UNIT - 1

Linear waveshaping

High pass & Low pass RC circuits and their responses for sinusoidal, step, Pulse, square, ramp and exponential inputs. High pass RC circuit as Differentiator. Low pass RC circuit as integrator, Attenuators. RL and RLC circuits and their response for step input. Ringing circuit.

Linear network :-

The passive elements such as resistor, inductor and capacitor are linear, since the current through them is proportional to the voltage.

ie; there is a linear relationship between the applied voltage and the resulting current.

A network consisting of these linear elements is called as linear network.

Wave shaping :-

"It is the process of changing the shape of input signal with linear/ non-linear circuits".

Uses

1) Converting one waveform to another.
2) To clip the positive (or) negative amplitude of the waveform.
3) To hold the waveform to a certain d.c level.

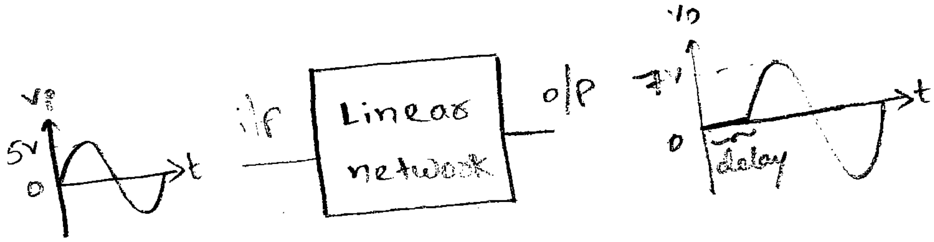
Types

1) Linear waveshaping.
2) Non-linear waveshaping.

Linear waveshaping :-

When a sinusoidal signal is applied to a transmission network consisting of linear elements, the output signal will be a true replica of the input signal except a change in amplitude & phase angle.

So, the feature of preserving waveshape in all linear network, the sinusoidal signal is unique.

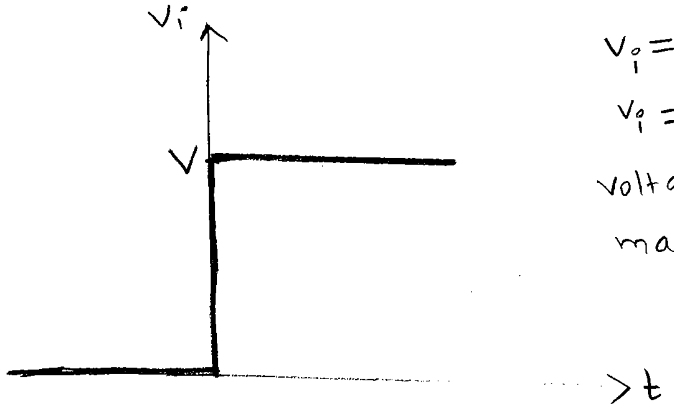


∴ "The process whereby the form of a non-sinusoidal signal is altered by transmission through a linear network is called Linear wave shaping".

The most important non-sinusoidal wave-forms are step, pulse, square wave, ramp and exponential wave-forms.

Step:

A step voltage is one which maintains the value zero for all times $t<0$ and maintains the value V for all times $t>0$.



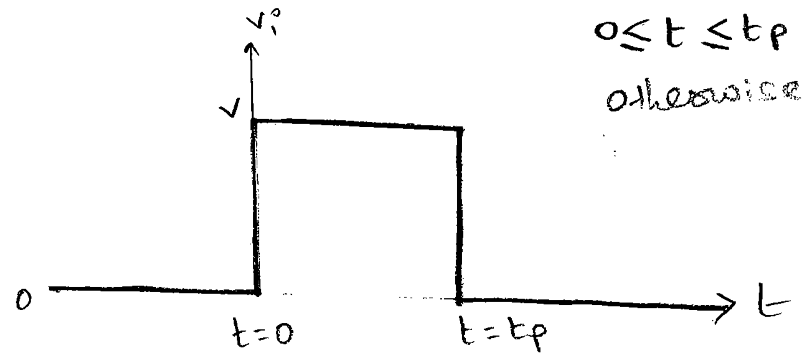
$V_i = 0, \quad t<0$

$V_i = V, \quad t>0$

voltage increases from 0 to max, $t=0$

Pulse:

An ideal pulse waveform is shown below. The pulse amplitude is 'V' and the pulse duration is t_p.

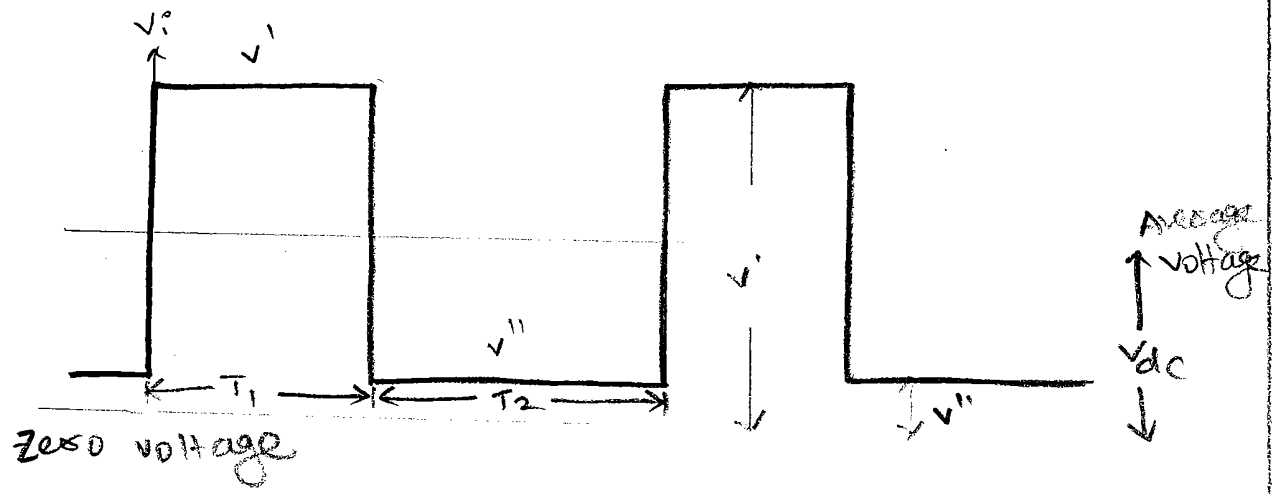


$0 \le t \le t_p, \quad V_i = V$

otherwise, $V_i = 0$

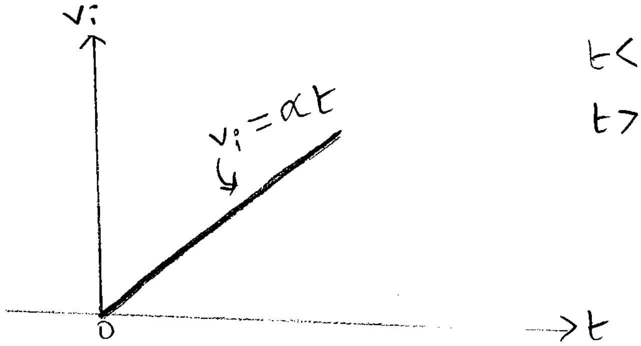
Square wave:

A waveform which maintains itself at one constant level V' for a time T_1 and at other constant level V'' for a time T_2 and which is repetitive with a period of $T = T_1 + T_2$ is called a square wave.



Ramp:

A waveform which is zero for $t<0$ and which increases linearly with time for $t>0$. is called ramp.



$t<0, \; v_i = 0$

$t>0, \; v_i$ increases linearly with time

$v_i = \alpha t$

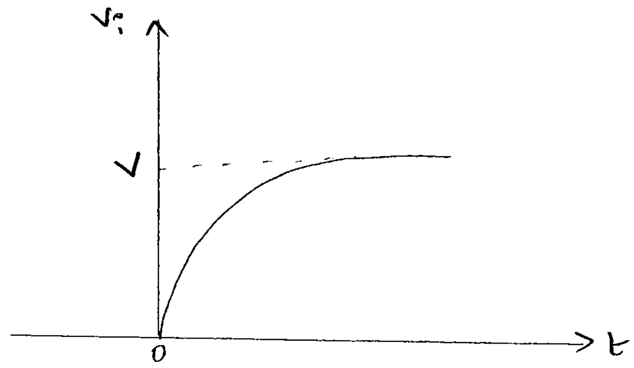
$\alpha = \frac{v_i}{t}$

Exponential signal:

The exponential input waveform is given by

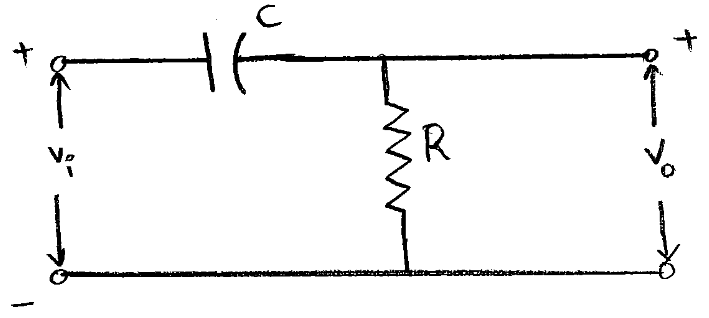
$$v_i = V(1 - e^{-t/\tau})$$

where τ = time constant of the exponential input



High-pass RC circuit

The High-pass RC circuit is shown below.



we know $X_C = \frac{1}{2\pi f C}$

"Voltage across a capacitor cannot reduce to zero instantaneously".

The reactance of the capacitor decreases with increasing frequency.

→ At higher frequencies, the capacitive reactance decreases and capacitor acts almost as a short circuit.

∴ "The higher frequency components in the input signal appear at the output with less attenuation. Due to this behaviour, the circuit is called as High pass filter."

→ At zero frequencies, the capacitor has infinite reactance and hence behaves as an open circuit.

i.e., if any D.C is present in the input voltage that is blocked by the capacitor.

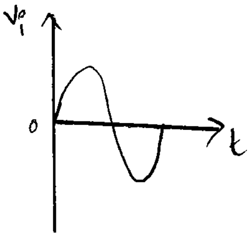
∴ capacitor C is called as blocking capacitor.

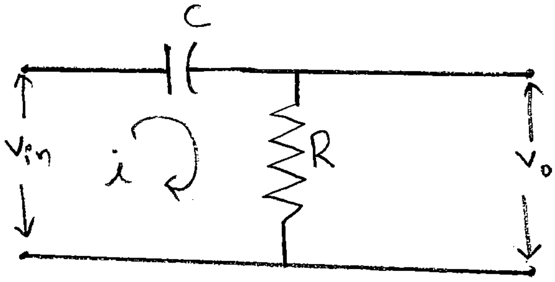
- High pass RC circuit is also called as

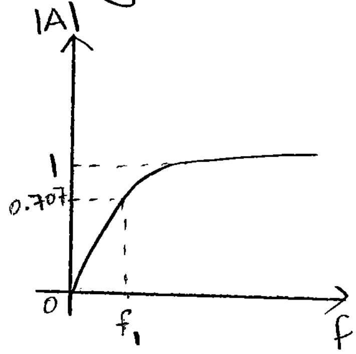
<u>Differentiator</u> (or) <u>DC isolation circuit</u>.

① <u>Sinusoidal</u> input :

for a sinusoidal input V_{in}, the output signal V_o increases in amplitude with increasing frequency.







$$V_0 = iR \quad \text{—①}$$

~~but~~ $\cancel{i = \frac{V_{in}}{R + \frac{1}{jX_c}} = \frac{V_{in}}{R + \frac{1}{j2\pi}}}$

$$\text{but } i = \frac{V_{in}}{R + jX_c} = \frac{V_{in}}{R + \frac{j}{2\pi f C}} = \frac{V_{in}}{R\left[1 + \frac{j}{2\pi f RC}\right]} \quad \text{—②}$$

sub equ ② in equ ①

$$V_0 = \frac{V_{in}}{\cancel{R}\left[1 + \frac{j}{2\pi f RC}\right]} \times \cancel{R} = \frac{V_{in}}{1 + \frac{j}{2\pi f RC}}$$

Let $f_1 = \frac{1}{2\pi RC}$

$$V_0 = \frac{V_{in}}{1 + j\frac{f_1}{f}}$$

$$A = \frac{V_0}{V_{in}} = \frac{1}{1 + j\left(\frac{f_1}{f}\right)}$$

$$|A| = \frac{1}{\sqrt{1+\left(\frac{f_1}{f}\right)^2}}$$

$$\theta = +\tan^{-1}\left(\frac{f_1}{f}\right)$$

at $f = f_1$, the magnitude of capacitive reactance is equal to the resistance

i.e; $X_C = R$

$$|A| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

$$\therefore \boxed{|A| = 0.707}$$

$$\theta = \tan^{-1} 1 = 45$$

$$\therefore \boxed{\theta = 45^\circ}$$

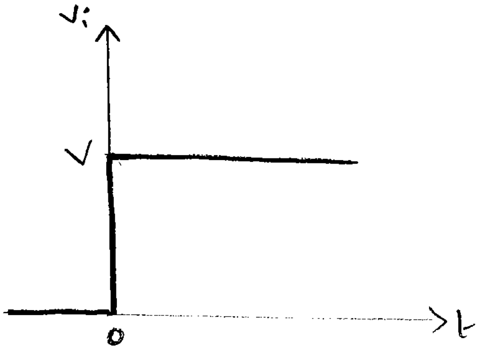
→ the gain $|A| = 0.707$ indicates signal reduction of 3 decibels (3dB).

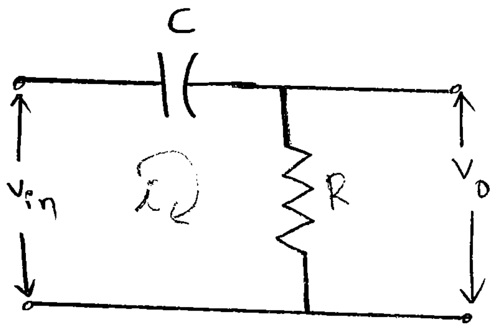
$\therefore f_1$ is called Lower 3-dB frequency.

→ The maximum possible value of the gain is unity at high frequencies.

② step input :

A step voltage is one which maintains the value zero for all times $t<0$ and maintains the value V for all times $t>0$.





The transition between the two voltages takes place at $t=0$.

$V_i=0$ immediately before $t=0$
[to be refferred as $t=0^-$]

$V_i=V$ immediately after $t=0$
[to be referred as $t=0^+$]

Apply KVL to the input side

$$V_i=\frac{1}{C}\int i\,dt+Ri$$

but $V_0=iR \Rightarrow i=\frac{V_0}{R}$

$$V_i=\frac{1}{C}\int\frac{V_0}{R}dt+V_0 \quad — ①$$

differentiate equ ① with respect to time

$$\frac{dV_i}{dt}=\frac{V_0}{RC}+\frac{dV_0}{dt}$$

$$\boxed{\frac{dV_i}{dt} = \left(\frac{1}{RC}\right)V_0 + \frac{dV_0}{dt}} \quad -(2)$$

steady state response   Transient response

Since $V_i = V$ (constant) at $t = 0^+$

$$\frac{dV_i}{dt} = \frac{dV}{dt} = 0$$

$$\therefore \frac{1}{RC}V_0 + \frac{dV_0}{dt} = 0$$

The General solution is

V_0 = complementary function + particular integral.

i) Complementary function

$$\frac{dV_i}{dt} = \frac{V_0}{RC} + \frac{dV_0}{dt}$$

let $\frac{d}{dt} = D$

$$DV_0 + \frac{V_0}{RC} = DV_i$$

$$V_0\left[D + \frac{1}{RC}\right] = DV_i \quad -(3)$$

L.H.S   R.H.S

to find Complementary function take L.H.S

$$V_0\left[D + \frac{1}{RC}\right]$$

$$D = -\frac{1}{RC} = \phi_1$$

$$C.F = K e^{+\phi_1 t} = K e^{-t/RC}$$

ii) Particular integral

to find P.I take R.H.S of eqn (3)

DV_i

$\Rightarrow \frac{d}{dt}V_i$

input is a step which is constant

$\therefore \frac{d}{dt}V = 0$

$\underline{P.I = 0}$

$V_0 = C.F + P.I$

$V_0 = Ke^{-t/RC} + 0$

$V_0 = Ke^{-t/RC}$ — (4)

To find the value of K, we have to apply initial conditions.

