\Omega$.

Required: $\Gamma_L = ?$, $S = ?$, $Y_L = ?$ $Z_{in}|_{at\,0.4\lambda} = ?$ $V_{max} = ?$ $V_{min} = ?|_{0.6\lambda}$

and $Z_{in}|_G = ?$

(a) To use Smith chart, first we normalize Z_L as

$$z_L = \frac{Z_L}{Z_0} = \frac{100 + j150}{75} = 1.33 + j2.$$

We locate this at point P on the chart, as the intersection point of $r = 1.33$ circle and $x = 2$ circle.

$\therefore |\Gamma_L| = \frac{OP}{OQ} = \frac{6\ cm}{9.1\ cm} = 0.659.$

$\theta_{\Gamma_L} =$ angle POS $= 40^\circ$.

$$\boxed{\therefore \Gamma = 0.659 \angle 40^\circ}$$

(b) Draw the Constant S- Circle passing through P and obtain $S = 4.82$

(c) To obtain Y_L, extend PO to POP' and mark point P' where the Constant S- circle meets POP'. At P' we obtain

$$y_L = 0.228 - j\,0.35.$$

$$\therefore Y_L = Y_0\, y_L = \frac{1}{75}(0.228 - j0.35) = \boxed{3.07 - j\,4.62\ ms}$$

(d) 0.4λ Corresponds to an angular movent of $0.4 \times 720^\circ = 288^\circ$ on the Constant S- Circle. From P, we move 288° towards generator (clock wise) on the S- Circle to reach point, R.

At R, $z_{in} = 0.3 + j0.63$

$$\therefore Z_{in} = Z_0\, z_{in} = 75(0.3 + j0.63)$$

$$\boxed{Z_{in} = 22.5 + j\,47.25\Omega}$$

(e) 0.6λ Corresponds to an angular movement of $0.6 \times 720^\circ$ $= 432^\circ =$ 1 revolution $+ 72^\circ$.

Thus we start from P. move along the S- Circle 432° and reach the generator at G. Note that to reach G

Solution:

(a) We can use the Smith chart to solve this problem. The normalized load impedance is

