

Piezo-electric Transducers: A Piezo-electric material is one in which an electrical ^{potential} appears across certain surfaces of a crystal if the dimensions of the crystal are changed by the application of a mechanical force. This potential is produced by the displacement of charges. The effect is reversible, i.e. conversely, if a varying potential is applied to the proper axis of the crystal, it will change the dimensions of the crystal thereby deforming it. This effect is known as piezo-electric effect. Elements exhibiting Piezo-electric qualities are called as electro-resistive element.

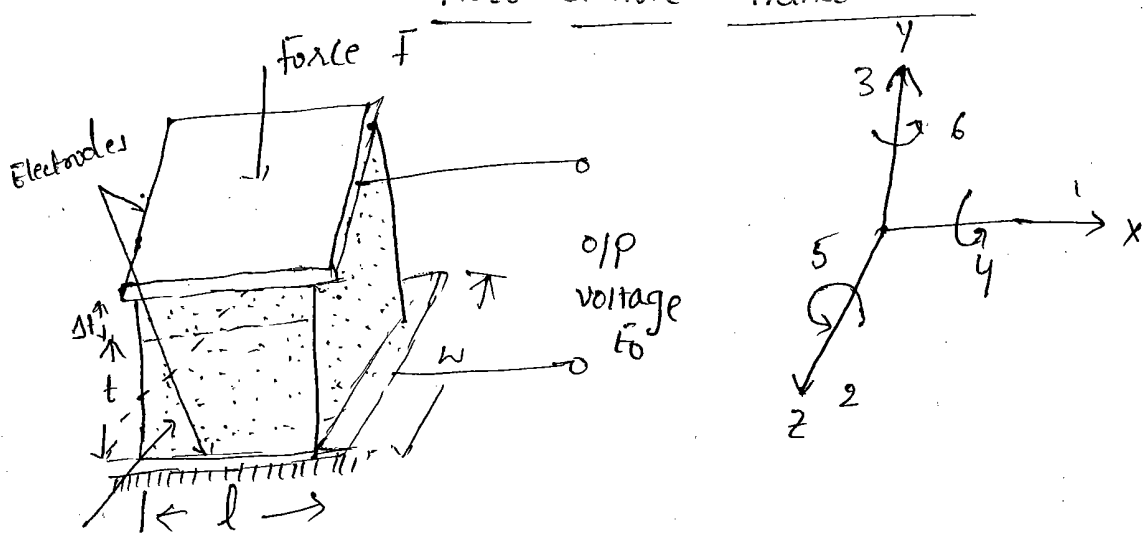
Common piezo-electric materials include Rochelle Salts, ammonium dihydrogen phosphate, lithium sulphate, dipotassium tartarate, Potassium dihydrogen phosphate, quartz and Ceramics A & B. Except for quartz and Ceramics A & B, the rest are man-made crystals & grown from aqueous solutions under carefully controlled conditions.

→ The ceramic materials are polycrystalline in nature. They are basically, made of barium titanate. They do not have piezo-electric properties in their original state but these properties are produced by special polarizing treatment.

→ The material that exhibit a significant and useful piezo-electric effect are divided into 2 categories

- ① natural group.
- ② synthetic group.

Quartz and Rochelle Salt belong to natural group while materials like lithium sulphate, ethylene diamine tartarate belong to the synthetic group.



Piezo electric crystal can be made to respond to (or cause) mechanical deformations of the material in many different modes. The mode can be: thickness expansion, transverse expansion, thickness shear and face shear. The mode of motion effected depends on the shape of the body relative to the crystal axis and location of the electrodes. A piezo-electric element used for converting mechanical motion to electrical signals may be thought as charge generator and capacitor. Mechanical deformation generates a charge and this charge appears as a voltage across the electrodes.

The voltage is $E = Q/C$

The piezo-electric crystal is shown above.