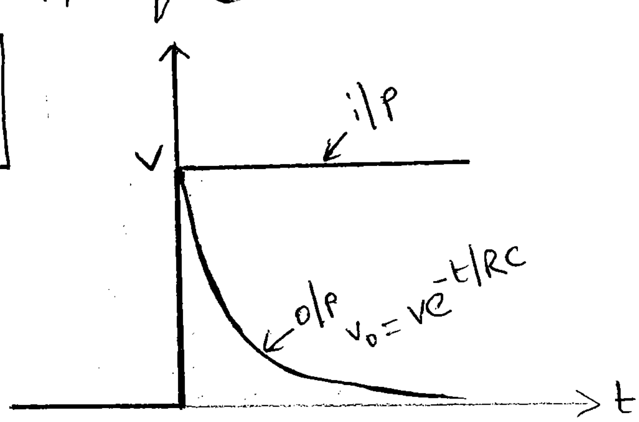
At t=0, capacitor acts as short circuit.

$V_0 = Ke^{-0/RC}$

$V_0 = K$, but $V_0(0^+) = V$ $\therefore K = V$

substitute K value in eqn (4)

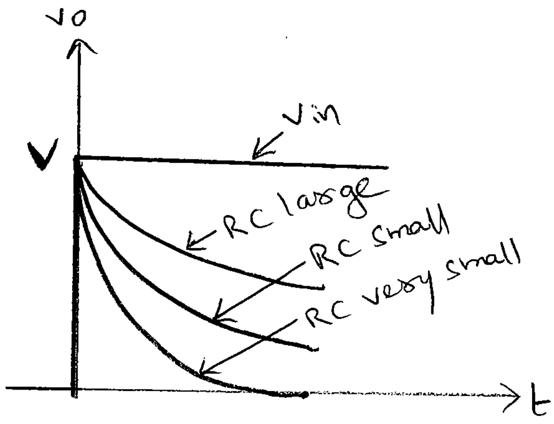
$$\boxed{V_0 = Ve^{-t/RC}}$$



$$V_0 = Ve^{-t/RC}$$

i) At $t = RC$

$$V_0 = Ve^{-RC/RC} = 0.3678V$$



ii) At $t = 2RC$

$$V_0 = Ve^{-2RC/RC} = 0.135V$$

iii) At $t = 3RC$

$$V_0 = Ve^{-3RC/RC} = 0.049V$$

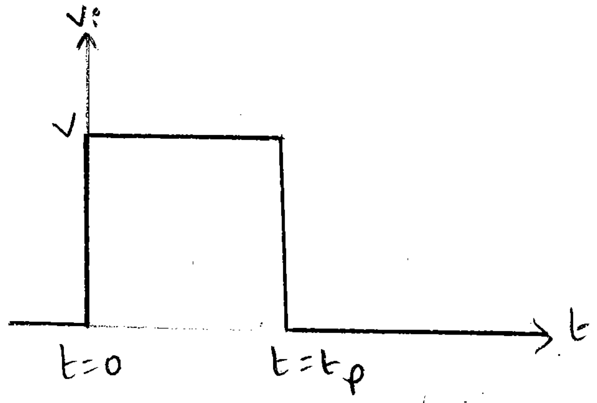
iv) At $t = 5RC$

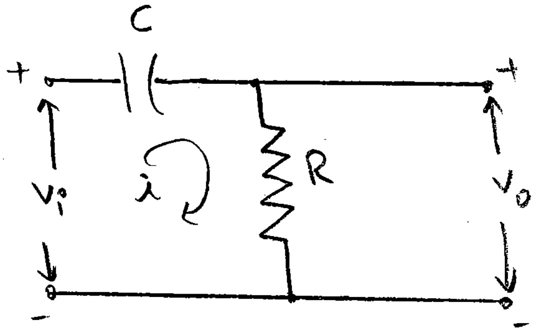
$$V_0 = Ve^{-5RC/RC} = 0.007V$$

∴ The output voltage V_0 becomes practically zero after 5 time-constants.

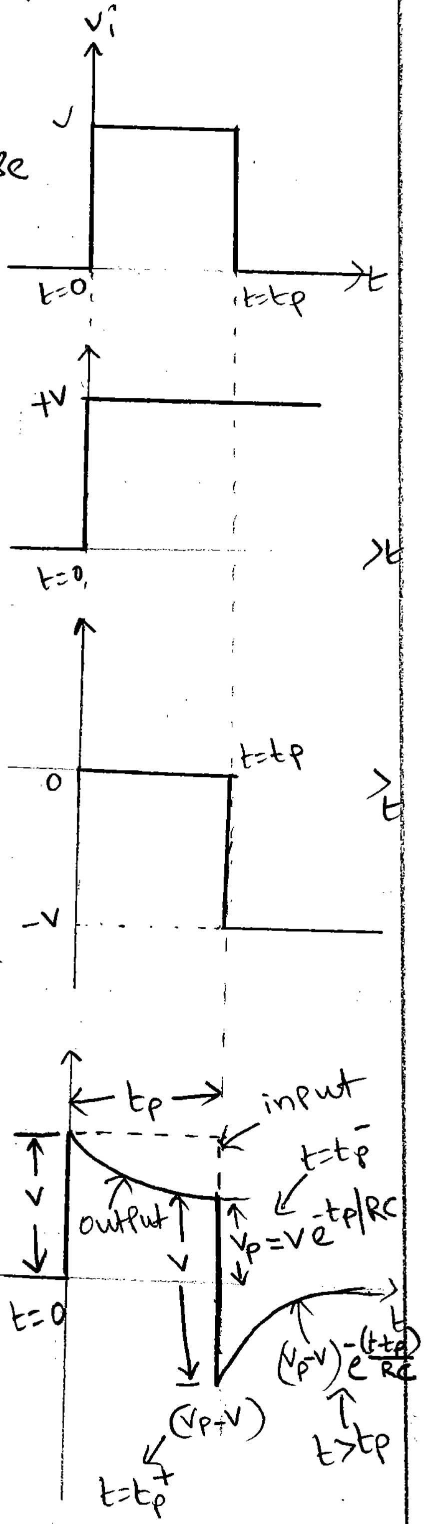
③ Pulse input:

An ideal pulse waveform is shown below. The pulse amplitude is V and pulse duration is t_p.





→ pulse may be considered to be the sum of a step voltage $+V$ volts starting at $t=0$ and a step voltage $-V$ volts starting at $t=t_p$.



i) → If the pulse is applied to the High pass RC circuit, the response for time $t<t_p$ is same as the step voltage input.

Hence the output at $t=t_p^-$ is given by

$$\boxed{V_p=V_0=Ve^{-t_p/RC}} \quad \text{—} \; ①$$

ii) → At the end of the pulse, the input falls abruptly by the amount V. For this sudden change the capacitor acts as short circuit and this change will appear at the output (the voltage across the capacitor cannot change instantaneously) so, the output also decreases by the same amount of V volts

At $\underline{t=t_p^+}$

$$V_0=V_p-V=Ve^{-t_p/RC}-V$$

$$\boxed{V_0=V\left[e^{-t_p/RC}-1\right]} \quad \text{—} \; ②$$

iii) t>tp

$$V_0 = (V_p - V)\, e^{-(t-t_p)/RC}$$

$$\boxed{V_0 = V\left(e^{-t_p/RC} - 1\right) e^{-(t-t_p)/RC}} \quad \text{—③}$$

∴ for $t > t_p$, the output rises exponentially towards zero with a time constant RC.

The response of High-pass RC circuit to a pulse if $RC \gg t_p$ and $RC \ll t_p$

→ Passing a pulse through an RC coupling network which results distortion in the output.

There is a tilt to the top of the pulse and an undershoot at the end of the pulse. If these distortions are to be minimized, if the time constant RC must be very large as compared to the pulse width tp.

→ If RC is greater means the time taken by the capacitor to charge is more and also the time taken by the capacitor to discharge is more.

→ However for all the values of ratio $\frac{RC}{t_p}$ the area below the axis will always equal to the area above.

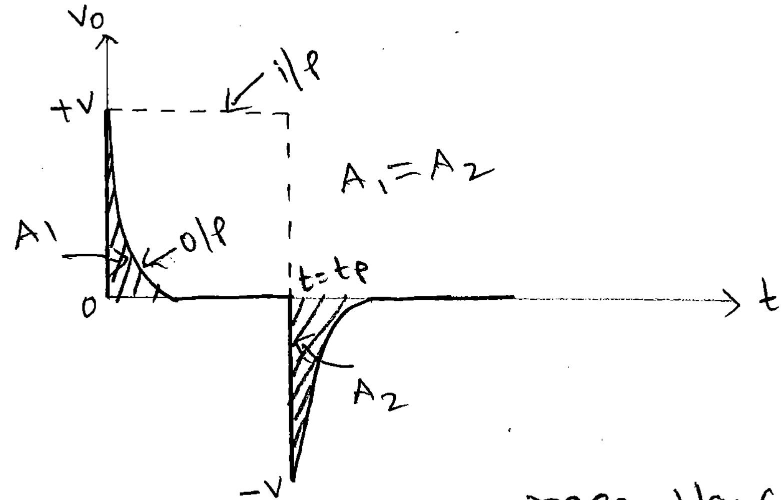
→ For High pass RC circuit the input and output are separated by the blocking capacitor C. So the d.c (or) average level of the output signal is zero for this linear circuit.

i) $RC \ll t_p$ i.e., $\frac{RC}{t_p} \ll 1$

If the time constant is very small, the output consists of a positive spike of amplitude V volts at the beginning of the pulse and a negative spike of the same size at the end of the pulse.

∴ "The process of converting pulses into spikes by means of a circuit of short time constant is called peaking".

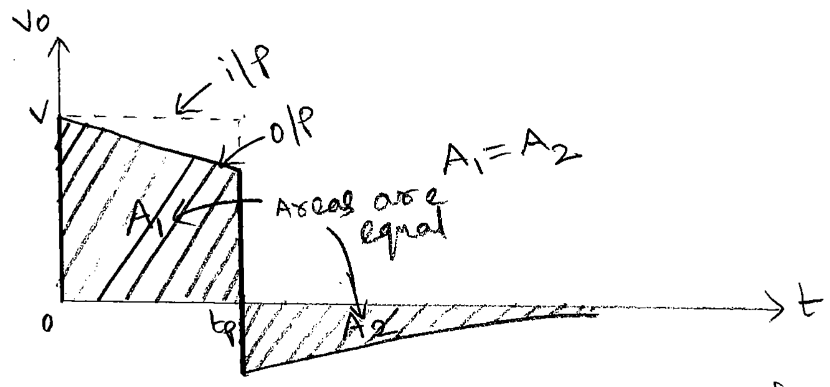
"So, the High pass circuit with a very small time constant is called peaking circuit".



The output waveform bears no resemblance to the input waveform. Smaller the time constant, larger the distortion and vice versa.

ii) $RC \gg t_p$ (or) $\frac{RC}{t_p} \gg 1$

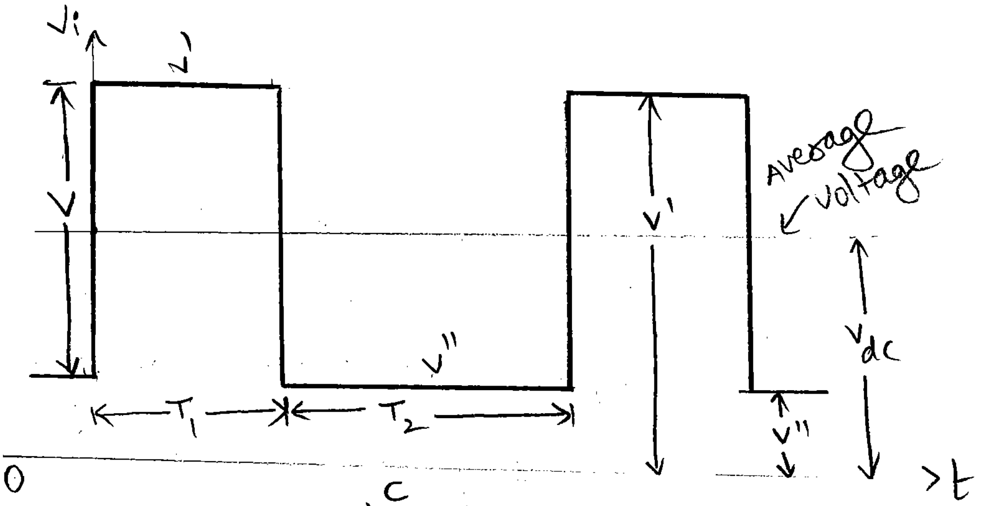
If the time constant is very large, there is only a slight tilt to the output pulse and the undershoot is very small, but for $t > t_p$, the output rises towards zero very slowly since its area must equal that of the positive portion.

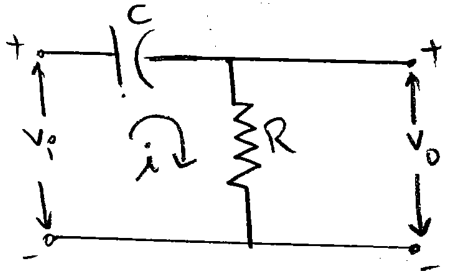


∴ In order to minimize distortion, it is clear that the time-constant of the circuit must be very large as compared to the pulse width.

④ square wave input:

A square wave is a periodic waveform which maintains itself at one constant level V' for a time T_1 and at another constant level V'' for a time T_2 and which is repetitive with a period $T = T_1 + T_2$.





Proof:-

By writing KVL at the input side

$$V_i = \frac{1}{C}\int i\,dt + iR$$

but $V_0 = iR$, $i = \frac{V_0}{R}$

$$V_i = \frac{1}{C}\int i\,dt + V_0$$

$$V_i = \frac{1}{C}\int \frac{V_0}{R}\,dt + V_0$$

differentiating the above equation with respect to 't' we get

$$\frac{dV_i}{dt} = \frac{1}{C}\times\frac{V_0}{R} + \frac{dV_0}{dt}$$

$$\frac{dV_i}{dt} = \frac{V_0}{RC} + \frac{dV_0}{dt}$$

multiplying by dt and integrating this equation over one period T we get

$$\int_0^T \frac{dV_i}{\cancel{dt}} \times \cancel{dt} = \int_0^T \frac{V_0}{RC}\, dt + \int_0^T \frac{dV_0}{\cancel{dt}} \times \cancel{dt}$$

$$\int_0^T dV_i = \frac{1}{RC}\int_0^T V_0\, dt + \int_0^T dV_0$$

$$V_i(T) - V_i(0) = \frac{1}{RC}\int_0^T V_0\, dt + V_0(T) - V_0(0)$$

under steady state conditions, the output waveform (as well as input waveform) is repetitive with a period T.

so

$$V_0(T) = V_0(0) \text{ and}$$

$$V_i(T) = V_i(0)$$

$$\therefore\ \cancel{V_i(T) - V_i(0)} = \frac{1}{RC}\int_0^T V_0\, dt + \cancel{V_0(T) - V_0(0)}$$

$$0 = \frac{1}{RC}\int_0^T V_0\, dt$$

(or)

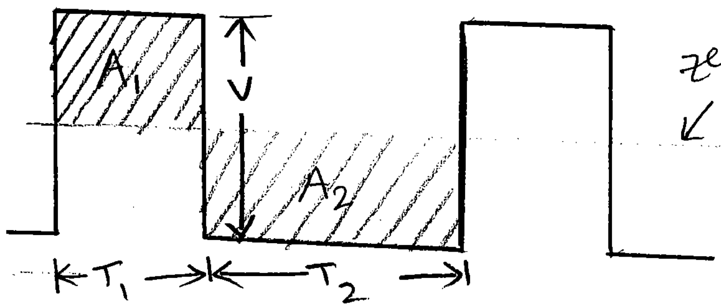
$$\boxed{\int_0^T V_0\, dt = 0}$$

$\therefore$ This integral represents the area under the output waveform over one cycle is zero.