$$z_L = \frac{Z_L}{Z_o} = \frac{100 + j150}{75} = 1.33 + j2$$

We locate this at point P on the Smith chart of Figure 11.16. At P, we obtain

$$|\Gamma| = \frac{OP}{OQ} = \frac{6 \text{ cm}}{9.1 \text{ cm}} = 0.659$$

$$\theta_\Gamma = \text{angle } POS = 40°$$

Hence,

$$\Gamma = 0.659 \underline{/40°}$$

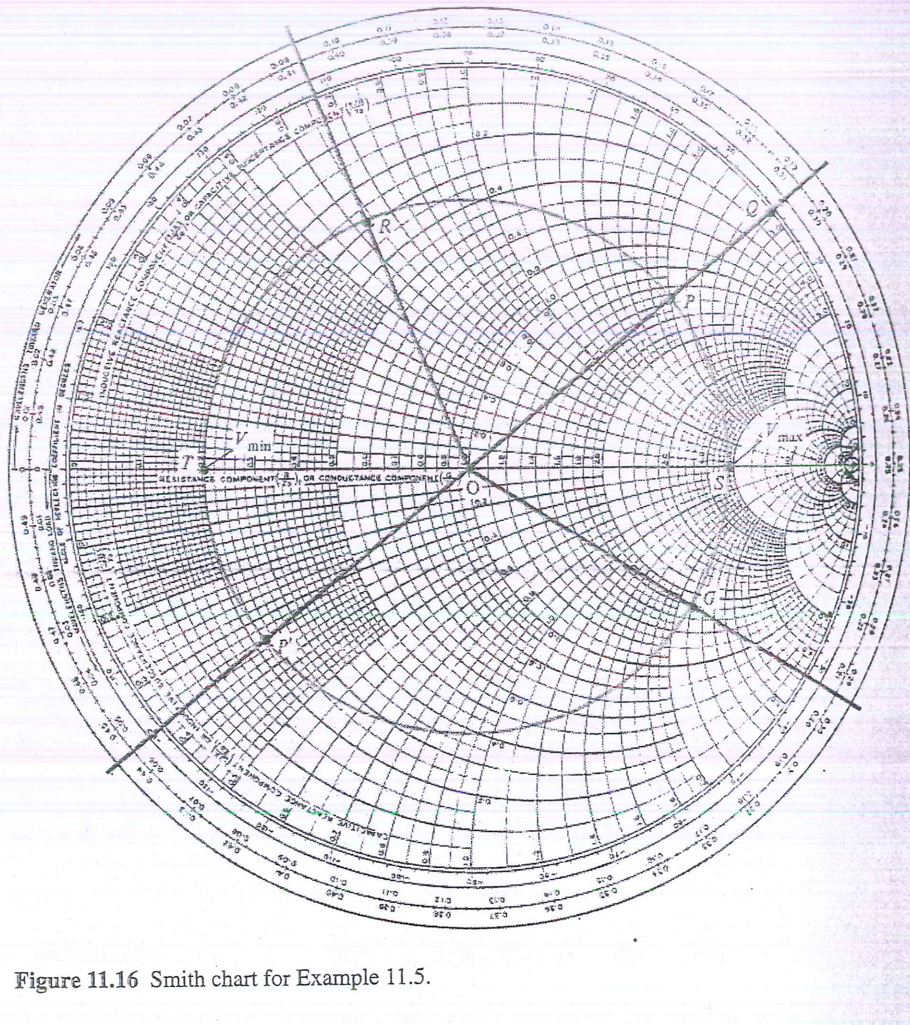


Figure 11.16 Smith chart for Example 11.5.

from P, we have passed through point T (location of V_{min}) once and point S (location of V_{max}) twice. Thus from the load.

1^{st} V_{max} is located at $\frac{40^\circ}{720^\circ}\lambda = 0.055\lambda$.

2^{nd} V_{max} is located at $0.055\lambda + \frac{\lambda}{2} = 0.555\lambda$

and the only V_{min} is located at $0.055\lambda + \frac{\lambda}{4} = 0.3055\lambda$.

(f) At G_1, $z_{in} = 1.8 + j2.2$ (from chart)

$Z_{in} = 75(1.8 - j2.2) = 135 - j165\,\Omega$.

Phase & Group velocities :– The phase velocity of the wave is the velocity of a "phase" which propagates. The group velocity is the velocity of the two or more waves Combined and travelled as a single wave. The group velocity is defined only to the Combination (super imposed) waves. whereas the phase velocity is defined for both the single and Superimposed waves. The group velocity is the velocity of the wave with low frequency, but the phase velocity is the velocity of the wave with high frequency. The phase velocity will be always greater than the group velocity.

$\frac{\lambda}{4}$ Matched line (or) Quarter wave Transformer :-

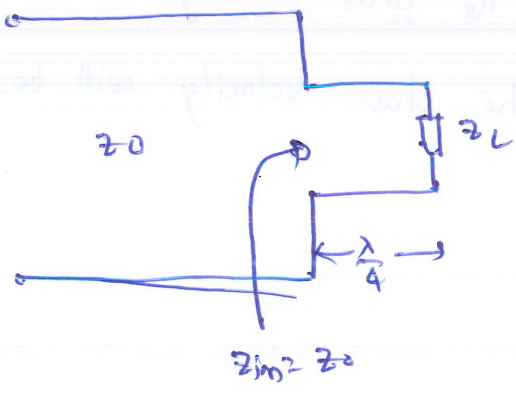
when $Z_0 \neq Z_L$, we can say that the load is mismatched and a reflected wave exists on the line. However, for maximum power transfer, it is desired that the load be matched to the transmission line ($Z_0 = Z_L$) so that there is no reflection ($|\Gamma_L| = 0$ or $S=1$). The matching can be achieved by using shorted sections of transmission Lines.

If $l = \frac{\lambda}{4}$, $\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

Then $Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\frac{\pi}{2}}{Z_0 + jZ_L \tan\frac{\pi}{2}} \right] = \frac{Z_0^2}{Z_L}$

That is $\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}$.