The magnitude and polarity of the induced

→ The piezo-electric effect is direction sensitive. A tensile force produces a voltage of one polarity while a compressive force produces a voltage of opposite polarity

The magnitude and polarity of the induced surface charges are proportional to the magnitude and direction of the applied force.

F. The polarity of induced charges depends upon the direction of applied force.

$$\text{charge } Q = dx F \quad \text{Coulomb} \quad \longrightarrow \textcircled{1}$$

d = charge sensitivity of the crystal

$\frac{C}{N}$: It is constant for a given crystal.

F = applied force, N.

→ The force F causes a change in thickness of the crystal.

$$F = \frac{AE}{t} \Delta t \quad \text{newton} \quad \longrightarrow \textcircled{2}$$

where A = area of crystal; m^2

t = thickness of crystal; m

E = Young's modulus, N/m^2

$$\text{Young's modulus } E = \frac{\text{Stress}}{\text{strain}} = \left(\frac{F}{A}\right) \cdot \frac{1}{\frac{\Delta t}{t}} = \frac{Ft}{A \Delta t} \quad N/m^2 \quad \longrightarrow \textcircled{3}$$

Area $A = wl$ where w = width of crystal; m

l = length of crystal; m

From eq. $\textcircled{1}$ & $\textcircled{2}$, we have,

$$\text{charge } Q = dAE \left(\frac{\Delta t}{F}\right) \quad \longrightarrow \textcircled{4}$$

The charge at the electrodes give rise to an output voltage E_0 .

$$E_0 = \frac{Q}{C_p} \quad ; \quad C_p = \text{Capacitance between the electrodes; F.} \quad \longrightarrow \textcircled{5}$$

$$C_p = \epsilon_r \epsilon_0 \frac{A}{t} \quad \longrightarrow \textcircled{6}$$

from eqns. (1), (5) & (6)

$$E_0 = \frac{Q}{C_p} = \frac{dF}{\epsilon_r \epsilon_0 \frac{A}{t}} = \frac{dt}{\epsilon_r \epsilon_0} \cdot \frac{F}{A} \rightarrow (7)$$

$\frac{F}{A} = p = \text{pressure (or stress in } N/m^2)$

$$E_0 = \frac{d}{\epsilon_r \epsilon_0} \cdot t \cdot p \rightarrow (8)$$

$$= g \cdot t \cdot p \rightarrow (9)$$

$$\text{where } g = \frac{d}{\epsilon_r \epsilon_0} \rightarrow (10)$$

where 'g' is the voltage sensitivity of the crystal.
This is constant for a given crystal cut.

Its units are Vm/N .

$$g = \frac{E_0}{t \cdot p} = \frac{E_0/t}{p} \rightarrow (11)$$

But $\frac{E_0}{t} = \text{electric field strength, } \frac{V}{m}$.

Let $E = \frac{E_0}{t} = \text{electric field}$

$$g = \frac{\text{electric field}}{\text{stress}} = \frac{E}{p} \rightarrow (12)$$

Now $\frac{E_0}{t}$ is the electric field intensity in the crystal and
p is the pressure (or the applied stress) to the crystal.
Crystal voltage sensitivity, g, can be defined as the ratio
of electric field intensity to pressure (or stress).

Now $\frac{E_0}{t} = E$ is the electric field intensity in the crystal and

The units of g are Vm/N .

from eq (10)

charge Sensitivity

$$d = \epsilon_r \epsilon_0 g \quad ; \quad \text{C/N} \quad \rightarrow \quad (13)$$

→ A quartz piezo-electric crystal having a thickness of 2mm & voltage sensitivity of 0.055 V-m/N is subjected to a pressure of 1.5 MN/m^2 . Calculate the voltage output.