(or)

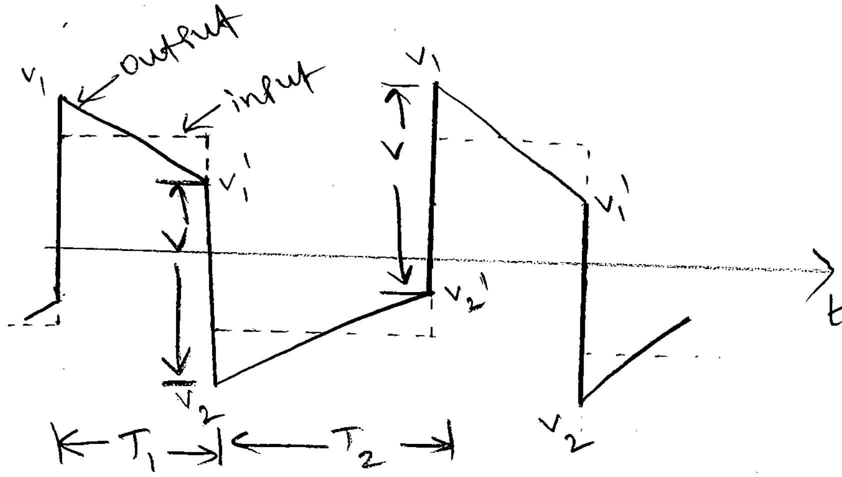
The average level (d.c component) of the output is always zero.

i) $RC >> T$ or $\frac{RC}{T} >> 1$



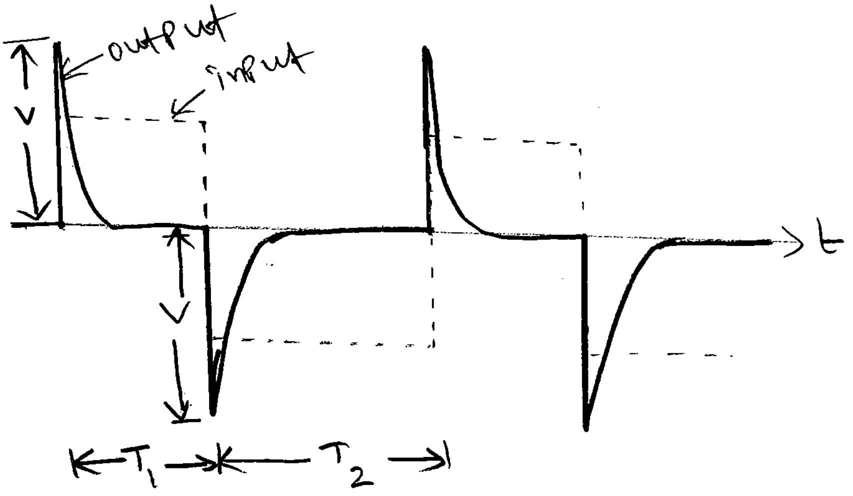
when the time constant is arbitrarily large (ie; $\frac{RC}{T_1}$ and $\frac{RC}{T_2}$ are very very large in comparision to unity) the output is same as the input but with zero d.c level.

ii) $RC > T$ or $\frac{RC}{T} > 1$



In this condition the output is in the form of a tilt.

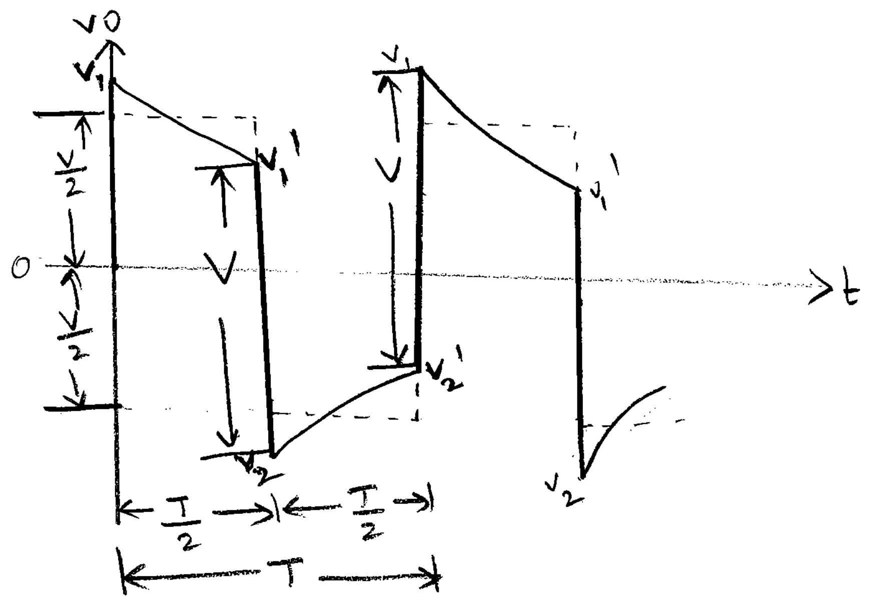
iii) $RC << T$ or $\frac{RC}{T} << 1$



if $RC << T$ (ie; $\frac{RC}{T_1}$ and $\frac{RC}{T_2}$ are very very small in comparison to unity), the output consists of alternate positive and negative spikes.

In this case, the peak-to-peak amplitude of the output waveform is twice the peak-to-peak value of the input waveform.

Expression for percentage tilt : ($RC > T$)



Consider the steady state response of a High pass RC circuit to a symmetrical square wave input.

"Tilt is defined as the decay in the amplitude of the output voltage wave, due to the input voltage maintaining a constant level."

$$\boxed{\text{percentage tilt } (P) = \frac{V_1 - V_1'}{V/2} \times 100}$$

i) At $t=0$, $V_0 = V_1$

ii) At $t>0$, $V_0 = V_1 e^{-t/RC}$

iii) At $t = T/2$, $V_0 = V_1'$

$$V_1' = V_1 e^{-T/2RC} \quad \text{——} \; (1)$$

iv) again at $t = T/2$, $V_0 = V_2'$

$$V_2' = V_2 e^{-T/2RC} \quad \text{——} \; (2)$$

$$V_1' - V_2 = V \quad \text{——} \; (3)$$

$$V_1 - V_2' = V \quad \text{——} \; (4)$$

Since input is symmetrical square wave, so

$$T_1 = T_2 = T/2$$

$$V_1 = -V_2$$

$$V_1' = -V_2'$$

substitute these in eqn (3)

$$V = V_1' - V_2$$

$$V = V_1 e^{-T/2RC} - V_2$$

$$V = V_1 e^{-T/2RC} + V_1$$

$$V = V_1\left[1 + e^{-T/2RC}\right]$$

$$\boxed{V_1 = \frac{V}{1+e^{-T/2RC}}} \quad \text{— (I)}$$

sub (I) in equ (1)

$$V_1' = V_1 e^{-T/2RC}$$

$$V_1' = \frac{V}{1+e^{-T/2RC}} \times e^{-T/2RC}$$

$$V_1' = \frac{V}{\left(1+e^{-T/2RC}\right) e^{T/2RC}} = \frac{V}{e^{T/2RC} + e^{0}}$$

$$\boxed{V_1' = \frac{V}{1+e^{T/2RC}}} \quad \text{— (II)}$$

sub (I) & (II) in percentage tilt

$$P = \frac{V_1 - V_1'}{V/2} \times 100$$

$$P = \frac{\frac{\cancel{V}}{1+e^{-T/2RC}} - \frac{\cancel{V}}{1+e^{T/2RC}}}{\cancel{V}/2} \times 100$$

$$P = \left[\frac{1}{1+e^{-T/2RC}} - \frac{1}{1+e^{T/2RC}}\right] \times 200$$

$$\frac{e^{-T/2RC}}{1+e^{-T/2RC}} = \frac{1}{(1+e^{-T/2RC})e^{T/2RC}} = \frac{1}{e^{T/2RC}+e^{0}} = \frac{1}{1+e^{T/2RC}}$$

$$P = \left[\frac{1}{1+e^{-T/2RC}} - \frac{e^{-T/2RC}}{1+e^{-T/2RC}}\right] \times 200$$

$$\boxed{P = \frac{1-e^{-T/2RC}}{1+e^{-T/2RC}} \times 200}$$

$$\boxed{P = \frac{1-e^{-1/2RCf}}{1+e^{-1/2RCf}} \times 200}$$

we know $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$

$$P = \frac{1-\left[1-\frac{T}{2RC}+\frac{\left(\frac{T}{2RC}\right)^2}{2!}-\right]}{1+\left[1-\frac{T}{2RC}+\frac{\left(\frac{T}{2RC}\right)^2}{2!}-\right]} \times 200$$

$$P = \frac{\cancel{1}-\cancel{1}+\frac{T}{2RC}}{1+1-\frac{T}{2RC}} \times 200$$

$$P = \frac{\frac{T}{2RC}}{2-\frac{T}{2RC}} \times 200$$

since $RC > T$ or $\frac{RC}{T} \gg 1 \Rightarrow \frac{T}{RC} < 1$ so $\frac{T}{2RC} < 1$

we can neglect

$$P = \frac{\frac{T}{2RC}}{\cancel{2}} \times \cancel{200}^{\,100}$$

$$\boxed{\% \text{ tilt } P = \frac{T}{2RC} \times 100}$$

we know $f_1 = \frac{1}{2\pi RC}$

$$2RC = \frac{1}{\pi f_1}$$

$$\% \text{ tilt } P = \frac{T}{1/\pi f_1} \times 100$$

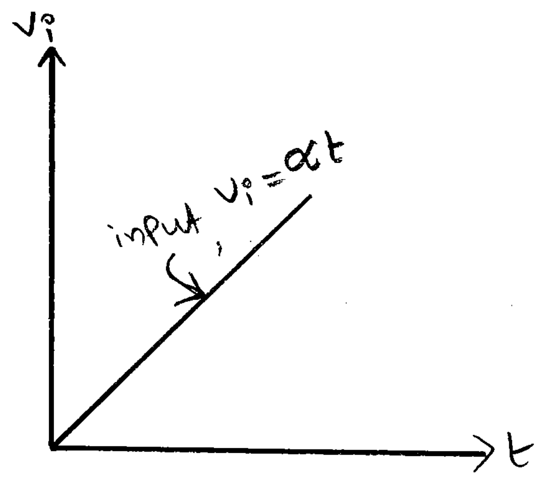
$$= T\pi f_1 \times 100 \quad , \text{ since } T = \frac{1}{f}$$

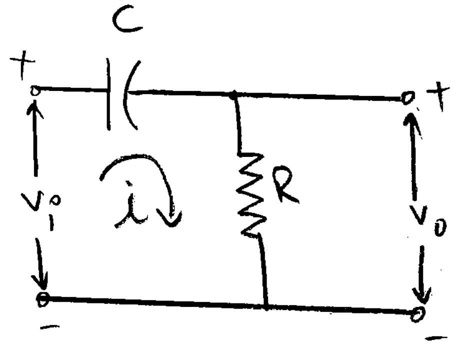
$$\boxed{\% \text{ tilt} = \frac{\pi f_1}{f} \times 100}$$

⑤ Ramp input :

A waveform which is zero for $t<0$ and which increases linearly with time for $t>0$.

$v_i = \alpha t$ is called a ramp or sweep voltage and is shown below:





By applying KVL to the input loop

$$v_i = \frac{1}{C}\int i\,dt + iR$$

$$v_0 = iR$$

$$i = \frac{v_0}{R}$$

$$v_i = \frac{1}{C}\int \frac{v_0}{R}\,dt + v_0$$

differentiate with respect to time

$$\frac{dv_i}{dt} = \frac{v_0}{RC} + \frac{dv_0}{dt}$$

$$v_i = \alpha t$$

$$\frac{dv_i}{dt} = \alpha$$

$$\therefore \boxed{\alpha = \frac{v_0}{RC} + \frac{dv_0}{dt}}$$

$$\alpha = v_0\left[\frac{1}{RC} + \frac{d}{dt}\right], \quad \text{but } \frac{d}{dt} = D$$

$$\therefore \left(D + \frac{1}{RC}\right) v_0 = \alpha$$

The solution of above Differential equation is

formula : $v_0(I.f) = \int \overset{R.H.S}{\frac{dv_i}{dt}} \times I.f \, dt$

where $\underset{\wedge}{I}.f = e^{\int \frac{1}{RC} dt} = e^{\frac{t}{RC}}$

Integral function

$$V_0 e^{t/RC} = \int \alpha e^{t/RC} dt + K$$

since $V_i = \alpha t$

$$\frac{dv_i}{dt} = \alpha$$

$$V_0 = \frac{\frac{\alpha e^{t/RC}}{1/RC} + K}{e^{t/RC}}$$

$$= \frac{\alpha \cancel{e^{t/RC}}}{1/RC} \cdot \cancel{e^{-t/RC}} + K e^{-t/RC}$$

$$V_0 = \alpha RC + K e^{-t/RC}$$

To find K value apply initial conditions

at $t=0$, $V_0 = 0$

$$0 = \alpha RC + K e^{-0}$$

$$\therefore K = -\alpha RC$$

$$V_0 = \alpha RC - \alpha RC e^{-t/RC}$$

$$\boxed{V_0 = \alpha RC\left[1 - e^{-t/RC}\right]}$$

i) If $RC \gg t$

$$V_0 = \alpha RC\left[1 - e^{-t/RC}\right]$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$

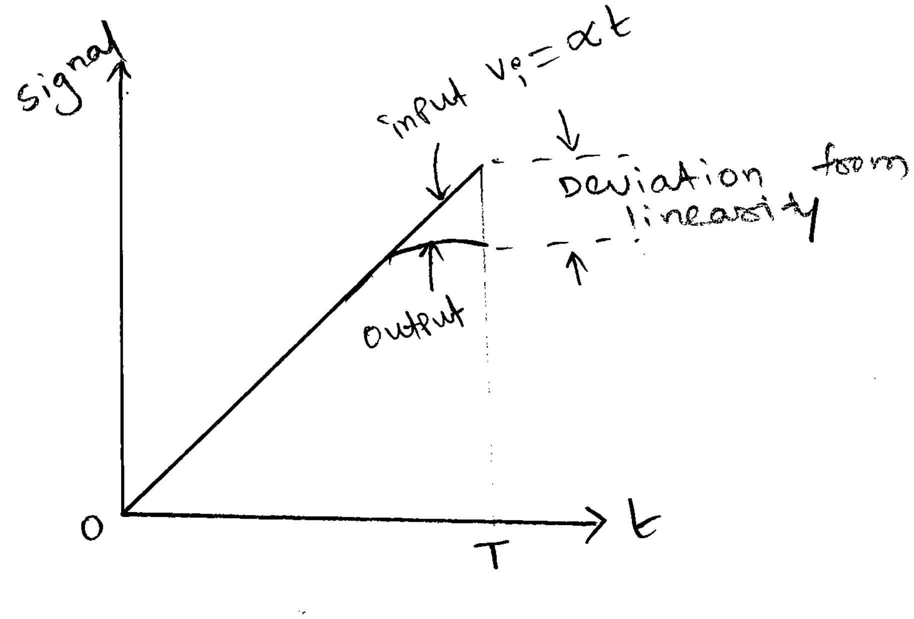
$$V_0 = \alpha RC\left[1 - \left(1 - \frac{t}{RC} + \frac{\left(\frac{t}{RC}\right)^2}{2!} - \cdots\right)\right]$$

$$= \alpha RC\left[\cancel{1} - \cancel{1} + \frac{t}{RC} - \frac{t^2}{2R^2C^2}\right]$$

$$= \alpha \cancel{RC}\left[\frac{t}{\cancel{RC}} - \frac{t^2}{2R^{\cancel{2}}C^{\cancel{2}}}\right]$$

$$= \alpha\left[t - \frac{t^2}{2RC}\right]$$

$$\boxed{V_0 = \alpha t\left[1 - \frac{t}{2RC}\right]}$$

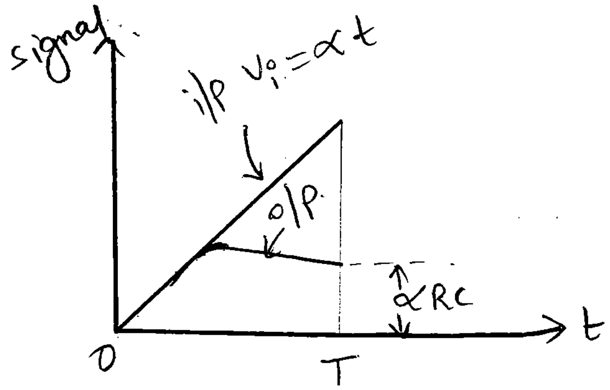


ii) If $RC \ll t$

$$V_0 = \alpha RC\left(1 - e^{-t/RC}\right)$$

$$V_0 = \alpha RC(1 - 0)$$

$$\therefore \boxed{V_0 = \alpha RC}$$



Transmission error (e_t):

for $RC \gg T$, the output signal falls away slightly from the input.