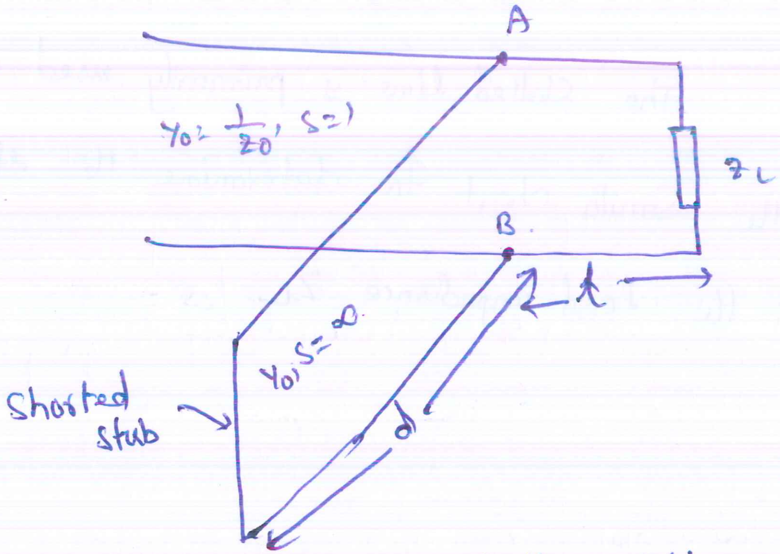
Thus by adding a $\frac{\lambda}{4}$ line on smith chart, we obtain the input admittance corresponding to a given load impedance. Due to this a $\frac{\lambda}{4}$ line is called as a matched line or Quarter wave transformer.



$Z_0' = \sqrt{Z_0 Z_L}$ [if $Z_{in} = Z_0$]

The disadvantage of quarter-wave transformer is that it is a narrow-band or frequency sensitive device.

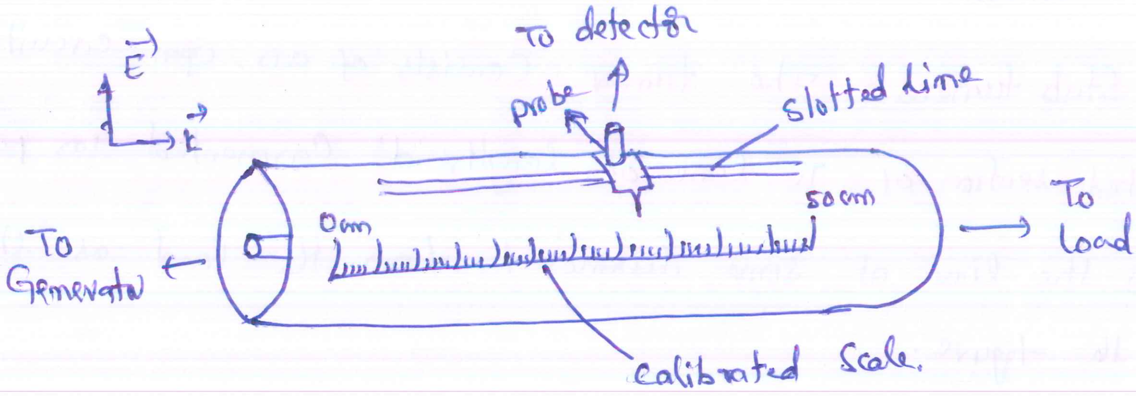
Stub Lines or Stub Tuners :- The major draw back of quarter-wave transformer can be eliminated by using the stub lines or stub tuners. The tuner consists of an open circuited or shorted section of Tx line of length 'd' connected in parallel with the line at some distance l from the load as shown in the figure.



If the matching ~~was~~ (maximum power transferred to load) was not done with a single stub then we will go for double-stub matching also.

Slotted Line (Impedance Measurement) :- The slotted line is a simple device used in determining the impedance of an unknown load at high frequencies upto the range of Giga Hertz.

It consists of a section of an air (lossless) line with a slot in the outer conductor as shown in the figure.. which samples the $\vec{E}$ field and Consequently measures the potential difference b/w the probe and its outer shield.



The Slotted line is primarily used in conjunction with the Smith chart to determine the standing wave ratio s. and the load impedance Z_L.