If the permittivity of quartz is $40.6 \times 10^{-12} \text{ F/m}$. Calculate its charge sensitivity.

sol o/p voltage $E_o = g t p$

$$= 0.055 \times 2 \times 10^{-3} \times 1.5 \times 10^6 = 165 \text{ V}$$

charge sensitivity $d = \epsilon_0 \epsilon_r g$

$$= 40.6 \times 10^{-12} \text{ F/m} \times 0.055$$

$$= 2.23 \times 10^{-12} \text{ C/N}$$

② A piezo-electric crystal having dimensions of $5 \text{ mm} \times 5 \text{ mm} \times 1.5 \text{ mm}$ and a voltage sensitivity of 0.055 V-m/N is used for force measurement. Calculate force if the voltage developed is 100 V

sol From o/p voltage,

$$p = \frac{E_o}{g t} = \frac{100}{0.055 \times 1.5 \times 10^{-3}} \text{ N/m}^2 = 1.2 \text{ MN/m}^2$$

$$F = p \cdot A \rightarrow 1.2 \times 10^6 \times 5 \times 5 \times 10^{-6} = 30 \text{ N}$$

A barium titanate pickup has the dimensions of

$5\text{mm} \times 5\text{mm} \times 1.25\text{mm}$. The force acting on it is 5N .

The charge sensitivity of barium titanate is 150 pC/N .

and its permittivity is $12.5 \times 10^{-9}\text{ F/m}$.

→ If the modulus of elasticity of barium titanate is $12 \times 10^6\text{ N/m}^2$. Calculate the strain. Also calculate the charge and the capacitance

→ sol

Area of plates $A = 5 \times 5 \times 10^{-6} = 25 \times 10^{-6}\text{ m}^2$

Pressure $P = \frac{F}{A} \rightarrow \frac{5\text{N}}{25 \times 10^{-6}} \left[\frac{\text{N}}{\text{m}^2} \right] = 0.2 \times 10^6\text{ N/m}^2$

voltage sensitivity $g = \frac{d}{\epsilon_0 \epsilon_r} = \frac{d}{\epsilon} = \frac{150 \times 10^{-12}}{12.5 \times 10^{-9}}\text{ Vm/N}$
 $= 12 \times 10^{-3}$

voltage generated

$$E_0 = g \cdot P = 12 \times 10^{-3} \times 0.2 \times 10^6 = 3\text{V}$$

Strain $\rightarrow E = \frac{\text{Stress}}{\text{strain}} \rightarrow \frac{P}{\text{strain}} \rightarrow \text{strain} = \frac{P}{E} = \frac{0.2 \times 10^6}{12 \times 10^6}$
Young's Module.

$$\text{Strain} = 0.0167$$

Charge $\rightarrow Q = dF = 150 \times 10^{-12} \times 5 = 750\text{ pC}$

$$C_p \rightarrow \frac{Q}{E_0} \rightarrow \frac{750 \times 10^{-12}}{3\text{V}} = 250\text{ pF}$$

→ A resistance strain gauge with gauge factor of 2 is cemented to a steel member, which is subjected to a strain of 1×10^{-6} . If the original resistance value of the gauge is 130Ω , calculate the change in resistance.

$$\Rightarrow k = \frac{\frac{\Delta R}{R}}{\frac{\Delta l}{l}} \quad \therefore \Delta R = k \cdot R \cdot \frac{\Delta l}{l}$$

$$= 2 \times 130 \times 1 \times 10^{-6}$$

$$= 260 \mu\Omega$$

② An AC LVDT has the following data.

Input = 6.3V, output = 5.2V; range ± 0.5 in.

Determine

- (i) Calculate the o/p voltage vs core position for a core movement going from $+0.45$ in to -0.30 in
- (ii) The o/p voltage when the core is -0.25 in from the centre.

Sol 0.5 in core displacement produces 5.2V, therefore a 0.45 in core movement produces

$$\frac{(0.45 \times 5.2)}{0.5} = 4.68V$$

Similarly a -0.30 in core movement produces

$$\frac{(-0.30 \times 5.2)}{-0.5} = -3.12V$$

- (ii) -0.25 in core movement produces
- $$\frac{(-0.25 \times 5.2)}{-0.5}$$