"A measure of the departure from linearity, expressed as the transmission error (e_t) is defined as the difference between input and output divided by the input."

The error at time $t = T$ is given by

$$e_t = \frac{V_i - V_0}{V_i} = \frac{\alpha T - \alpha T\left(1 - \frac{T}{2RC}\right)}{\alpha T}$$

$$e_t = \frac{\cancel{\alpha T}\left[\cancel{1} - \cancel{1} + \frac{T}{2RC}\right]}{\cancel{\alpha T}}$$

$$\boxed{e_t = \frac{T}{2RC}}$$

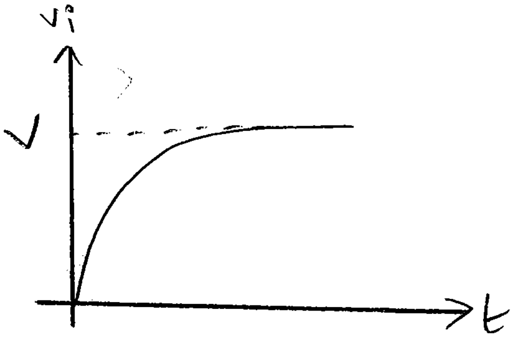
we know $f_1 = \frac{1}{2\pi RC}$

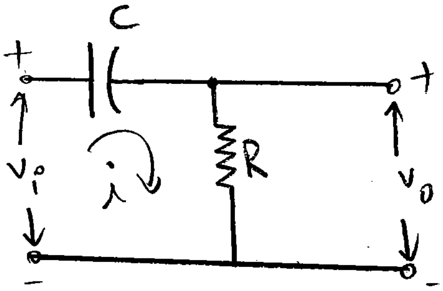
$$2RC = \frac{1}{\pi f_1}$$

$$e_t = \frac{T}{1/\pi f_1} = T\pi f_1$$

$$\boxed{e_t = \frac{\pi f_1}{f}}$$

(6) <u>Exponential signal</u>:





→ for all non-sinusoidal inputs like step, ramp, square, pulse, the output voltage is exponential in nature. In particular, for a Ramp input, the output voltage is exponentially rising and its waveform practically coincides with the input waveform near $t = 0$.

→ If the forcing function (particular integral) also an exponentially rising voltage, then the output and input waveforms are coincident at and near

$t=0$. The peak value of the output is decided by the time constant of the circuit.

→ Smaller the time constant, smaller is the output peak.

As $\tau(RC)$ increases in comparison with the time required for the input voltage to attain its steady state value, the output peak also increases. However the output voltage decays exponentially to zero after attaining peak value.

The exponential input waveform is given by

$$V_i = V\left[1 - e^{-t/\tau}\right] \quad \text{— (1)}$$

where τ = Time constant of the exponential input.

Apply KVL at the input and output loops

$$v_i = \frac{1}{C}\int i\,dt + iR$$

$$V_o = iR$$

$$i = \frac{V_o}{R}$$

$$v_i = \frac{1}{C}\int \frac{v_o}{R}\,dt + v_o$$

differentiating with respect to 't'

$$\frac{dv_i}{dt} = \frac{v_o}{RC} + \frac{dv_o}{dt} \quad \text{— (2)}$$

$$v_i = V\left[1 - e^{-t/\tau}\right] = V - Ve^{-t/\tau}$$

$$\frac{dv_i}{dt} = 0 - \frac{Ve^{-t/\tau}}{\cancel{\tau}} \times \frac{-1}{\tau} =$$

$$\frac{dv_i}{dt} = \frac{V}{\tau} e^{-t/\tau}$$

$$\frac{V}{\tau} e^{-t/\tau} = \frac{v_0}{RC} + \frac{dv_0}{dt}$$

$$\frac{V}{\tau} e^{-t/\tau} = v_0\left[\frac{1}{RC} + \frac{d}{dt}\right], \quad \text{but } \frac{d}{dt} = D$$

$$\left(D + \frac{1}{RC}\right) V_0 = \frac{V}{\tau} e^{-t/\tau} \Rightarrow \left(D + \frac{1}{RC}\right) V_0 = K$$

The solution of the differential equation with constant K is given by

$$\boxed{\begin{aligned} &\text{formula: } v_0 \times (I.F) = \int \overset{R.H.S}{\frac{dv_i}{dt}} \times I.F\, dt + C \\ &\text{where } I.F = e^{\int \frac{1}{RC} dt} = e^{t/RC} \end{aligned}}$$

$$V_0 \cdot e^{t/RC} = \int \frac{V}{\tau} \cdot e^{-t/\tau} \cdot e^{t/RC} dt + C$$

$$V_0 \cdot e^{t/RC} = \frac{V}{\tau} \int e^{t\left(\frac{1}{RC} - \frac{1}{\tau}\right)} dt + C$$

$$V_0 = \frac{\frac{V}{\tau} \int e^{t\left(\frac{1}{RC} - \frac{1}{\tau}\right)} dt + C}{e^{t/RC}}$$

$$V_0 = \frac{V}{\tau} e^{-t/RC} \int e^{t\left(\frac{1}{RC} - \frac{1}{\tau}\right)} dt + e^{-t/RC} \cdot C \quad \text{— (3)}$$

i) If $\tau \neq RC$ $(n \neq 1)$ $\Rightarrow \frac{RC}{\tau} \neq 1$

then the solution of equ ③ is

$$v_0 = \frac{V}{\cancel{\tau}} \cancel{e^{-t/RC}} \cdot \frac{\cancel{e^{t/RC}} \cdot e^{-t/\tau}}{\frac{(\tau - RC)}{RC\cancel{\tau}}} + c \cdot e^{-t/RC}$$

$$v_0 = \frac{VRC}{(\tau - RC)} \cdot e^{-t/\tau} + c \cdot e^{-t/RC}$$

To find the value of c, apply initial conditions

At $t = 0$, $v_0 = 0$

$$0 = \frac{VRC}{\tau - RC} e^{-0} + c \cdot e^{-0}$$

$$c = -\frac{VRC}{\tau - RC}$$

$$v_0 = \frac{VRC}{\tau - RC} e^{-t/\tau} - \frac{VRC}{\tau - RC} e^{-t/RC}$$

$$v_0 = \frac{VRC}{\tau - RC} \left\{ e^{-t/\tau} - e^{-t/RC} \right\}$$

let $x = \frac{t}{\tau}$, $n = \frac{RC}{\tau}$, $\frac{x}{n} = \frac{t}{RC}$

$$v_0 = \frac{VRC}{\tau\left(1 - \frac{RC}{\tau}\right)} \left\{ e^{-x} - e^{-x/n} \right\}$$

$$v_0 = \frac{V \cdot n}{1 - n} \left\{ e^{-x} - e^{-x/n} \right\}$$

$$\therefore \boxed{V_0 = \frac{V.\eta}{(\eta-1)}\left\{e^{-x/\eta} - e^{-x}\right\}}$$

ii) $\tau = RC \ (n=1) \Rightarrow \frac{RC}{\tau} = 1$

substitute this condition in equ③

$$V_0 = \frac{V}{\tau} e^{-t/RC} \int e^{\left(\frac{1}{RC} - \frac{1}{\tau}\right)t} dt + e^{-t/RC} \cdot c$$

$$V_0 = \frac{V}{\tau} e^{-t/RC} \int e^{\left(\frac{1}{RC} - \frac{1}{RC}\right)t} dt + e^{-t/RC} \cdot c$$

$$V_0 = \frac{V}{\tau} e^{-t/RC} \int dt + e^{-t/RC} \cdot c$$

$$V_0 = \frac{Vt}{\tau} e^{-t/RC} + c \cdot e^{-t/RC}$$

To find the value of c, apply initial conditions

at $t=0$, $V_0 = 0$

$$0 = \frac{V \cdot 0}{\tau} e^{-0} + c \cdot e^{-0}$$

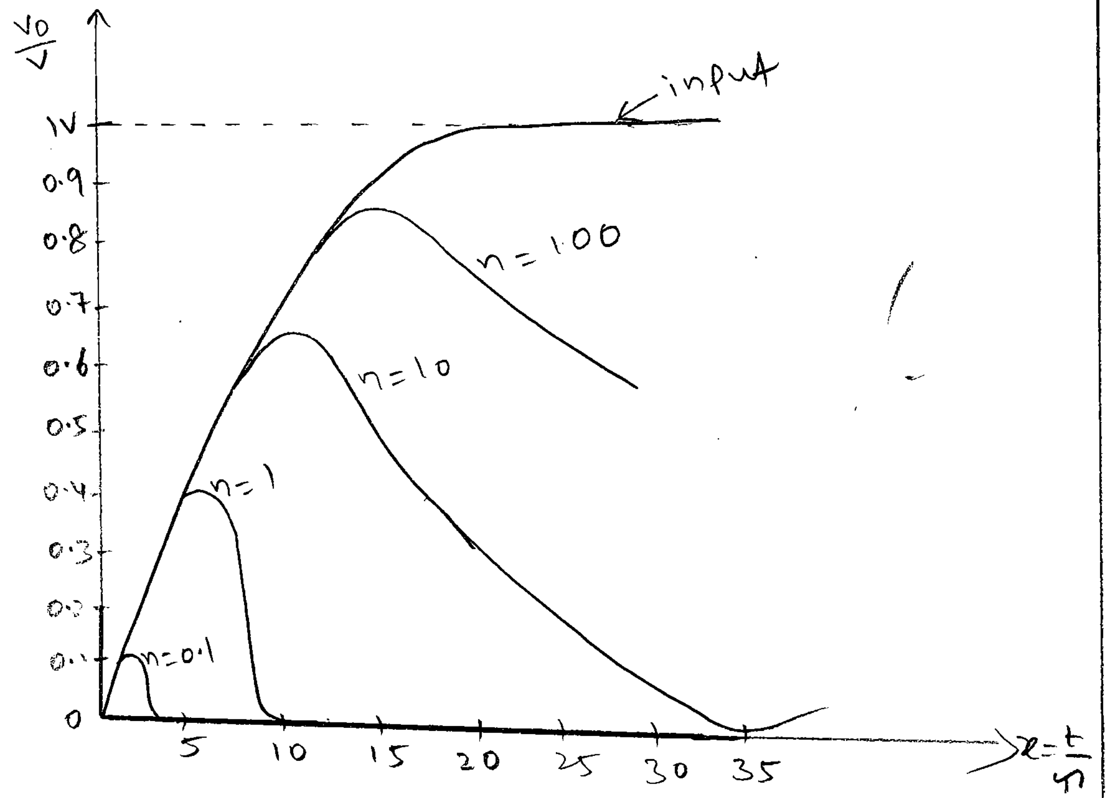
$$c = 0$$

$$V_0 = \frac{Vt}{\tau} e^{-t/RC}$$

if $x = \frac{t}{\tau}$, $n = \frac{RC}{\tau}$, $\frac{x}{n} = \frac{t}{RC}$

$$V_0 = V \cdot x \, e^{-t/\eta}$$

$$\boxed{V_0 = V \cdot x \, e^{-x}}$$



High pass RC circuit acts as differentiator

"If the time constant (τ = RC) is very small in comparison with the time (T) required for the input signal to make an appreciable change, the circuit is called a differentiator."

Under these circumstances the voltage drop across R will be very small in comparision with the drop across C. So, the current is determined entirely by the capacitance.

Then the current i is

$$i = C\frac{dv_i}{dt}$$

output voltage across R is

$$V_0 = i R$$

$$\boxed{V_0 = RC\frac{dv_i}{dt}}$$

hence the output is proportional to the Derivative of the input.

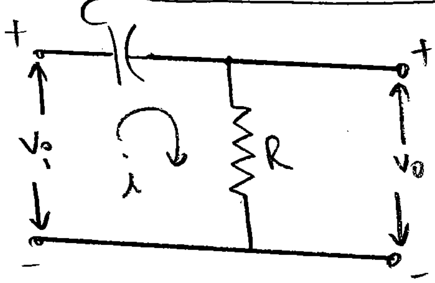
(or)

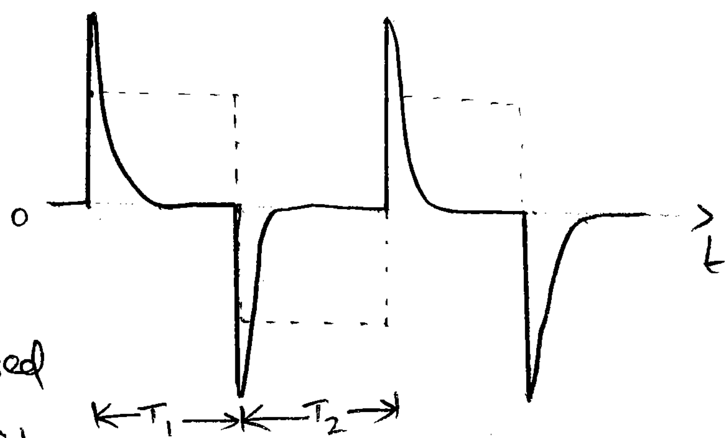
$$\frac{dv_i}{dt} = \underbrace{\frac{1}{RC} v_0}_{\text{steady state response (forced response)}} + \underbrace{\frac{dv_0}{dt}}_{\text{transient response (natural response)}}$$

for $RC \ll T$, transient response becomes insignificant

$$\frac{dv_i}{dt} = \frac{1}{RC} v_0 + 0$$

$$\boxed{V_0 = RC\frac{dv_i}{dt} \Rightarrow V_0 \propto \frac{dv_i}{dt}}$$





If a Ramp input is applied to a differentiator circuit

$$V_0 = RC\frac{dv_{in}}{dt}$$

$$V_{in} = \alpha t$$

$$\frac{dv_{in}}{dt} = \alpha$$

$RC \ll T$ $\boxed{V_0 = \alpha RC}$

→ so, the condition for High-pass RC circuit to act as differentiator is

$$\boxed{RC \ll T}$$

→ If a sine wave is applied to the high pass RC circuit, the output will be a sine wave shifted by a leading angle θ such that

$$\tan\theta = \frac{1}{\omega RC} = \frac{f_1}{f}$$

and the output will be proportional to $\sin(\omega t+\theta)$

In order to have true differentiation we must obtain $\cos\omega t$.

i.e; θ must be equal to 90°

this result can be obtained only if $R=0$ or $C=0$.

if $\omega RC = 0.1$ then $\theta = 84.3^\circ$

if $\omega RC = 0.01$ then $\theta = 89.4^\circ$ which is close to 90°

If the peak value of the input is V_m, then the output is

$$\frac{V_m R}{\sqrt{R^2+\frac{1}{\omega^2 C^2}}}\sin(\omega t+\theta)$$

and if $\omega RC \ll 1$, then the output is approximately equal to $V_m \omega RC \cos\omega t$.

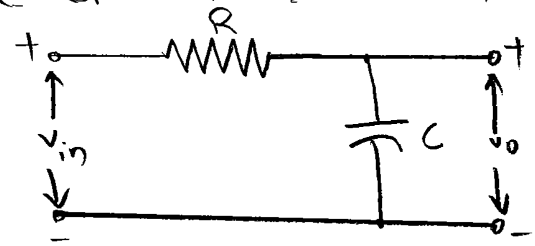
this result agrees with the expected value,

$$RC\frac{dv_{in}}{dt}.$$

∴ The condition for differentiation is $\underline{RC \ll T}$.

Low pass RC circuit

Low pass RC circuit is shown below:



"A Low pass RC circuit is a circuit which transmits only low-frequency signals and attenuates high-frequency signals".

The capacitance reactance is

$$X_C = \frac{1}{2\pi f C}$$

The reactance of capacitor decreases with increasing frequency.

i) At higher frequencies, the reactance becomes small and capacitor almost acts a short circuit.

$$\therefore V_0 = 0$$

ii) At low frequencies, the reactance becomes large and capacitor acts as open circuit, and all input appears at the output with less attenuation.

Apply KVL at input and output side

$$V_i = iR + \frac{1}{C}\int i\,dt$$

$$v_0 = \frac{1}{C}\int i\,dt$$

$$i = C\frac{dv_0}{dt}$$

$$V_i = RC\frac{dV_o}{dt} + V_o$$

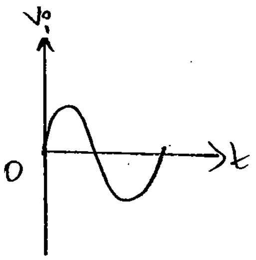
divide throughout by RC

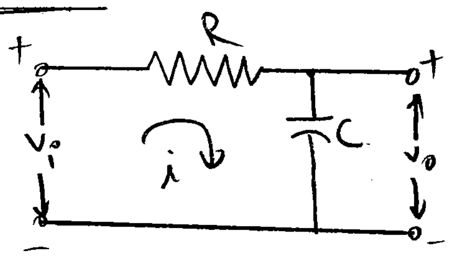
$$\boxed{\frac{V_i}{RC} = \frac{1}{RC}V_o + \frac{dV_o}{dt}}$$

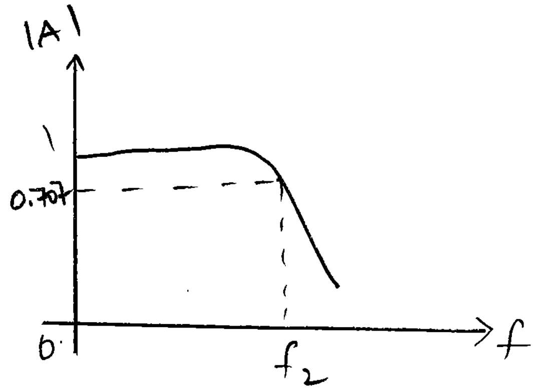
$\frac{1}{RC}V_o$ → steady state response (forced response)

$\frac{dV_o}{dt}$ → Transient response (natural response)

① sinusoidal input :







→ At low frequencies, the output is equal to the input and hence the gain is unity.

→ As the frequency increases, the output decreases and hence the gain decreases.

$$V_o = \frac{X_c}{J} I$$

$$I = \frac{V_{in}}{R + \frac{X_c}{J}},$$

$$V_o = \frac{V_{in} \times \frac{X_c}{j}}{R + \frac{X_c}{j}}, \text{ where } X_c = \frac{1}{2\pi f C}$$

In laplace transform

$C \rightarrow \frac{1}{CS}$

$s = j\omega$

$\frac{1}{j\omega C} = \frac{1}{j2\pi f C}$

$X_c = \frac{1}{2\pi f C} \Rightarrow \frac{X_c}{j}$

$$V_0 = \frac{V_{in} \times \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{V_{in} \times \frac{1}{\cancel{j\omega C}}}{\frac{j\omega RC + 1}{\cancel{j\omega C}}}$$

$$V_0 = \frac{V_{in}}{1 + j\omega RC} = \frac{V_{in}}{1 + j2\pi f RC}$$

let $f_2 = \frac{1}{2\pi RC}$ = upper 3dB-frequency

$$V_0 = \frac{V_{in}}{1 + j\frac{f}{f_2}}$$

$$A = \frac{V_0}{V_{in}} = \frac{1}{1 + j\frac{f}{f_2}}$$

$$|A| = \frac{1}{\sqrt{1 + (f/f_2)^2}}$$

$$\theta = -\tan^{-1}\left(\frac{f}{f_2}\right)$$

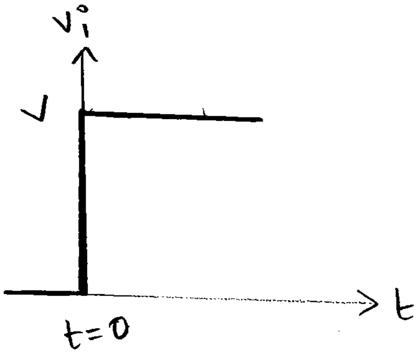
<u>At $f = f_2$</u>

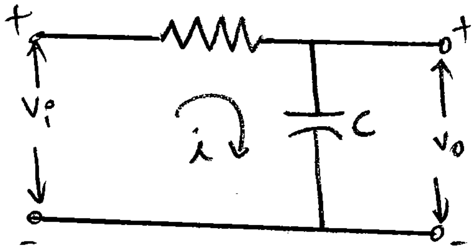
$$|A| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

$$\theta = \tan^{-1}(1) = 45^0$$

(2) Step input :

A step voltage is one which maintains the value zero for all times $t<0$ and maintains the value V for all times $t>0$.





Applying KVL to the input side and output side

$$v_i = iR + \frac{1}{C}\int i\,dt \quad \text{—— (1)}$$

$$v_0 = \frac{1}{C}\int i\,dt \quad \text{—— (2)}$$

differentiating equ (1) with respect to t

$$\frac{dv_i}{dt} = R\frac{di}{dt} + \frac{i}{C}$$

$$v_i = V, \quad t \geq 0$$

$$\frac{dv_i}{dt} = 0$$

$$0 = R\frac{di}{dt} + \frac{i}{C}$$

$$\frac{di}{dt} + \frac{i}{RC} = 0$$

$$\left(D + \frac{1}{RC}\right) i = 0, \quad \text{since } \frac{d}{dt} = D$$

By solving above differential equation

$$\boxed{\begin{gathered}\text{formula: } i \times (I.f) = \int \frac{dv_i}{dt} \times I.f \, dt + K \\ I.f = e^{\int \frac{1}{RC} dt} = e^{t/RC}\end{gathered}}$$

$$i \times e^{t/RC} = \overbrace{\int 0 \times e^{t/RC} dt + K}^{R.H.S}$$

$$i = \frac{K}{e^{t/RC}}$$

$$i = K e^{-t/RC}$$

To find K value, we have to apply initial conditions.

At $t=0$, capacitor acts as short circuit and voltage across it is zero

$$\therefore V_{in} = iR$$

$$i = \frac{V_{in}}{R}$$

$$\frac{V_{in}}{R} = K e^{-0}$$

$$K = \frac{V_{in}}{R}$$

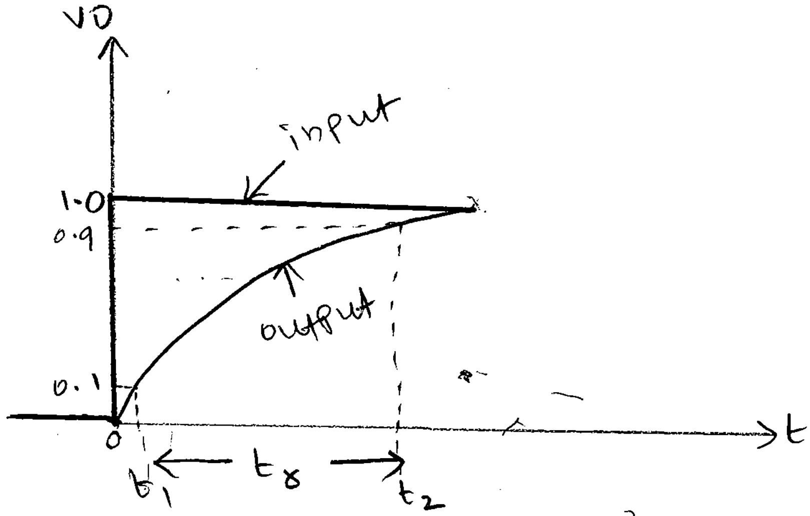
$$\therefore i = \frac{V_{in}}{R} e^{-t/RC} \quad \text{--- (3)}$$

output voltage $(V_0) = V_i - iR$ (i.e. $V_i = iR + V_0$)

$$V_0 = V - \left(\frac{V_i}{\cancel{R}} e^{-t/RC}\right)\cancel{R}$$

$$V_0 = V - Ve^{-t/RC}$$

$$\boxed{V_0 = V\left[1 - e^{-t/RC}\right]}$$



Rise time (t_r):

"It is defined as the time it takes the voltage to rise from 0.1 to 0.9 of its final value".

Proof:

$$V_0 = V(1 - e^{-t/RC})$$

$$0.1\cancel{V} = \cancel{V}\left[1 - e^{-t_1/RC}\right]$$

$$0.1 = 1 - e^{-t_1/RC}$$

$$e^{-t_1/RC} = 0.9$$

$$t_1 = -RC \ln 0.9 \quad \text{—(I)}$$

$$0.9\cancel{V} = \cancel{V}(1 - e^{-t_2/RC})$$

$$0.9 = 1 - e^{-t_2/RC}$$

$$e^{-t_2/RC} = 0.1$$

$$t_2 = -RC \ln 0.1 \quad \text{—(II)}$$

$$\text{Rise time } (t_r) = t_2 - t_1$$

$$= -RC \ln 0.1 - (-RC \ln 0.9)$$

$$= RC\left[\ln 0.9 - \ln 0.1\right]$$

$$\boxed{t_r = 2.2RC}$$

we know $f_2 = \frac{1}{2\pi RC}$

$$RC = \frac{1}{2\pi f_2}$$

$$t_r = 2.2 \times \frac{1}{2\pi f_2}$$

$$\boxed{t_r = \frac{0.35}{f_2}}$$

Thus the rise time is proportional to time constant T and inversely proportional to upper 3-dB frequency.

Delay time:

"Time required for the output voltage to reach 50% of its final value starts from initial value".

$$V_0 = V\left[1 - e^{-t/RC}\right]$$

$$0.5\cancel{V} = \cancel{V}\left[1 - e^{-t_d/RC}\right]$$

$$0.5 = 1 - e^{-t_d/RC}$$

$$e^{-t_d/RC} = 0.5$$

$$t_d = -RC \ln 0.5$$

$$\boxed{t_d = 0.69\,RC}$$

Time constant:

Time required for the output voltage to reach 63.2% of its final value starts from initial value.

$$0.632\cancel{V} = \cancel{V}[1-e^{-t/RC}]$$

$$0.632 = 1-e^{-t/RC}$$

$$e^{-t/RC} = 1-0.632$$

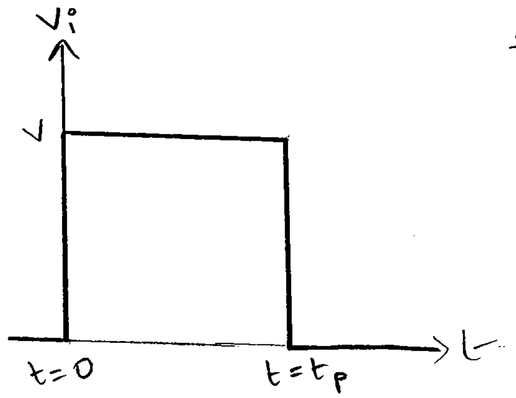
$$e^{-t/RC} = 0.3678$$

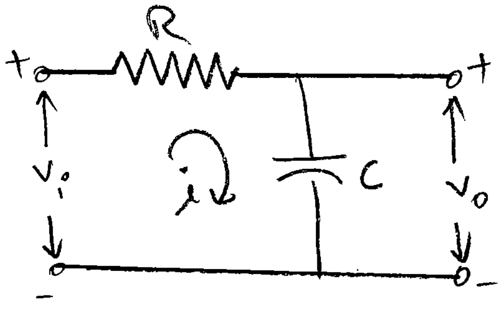
$$t = -RC \ln 0.3678$$

$$\boxed{T = RC}$$

(3) Pulse input:

An ideal pulse waveform is shown in below fig. The pulse amplitude is V and pulse duration is t_p.



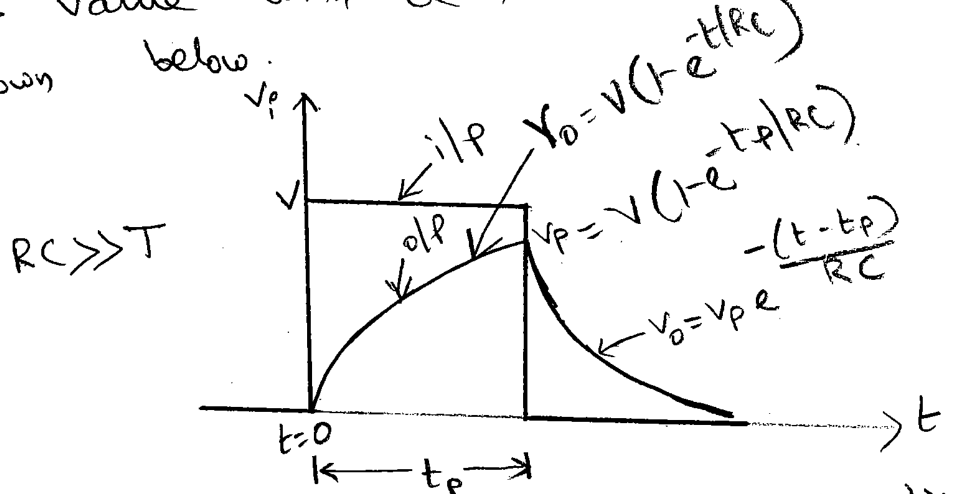


→ Pulse may be considered to be the sum of step voltage +V whose discontinuity occurs at t=0 and a step voltage whose discontinuity occurs at $t = t_p$.

→ The response to a pulse, for times less than the pulse width t_p, is same as that for a step input and is given by

$$v_p = v_0 = V\left[1 - e^{-t/RC}\right]$$

→ At the end of the pulse the voltage is v_p and the output must decreases to zero from this value with a time constant RC and is shown below.

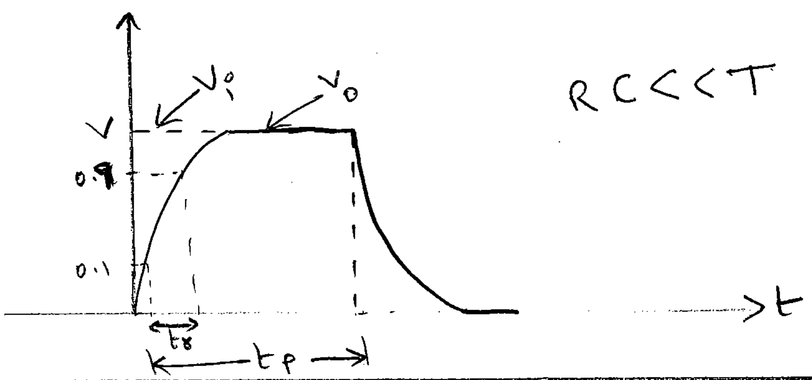


→ To minimize the distortion, the rise time must be small compared with the pulse width.

if f_2 is choosen equal to $\frac{1}{t_p}$, then

$$t_r = \frac{0.35}{f_2} = 0.35\,t_p$$

the output waveform with $t_r = 0.35\,t_p$ is shown in fig.



"A pulse shape will be preserved if 3-dB frequency is approximately equal to the reciprocal of the pulse width".

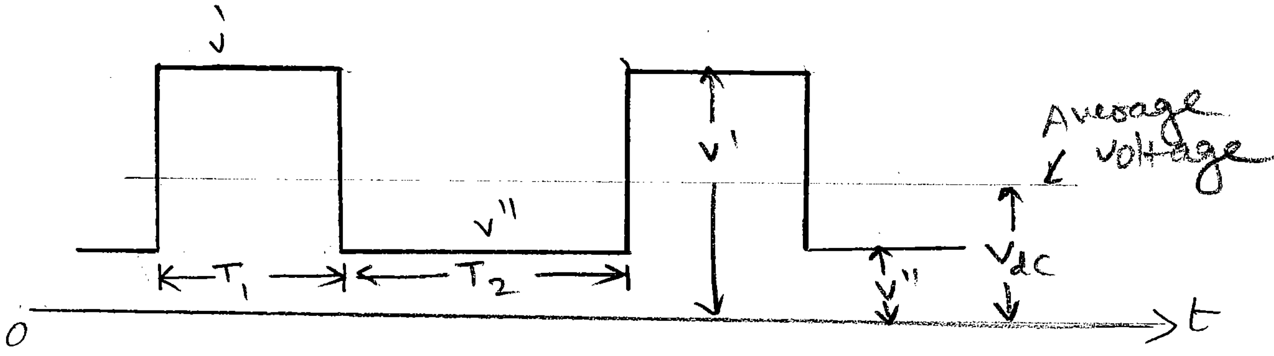
Thus to pass a 0.5 μsec pulse reasonably will requires a circuit with an upper 3-dB frequency of the order of 2 MHz.

(4) square wave input:

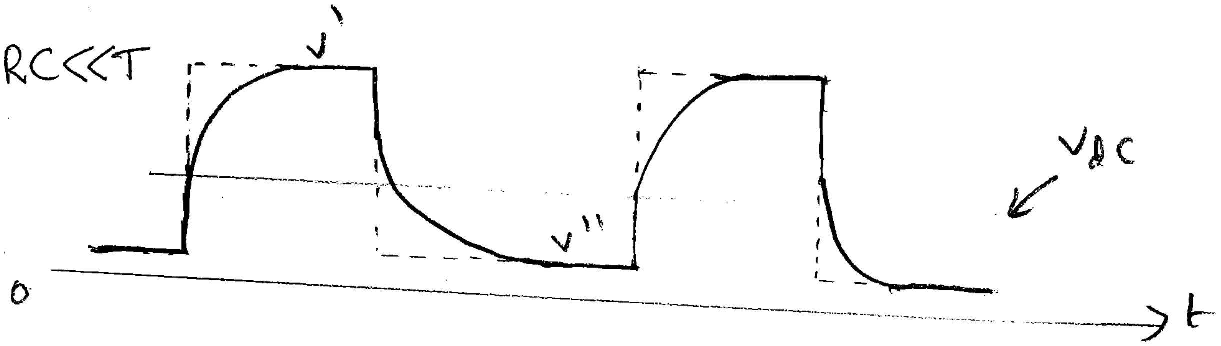
A square wave is a periodic waveform which maintains itself at one constant level V' with respect to ground for a time T_1 and then changes abruptly to another level V'' and remains constant at that level for a time T_2 and repeats itself at regular intervals of time $T = T_1 + T_2$.

A square wave may be treated as a series of positive and negative steps. The shape of the output waveform for a square wave input depends on time constant of the circuit.

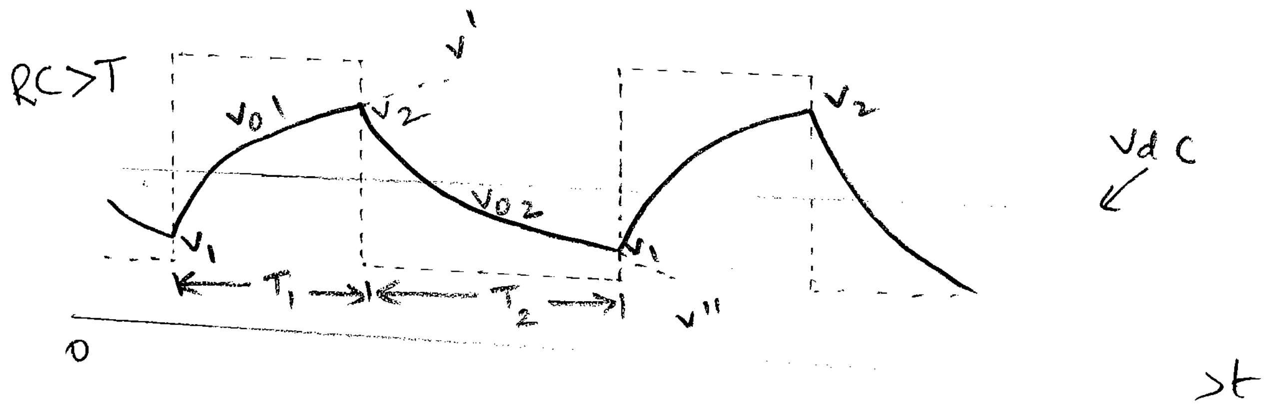
If the time constant is very small, the rise time will also be small and a reasonable reproduction of the input may be obtained.



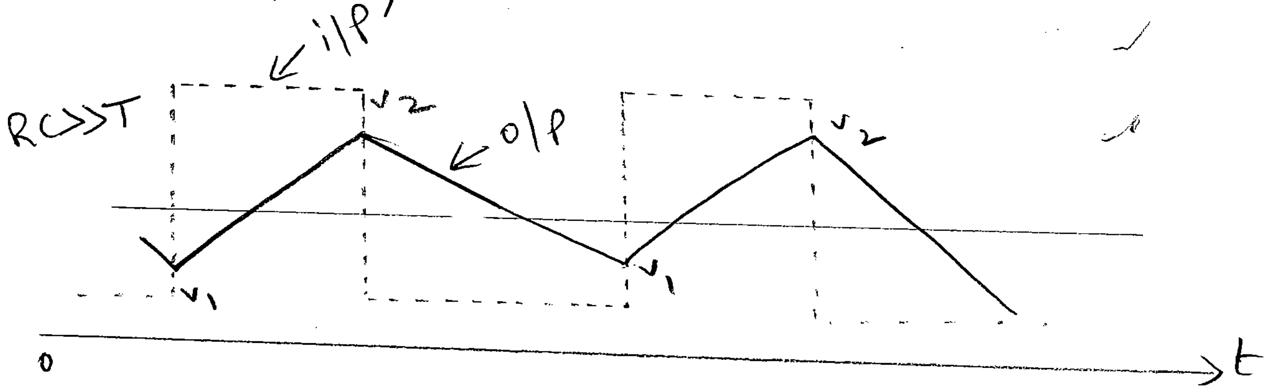
→ if rise time t_r is very small as compared with pulse width, the steady state response is shown below



→ If the time constant RC is greater in comparable with the period of the input square wave, the output is shown below.



→ If the time constant is very large compared with the period of the input, square wave the output consists of exponential sections which are essentially linear.



→ At any instant, to find the output voltage for a single time constant circuit having initial and final values V_i and V_f respectively.

The output voltage is given by

$$\boxed{V_0 = V_f + (V_i - V_f)\, e^{-t/RC}}$$

The output voltage V_{O_1} and V_{O_2} is given by

<u>for rising portion</u>

$$V_{O_1} = V' + (V_1 - V')\cdot e^{-T_1/RC} \quad \text{— (1)}$$

<u>for falling portion</u>

$$V_{O_2} = V'' + (V_2 - V'')\cdot e^{-T_2/RC} \quad \text{— (2)}$$

if we set $v_{o1} = V_2$ at $t = T_1$

and $v_{o2} = V_1$ at $t = T_1 + T_2$

$$V_2 = V' + (V_1 - V')e^{-T_1/RC}$$

$$V_1 = V'' + (V_2 - V'')e^{-T_2/RC}$$

→ Since the average voltage across R is zero, then the d.c voltage at the output is same as that of the input. This average value is indicated as $V_{d.c.}$

Consider a symmetrical square wave with zero average value, so that

$$T_1 = T_2 = \frac{T}{2}$$

$$V' = -V'' = \frac{V}{2}$$

and $V_1 = -V_2$

$$V_2 = \frac{V}{2} + \left(-V_2 - \frac{V}{2}\right)e^{-T/2RC}$$

$$V_2 = \frac{V}{2} - V_2 e^{-T/2RC} - \frac{V}{2}e^{-T/2RC}$$

$$V_2\left[1 + e^{-T/2RC}\right] = \frac{V}{2}\left[1 - e^{-T/2RC}\right]$$

$$V_2 = \frac{V}{2}\left[\frac{1 - e^{-T/2RC}}{1 + e^{-T/2RC}}\right] = \frac{V}{2}\left[\frac{1 - \frac{1}{e^{T/2RC}}}{1 + \frac{1}{e^{T/2RC}}}\right]$$

$$V_2 = \frac{V}{2}\left[\frac{e^{T/2RC} - 1}{e^{T/2RC} + 1}\right]$$

$$V_2 = \frac{V}{2}\left(\frac{e^{2x}-1}{e^{2x}+1}\right), \quad \text{where } x = \frac{T}{4RC}$$

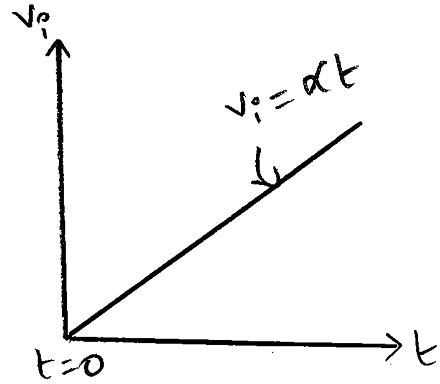
$$\therefore \boxed{V_2 = \frac{V}{2}\tanh x}$$

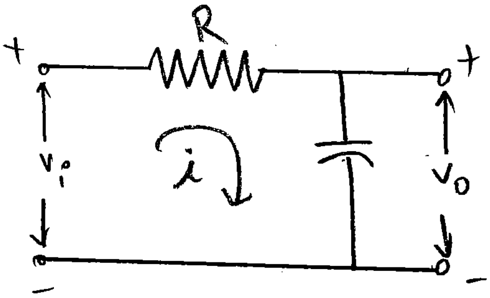
where T = Time period of square wave

(5) Ramp input:

A waveform which is zero for $t<0$ and which increases linearly with time for $t>0$.

$V_i = \alpha t$ is called as ramp input.





Apply KVL to the input loop & output loop

$$V_i = iR + \frac{1}{C}\int i\,dt$$

$$V_0 = \frac{1}{C}\int i\,dt$$

$$V_i = iR + V_0$$

$$V_0 = V_i - iR$$

$$V_0 = V_i - V_R$$

From High pass RC circuit, for a ramp input the voltage across resistor R is given by

$$V_R = \alpha RC\left[1-e^{-t/RC}\right]$$

∴ The output voltage across capacitor 'c' for Low pass RC circuit is given by

$$V_0 = V_i - V_R$$

$$V_0 = \alpha t - \alpha RC\left[1-e^{-t/RC}\right]$$

i) If $RC \ll T$, Transients are negligible ($e^{-t/RC} \approx 0$)

$$V_0 = \alpha t - \alpha RC$$

$$\boxed{V_0 = \alpha(t-RC)}$$

ii) If $RC \gg T$, Transients are predominant.

$$V_0 = \alpha t - \alpha RC\left[1-e^{-t/RC}\right]$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$V_0 = \alpha t - \alpha RC\left[1-\left(1-\frac{t}{RC}+\frac{t^2}{2R^2C^2}-\cdots\right)\right]$$

$$V_0 = \alpha t - \alpha RC\left[\cancel{1}-\cancel{1}+\frac{t}{RC}-\frac{t^2}{2R^2C^2}\right]$$

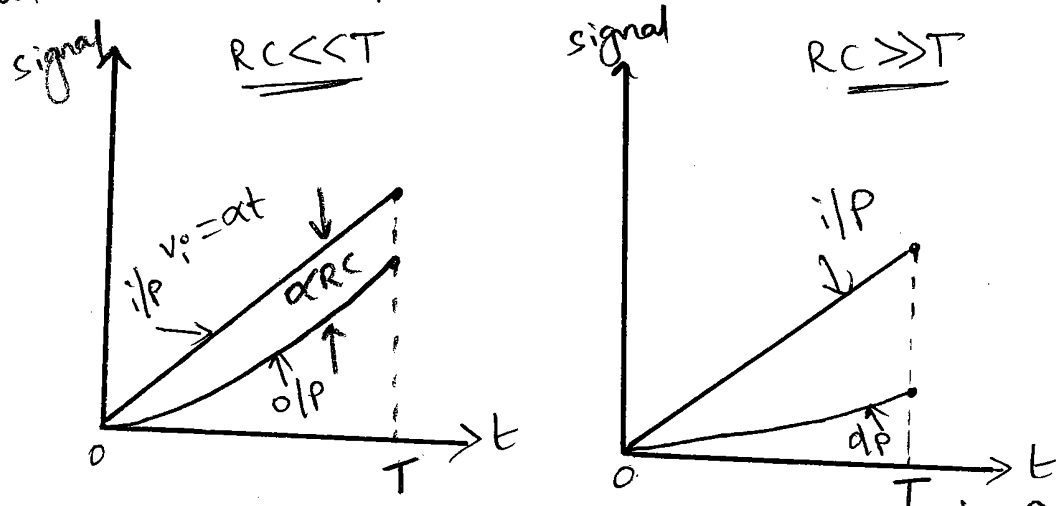
$$V_0 = \alpha t - \alpha RC\left[\frac{t}{RC}-\frac{t^2}{2R^2C^2}\right]$$

$$V_0 = \alpha t - \alpha \cancel{RC} \times \frac{t}{\cancel{RC}} + \alpha \cancel{RC} \times \frac{t^2}{2R^2C^2}$$

$$V_0 = \cancel{\alpha t} - \cancel{\alpha t} + \frac{\alpha t^2}{2RC}$$

$$\boxed{V_0 \simeq \frac{\alpha t^2}{2RC}}$$

The input and output waveforms for a ramp input to a Low pass RC circuit is shown below.



To transmit the ramp signal with little distortion, a small time constant must be used relative to the total ramp input with time T.

The transmission error e_t is defined as the difference between input and output divided by the input at t = T.

For $\underline{RC << T}$

$$e_t = \left.\frac{v_i - v_o}{v_i}\right|_{t=T}$$

$$e_t = \left.\frac{\alpha t - (\alpha t - \alpha RC)}{\alpha t}\right|_{t=T}$$

$$e_t = \frac{\cancel{\alpha T} - \cancel{\alpha T} + \alpha RC}{\alpha T}$$

$$e_t = \frac{\cancel{\alpha} RC}{\cancel{\alpha} T}$$

$$\boxed{e_t = \frac{RC}{T}}$$

$$f_2 = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_2}$$

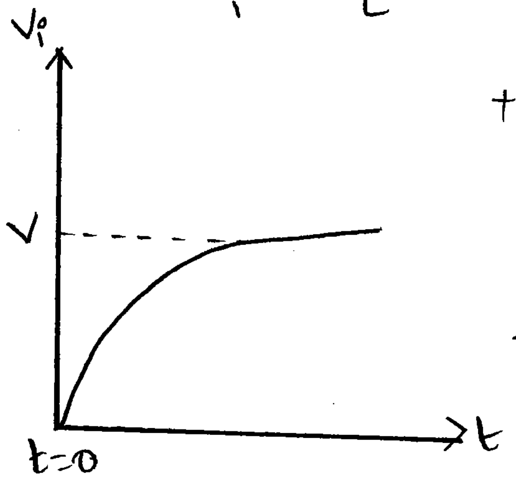
$$\therefore \boxed{e_t = \frac{1}{2\pi f_2 T}}$$

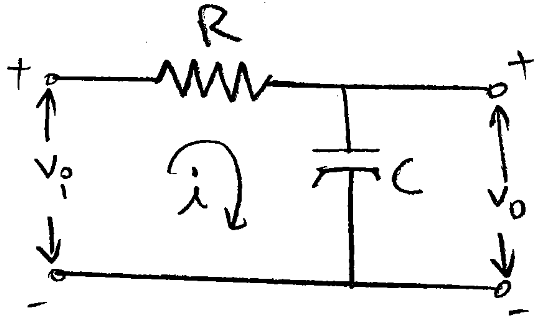
where f_2 = upper 3-dB frequency.

⑥ Exponential input:

The exponential input waveform is given by

$$V_i = V\left[1 - e^{-t/\tau}\right]$$





By applying KVL to the input and output loop

$$V_i = iR + \frac{1}{C}\int i\,dt$$

$$V_0 = \frac{1}{C}\int i\,dt$$

$$V_i = iR + V_0$$

$$V_0 = V_i - iR$$

$$V_0 = V_i - V_R$$

where V_R = voltage drop across R.

The voltage drop across R in case of High pass RC circuit is

for $\underline{n=1}$, $V_R = V.x.e^{-x}$

for $\underline{n \neq 1}$, $V_R = \frac{V.n}{n-1}\left[e^{-x/n} - e^{-x}\right]$

∴ The voltage drop across 'C' in case of Low pass RC circuit is

$$V_0 = V_i - V_R$$

for $\underline{n=1}$ $(RC = \tau)$

$$V_0 = V_i - V_R$$

$$V_0 = V\left[1 - e^{-t/RC}\right] - Vxe^{-x}$$

$$V_0 = V - Ve^{-t/RC} - Vxe^{-x}$$

$$x = \frac{t}{\tau}, \quad n = \frac{RC}{\tau}, \quad \frac{x}{n} = \frac{t}{RC}$$

$$V_0 = V - Ve^{-x/n} Vxe^{-x} \quad \left(\text{since } \frac{t}{RC} = \frac{t}{\tau}\right)$$

$$\cancel{V_0 = V - Ve^{-x}(1+x)}$$

$$\boxed{\frac{V_0}{V} = 1 - e^{-x/n}(1+x)}$$

$$\boxed{\frac{V_0}{V} = 1 - e^{-x/n} - e^{-x}.}$$

if $n \neq 1$ $(RC \neq \tau)$

$$V_0 = V_{in} - V_R$$

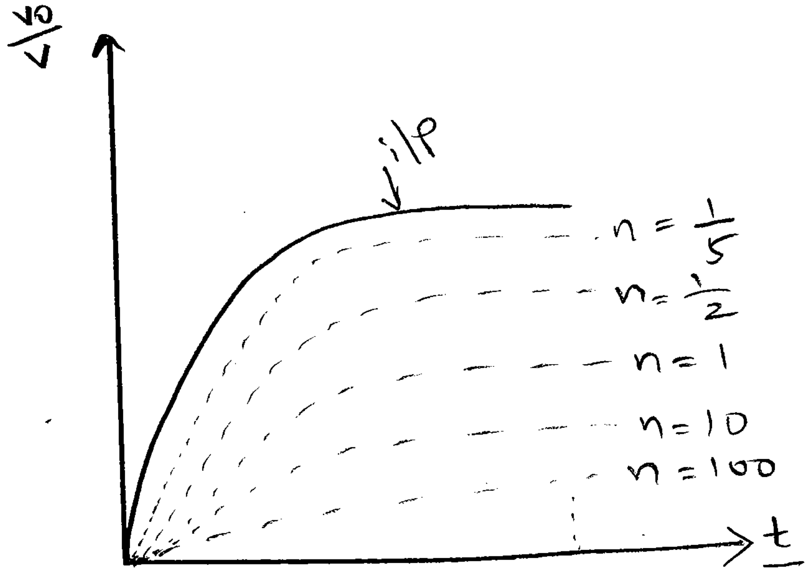
$$V_0 = V\left[1 - e^{-t/\tau}\right] - \frac{V n}{n-1}\left[e^{-x/n} - e^{-x}\right]$$

$$V_0 = V - V e^{-t/\tau} - \frac{V n}{n-1} e^{-x/n} + \frac{V n}{n-1} e^{-x}$$

$$V_0 = V\left[1 - e^{-x} - \frac{n e^{-x/n}}{n-1} + \frac{n e^{-x}}{n-1}\right]$$

$$\boxed{\frac{V_0}{V} = 1 - e^{-x} - \frac{n}{n-1} \cdot e^{-x}\left(e^{1/n} - 1\right)}$$

The input and output waveforms for a given exponential input to a Low pass RC circuit are shown below:



Low pass RC circuit acts as an Integrator

"If the time constant of an RC low-pass circuit is very large in comparison with the time required for the input signal to make an appreciable change, the circuit is called as integrator."

If the time constant of an RC Low pass circuit is very large, the capacitor charges very slowly and so almost all the input voltage appears across the resistor for small values of time.

As $RC \gg T$, the voltage drop across C will be very small in comparison to the voltage drop across R and we may consider that the total input V_i appears across R, then

$$V_i = iR$$

$$i = \frac{V_i}{R}$$

for Low pass RC circuit, the output voltage V_o is given by

$$V_o = \frac{1}{C}\int i\,dt$$

$$V_o = \frac{1}{C}\int \frac{V_i}{R}\,dt$$

$$\boxed{V_o = \frac{1}{RC}\int V_i\,dt}$$

(or)

Apply KVL

$$V_i = iR + \frac{1}{C}\int i\,dt$$

$$V_o = \frac{1}{C}\int i\,dt$$

$$i = C\frac{dV_o}{dt}$$

$$V_i = RC\frac{dV_o}{dt} + V_o$$

If $RC >> t$, transient is predominant

$$\cancel{V_i = iR + \frac{1}{C}}$$

$$V_i = RC\frac{dV_o}{dt}$$

$$\boxed{V_o = \frac{1}{RC}\int V_i\,dt}$$

In terms of steady - state analysis, if we define satisfactory integration as meaning that an input sinusoid has been shifted at least 89.4°.

Then it is necessary, $RC > 15T$

where T = period of the sine wave

Advantages of Integrator over Differentiator.

Integrators are almost invariably preferred over differentiators in analog computer applications for the following reasons.

1) It is easier to stabilize an integrator than differentiator because the gain of an integrator decreases with frequency, where as the gain of differentiator increases with frequency.
2) An integrator is less sensitive to noise voltages than a differentiator because of its limited bandwidth.
3) The amplifier of a differentiator may overload if the input waveform changes very rapidly.
4) It is more convenient to introduce initial conditions in an amplifier.

Applications of High pass RC circuit :

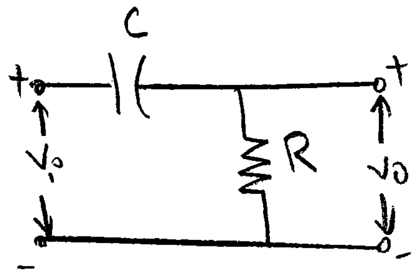
1) As a High pass filter, when $RC \gg T$
2) As a Differentiator, when $RC \ll T$
3) As a peaking circuit, when $RC < T$
4) As a lead network.
5) To generate square wave from triangular wave
6) To generate step from Ramp
7) To generate spikes from Rectangular or square wave form — The spikes are used as Trigger pulses or synchronization pulses in circuits used in television and CRO's.

Applications of Low pass RC circuit :

1) As a Low pass filter, when $RC \ll T$
2) As an integrator, when $RC \gg T$
3) As a lag network.
4) To perform mathematical integration in Analog Computers.
5) To generate triangular wave from square wave.
6) To generate sawtooth wave from rectangular wave
7) To Trigger electronic devices.

Problems High pass RC circuit

(2.2) A 10HZ Symmetrical square wave whose peak-to-peak amplitude is 2V is impressed upon high pass RC circuit whose lower 3-dB frequency is 5HZ. calculate and sketch the output wave form. In particular what is the peak-to-peak output amplitude?



Sol: Given data:

$V_{P-P} = 2V$

$f = 10\,HZ$

$f_L = 5\,HZ$

$V_1 = V_1' = V_2 = V_2' = ?$

$V_{OP-P} = ?$

Time period T of the square wave is

$$T = \frac{1}{f} = \frac{1}{10} = 0.1\,sec$$

$$f_L = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_L} = \frac{1}{2\pi \times 5} = \frac{1}{10\pi} = 0.0318\,sec.$$

given symmetrical square wave

$$T_1 = T_2 = \frac{T}{2} = \frac{0.1}{2} = 0.05\,sec$$

$RC < \frac{T}{2}$, so time taken by the capacitor to charge is less and discharge is also less.

for a Symmetrical square wave to a High pass RC ckt

$$V_1 = -V_2$$
$$V_1' = -V_2'$$

$$V_1 = \frac{V}{1+e^{-T/2RC}} = \frac{2}{1+e^{-0.05/0.0318}} = 1.656V$$

$$V_1' = V_1 e^{-T/2RC} = 1.656 \times e^{-0.05/0.0318} = 0.343V$$

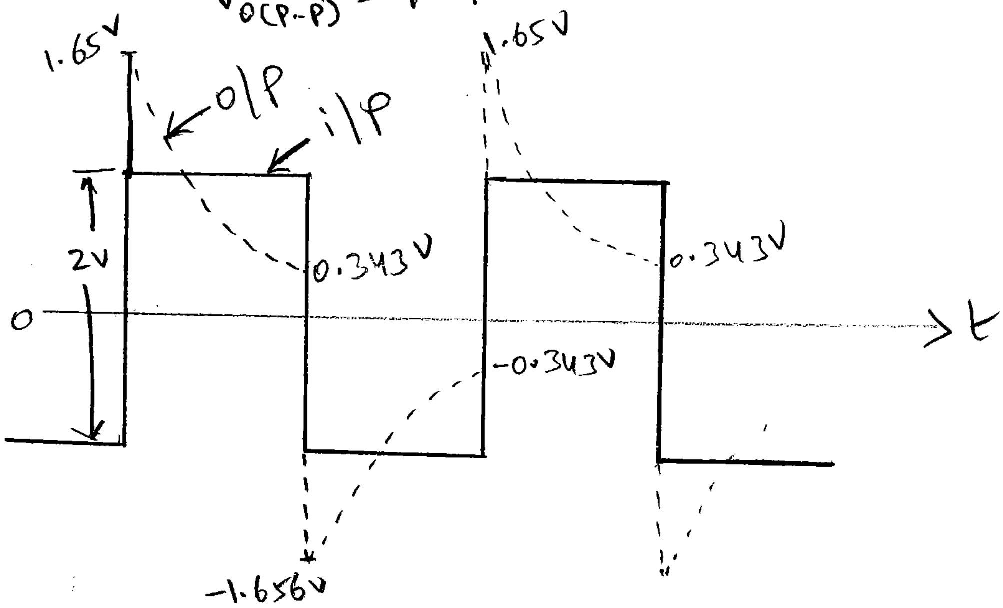
(or)

$$V_1' = \frac{V}{1+e^{T/2RC}} = \frac{2}{1+e^{0.05/0.0318}} = 0.34V$$

$\therefore V_1 = 1.66V,\ V_2 = -1.66V,\ V_1' = 0.34V,\ V_2' = -0.34V$

The peak-to-peak output voltage

$$V_{o(p-p)} = |V_1| + |V_2| = 1.66 + 1.66 = \underline{3.32V}$$



(2-6) A 10-HZ square wave is fed to an amplifier. Calculate and plot the output wave form under the following conditions: The lower 3-dB frequency is a) 0.3HZ b) 3.0HZ c) 30HZ.

Sol: Given data

$f = 10HZ$

$T = \frac{1}{f} = \frac{1}{10} = 0.1\ sec$

$\frac{T}{2} = \frac{0.1}{2} = 0.05\ sec$

a) $f_L = 0.3HZ$

$f_L = \frac{1}{2\pi RC}$

$RC = \frac{1}{2\pi f_L} = \frac{1}{2\pi \times 0.3} = 0.53\ sec$

$RC \gg \frac{T}{2}$, Assume input peak-to-peak voltage is V volts and is symmetrical.

$$V_1 = \frac{V}{1+e^{-T/2RC}} = \frac{V}{1+e^{\frac{-0.05}{0.53}}} = 0.52V$$

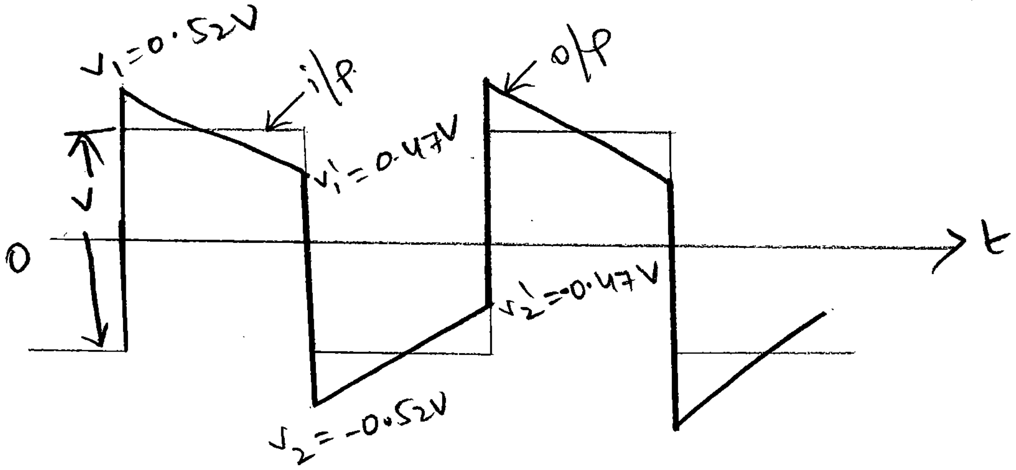
$$V_1' = V_1 e^{-T/2RC} = 0.52 \cdot e^{\frac{-0.05}{0.53}} = 0.47V$$

$$V_2 = -V_1 = -0.52V$$

$$V_2' = -V_1' = -0.47V$$

Peak-to-peak output voltage $= |V_1| + |V_2|$

$= |0.52| + |0.52| = 1.04V$



b) $f_2 = 30\,Hz$

$$\frac{T}{2} = 0.05\,sec$$

$$RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 3} = 0.053\ sec$$

$$RC \simeq \frac{T}{2}$$

$$V_1 = \frac{V}{1+e^{-T/2RC}} = \frac{V}{1+e^{-\frac{0.05}{0.053}}} = 0.719\,V$$

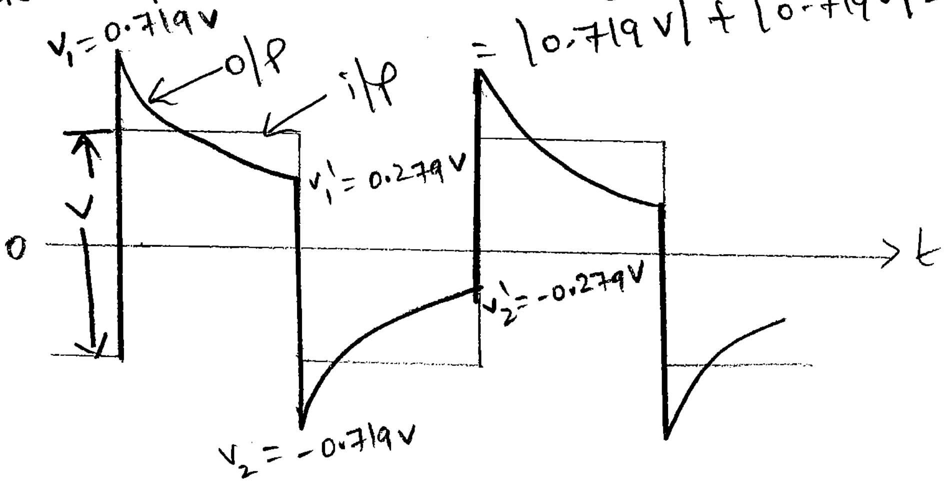
$$V_1' = V_1\, e^{-T/2RC} = 0.719V \cdot e^{\frac{-0.05}{0.053}} = 0.279\,V$$

$$V_2 = -V_1 = -0.719\,V$$

$$V_2' = -V_1' = -0.279\,V$$

Peak-to-peak output voltage $= |V_1| + |V_2|$

$= |0.719\,V| + |0.719\,V| = 1.438V$ volts



c) $f_L = 30Hz$

$\frac{T}{2} = 0.05 sec$

$RC = \frac{1}{2\pi f_L} = \frac{1}{2\pi \times 30} = 0.0053$ sec

$RC << \frac{T}{2}$

$V_1 = \frac{V}{1+e^{-T/2RC}} = \frac{V}{1+e^{-\frac{0.05}{0.0053}}} = 0.99V$ volts

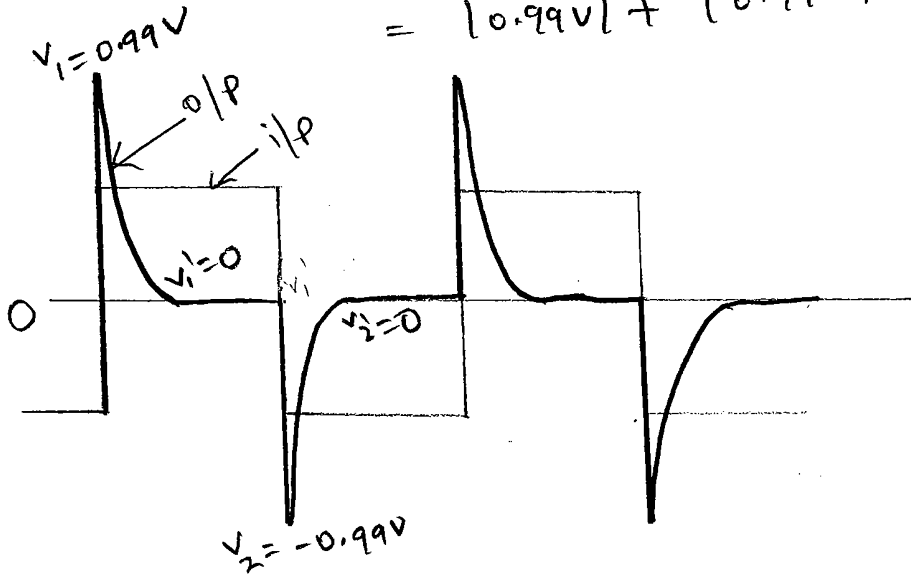
$V_1' = V_1 e^{-T/2RC} = 0.99V \times e^{-\frac{0.05}{0.0053}} = 0.000079V \simeq 0V$

$V_2 = -V_1 = -0.99V$

$V_2' = -V_1' = -0.000079V \simeq 0V$

Peak-to-peak voltage $V_{p-p} = |V_1| + |V_2|$

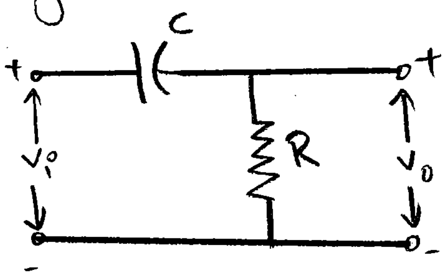
$= |0.99V| + |0.99V| = 1.98V$



(2.4) A square wave whose peak-to-peak value is 1V extends $\pm 0.5V$ with respect to ground. The duration of positive section is 0.1 sec and negative section is 0.2 sec. If this waveform is impressed upon an RC differentiating circuit whose time constant is 0.2 sec. What are the steady state maximum and minimum values of the output waveform?

Sol: Given data:

Unsymmetrical square wave High pass RC circuit



$V_{i(p-p)} = 1V$

$V_{d.c} = 0$

$T_1 = 0.1 sec$

$T_2 = 0.2 sec$

$RC(\tau) = 0.2 sec$

$V_1' = V_1 e^{-T_1/RC} \longrightarrow (1)$

$V_2' = V_2 \cdot e^{-T_2/RC} \longrightarrow (2)$

$V_1' - V_2 = V \longrightarrow (3)$

$V_1 - V_2' = V \longrightarrow (4)$

sub eqn (1) in eqn (3)

$V_1 \cdot e^{-T_1/RC} - V_2 = 1$

sub eqn (2) in eqn (4)

$V_1 - V_2 \cdot e^{-T_2/RC} = 1$

By solving above two equations.

$$v_1 \cdot e^{-T_1/RC} - v_2 = 1$$

$$v_1 - v_2 e^{-T_2/RC} = 1$$

$$\cancel{v_1 \cdot e^{-T_1/RC}} - v_2 = 1$$

$$\cancel{v_1 e^{-T_1/RC}} - v_2 \cdot e^{-T_1/RC} \cdot e^{-T_2/RC} = e^{-T_1/RC}$$

$$v_2\left[e^{-T_1/RC} \cdot e^{-T_2/RC} - 1\right] = 1 - e^{-T_1/RC}$$

$$v_2 = \frac{1 - e^{-\frac{0.1}{0.2}}}{e^{-\frac{0.1}{0.2}} \cdot e^{-\frac{0.2}{0.2}} - 1} = \frac{0.393}{(0.606 \times 0.367 - 1)}$$

$$v_2 = -0.505 \text{ volts}$$

$$v_2' = v_2 \cdot e^{-T_2/RC} = -0.505\, e^{-\frac{0.2}{0.2}}$$

$$v_2' = -0.185 \text{ volts}$$

$$v_1 - v_2' = 1$$

$$v_1 = 1 + v_2'$$

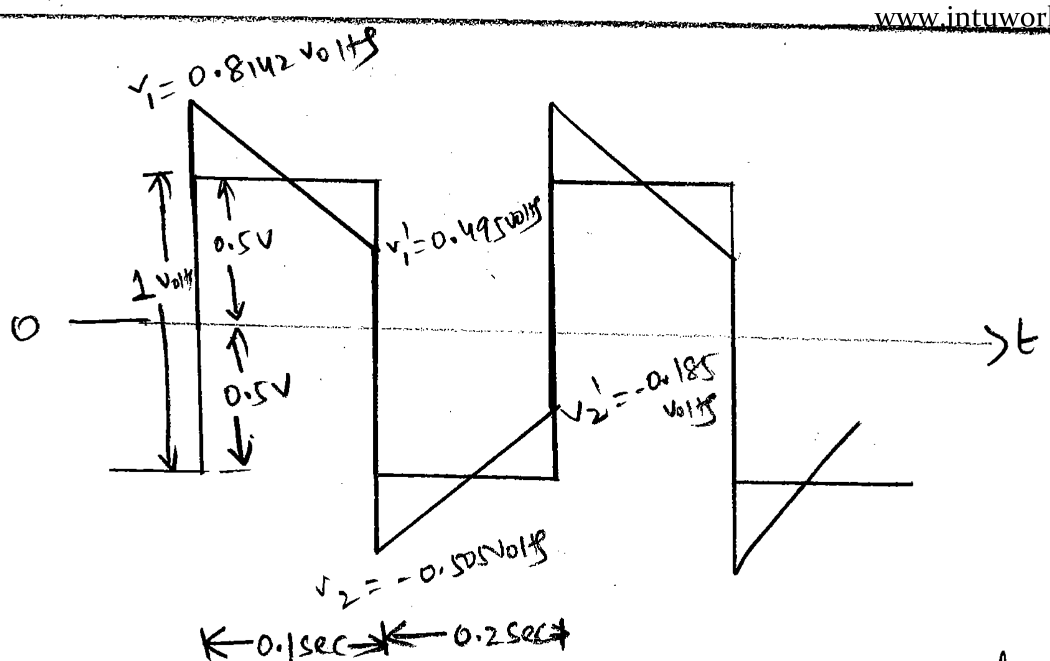
$$v_1 = 1 - 0.185 = 0.8142 \text{ volts}$$

$$v_1' - v_2 = 1$$

$$v_1' = 1 + v_2 = 1 - 0.505 = 0.495 \text{ volts}$$

Peak-to-peak output voltage $= |v_1| + |v_2|$

$$= |0.8142| + |0.505| = 1.319 \text{ volts}$$



(2.8) A square wave whose peak-to-peak value is 1V, extends ±0.5 with respect to ground. The half period is 0.1sec. This voltage is impressed upon R.C differentiating circuit whose time constant is 0.2 sec. Determine the maximum and minimum values of the output voltage in the steady state?

Sol: <u>Given data</u>

$V_{i(p-p)} = 1V$

$T_1 = 0.1sec$

$T_2 = 0.1sec$

$RC(\tau) = 0.2sec$

$V_{dc} = 0V$

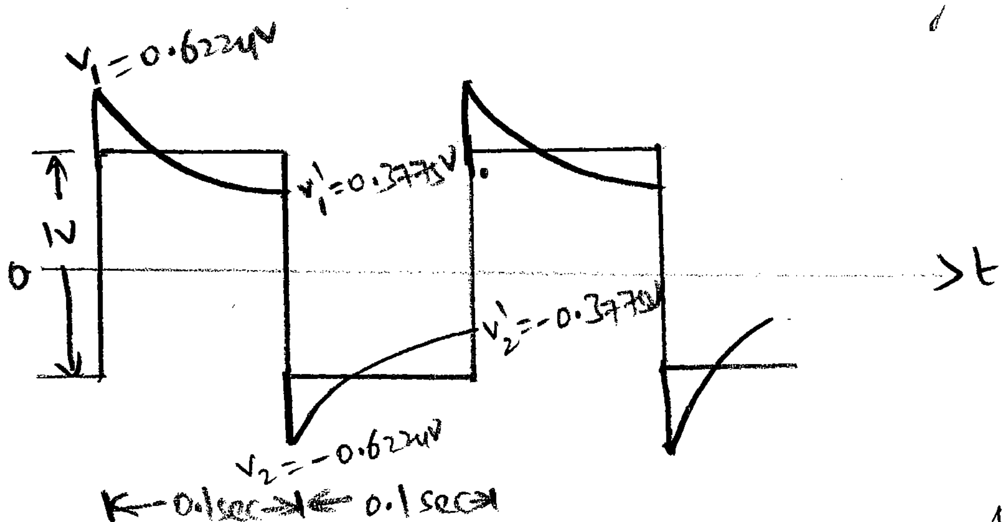
$V_1 = V_1' = V_2 = V_2' = ?$

$RC > T_1$ ie; $RC > \frac{T}{2}$

$\boxed{\cancel{V_1' = V_1 e^{-T_1/RC}}}$

$$V_1 = \frac{V}{1+e^{-T/2RC}} = \frac{1}{1+e^{-\frac{0.2}{2\times 0.2}}} = 0.6224 \text{ volts} = -V_2$$

$$V_1' = V_1 e^{-T/2RC} = 0.6224 \cdot e^{-0.5} = 0.3775 \text{ volts} = -V_2'$$



(2.5) An exponential input is applied to an high pass RC circuit

a) if $n \neq 1$ prove that the peak of the output pulse occurs at $x = 2.30 \frac{n}{n-1} \log n$.

b) if $n = 1$ prove that the peak of the output pulse occurs at $x = 1$.

Sol:

a) for High pass RC circuit by applying an exponential input if $\tau \neq RC$ (or) $n \neq 1$ is

$$V_0 = \frac{V \cdot n}{n-1}\left[e^{-x/n} - e^{-x}\right] \quad \text{——— (1)}$$

Peak value of the output pulse occurs when

$$\frac{dV_0}{dx} = 0$$

differentiate eqn (1) with respect to x

$$\frac{dv_0}{dx} = \frac{v.\eta}{\eta-1}\left[e^{-x/\eta}\cdot -\frac{1}{\eta} - e^{-x}(-1)\right]$$

$$0 = \frac{v.\eta}{\eta-1}\left[-\frac{e^{-x/\eta}}{\eta} + e^{-x}\right]$$

$$0 = -\frac{v\eta}{\eta-1}\frac{e^{-x/\eta}}{\eta} + \frac{v\eta}{\eta-1}e^{-x}$$

$$\cancel{\frac{v\eta}{\eta-1}}\frac{e^{-x/\eta}}{\eta} = \cancel{\frac{v\eta}{\eta-1}}e^{-x}$$

$$e^{-x/\eta} = \eta e^{-x}$$

Taking natural log on both sides

$$-\frac{x}{\eta}\log_e^e = \log_e \eta - x\log_e^e \quad , \text{since } \log_e e = 1$$

$$-\frac{x}{\eta} = \log_e \eta - x$$

$$-\frac{x}{\eta} + x = \log_e \eta$$

$$x\left[-\frac{1}{\eta} + 1\right] = \log_e \eta$$

$$x\left[\frac{\eta-1}{\eta}\right] = \log_e \eta$$

$$x = \frac{\eta}{\eta-1}\log_e \eta$$

$$\log_e \eta = \frac{\log_{10}\eta}{\log_{10}e} \quad , \text{but } \log_{10}e = \frac{\log_e e}{\log_e 10} = \frac{1}{2.3}$$

$\log_e 10$ ↙ $\ln 10$

$$x = \frac{\eta}{\eta-1}\cdot\frac{\log_{10}\eta}{1/2.3}$$

$$\boxed{x = 2.3 \cdot \frac{n}{n-1} \log_{10} n}$$

b) For High pass RC circuit by applying exponential input if $T = RC$ or $n = 1$ is

$$V_o = V \cdot x e^{-x}$$

differentiate with respect to x

$$\frac{dV_o}{dx} = V \cdot e^{-x} + V x e^{-x}(-1)$$

$$0 = V e^{-x} - V x e^{-x}$$

$$0 = V e^{-x}\left[1 - x\right]$$

$$1 - x = 0$$

$$\boxed{x = 1}$$

$\frac{d}{dt}(u \cdot v) = u\frac{dv}{dt} + v\frac{du}{dt}$

let $u = e^{-x}$

$v = Vx$

(2.2) The pulse from a high voltage generator (a magnetron) rises linearly for 0.05 μsec and then remains constant for 1 μsec. The rate of rise of the pulse is measured with an RC differentiating circuit whose time constant is 250 psec. If the positive output voltage from the differentiator has a maximum value of 50V, what is the peak voltage of the generator?

Sol: Given Data:

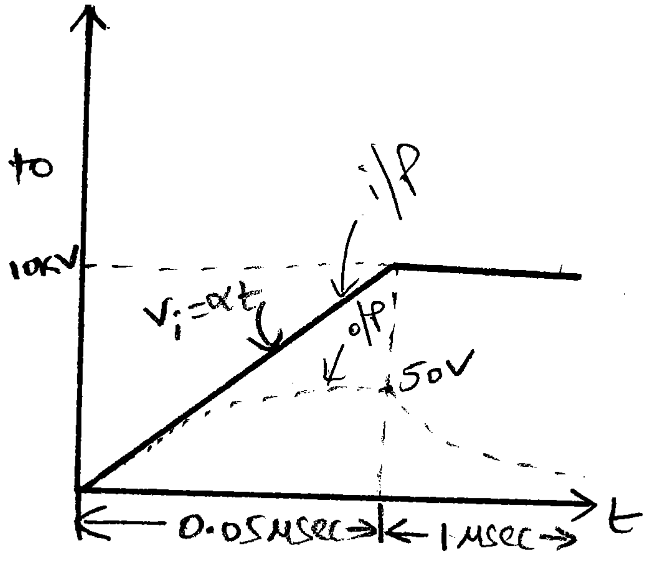
$T = 0.05\,\mu sec$

$RC = 250\,p\,sec = 250 \times 10^{-12}\,sec$

$V_{o(max)} = 50V$.

$RC << T$

By applying ramp input to high pass RC circuit



$$V_o = \alpha RC\left[1 - e^{-t/RC}\right]$$

for $\underline{RC << T}$,

$$V_o = \alpha RC$$

$$50 = \alpha RC$$

$$\alpha = \frac{50}{RC} = \frac{\cancel{50}}{\cancel{250} \times 10^{-12}} = 0.2 \times 10^{12}\ V/sec$$

The peak value of the generator is

$$V_i = \alpha T$$

$$V_i = 0.2 \times 10^{12} \times 0.05\mu sec$$

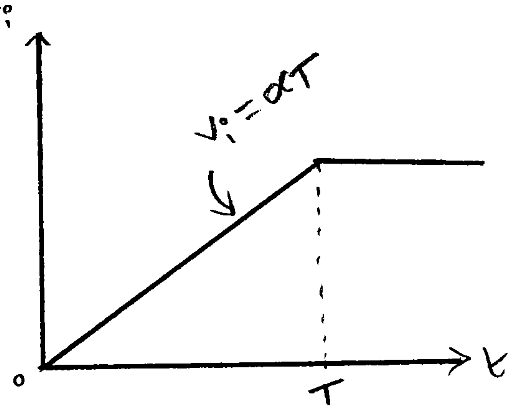
$$V_i = 10000V$$

$$\underline{\underline{V_{i(max)} = 10KV}}$$

(2.11) The Limited ramp is applied to an RC differentiator, what is the peak of the output waveform and draw to scale, the output waveform for the cases:

a) $T = RC$ b) $T = 0.2RC$ c) $T = 5RC$

Sol:



a) The ramp input is applied to an RC high pass circuit, the output voltage is given by

$$V_o = \alpha RC\left[1 - e^{-t/RC}\right]$$

$$V_i = \alpha T$$

$$\alpha = \frac{V_i}{T}$$

$$V_o = \frac{V_i RC}{T}\left[1 - e^{-t/RC}\right]$$

at $T = RC$

$$V_o = \frac{V_i \cancel{RC}}{\cancel{RC}}\left[1 - e^{-\frac{RC}{RC}}\right]$$

$$V_o = V_i\left[1 - e^{-1}\right] = V_i\left[1 - 0.368\right]$$

$$\therefore V_o = 0.632V_i$$

b) at $T = 0.2RC$

$$V_0 = \frac{V_i \cancel{RC}}{0.2\cancel{RC}}\left[1 - e^{-\frac{0.2\cancel{RC}}{\cancel{RC}}}\right]$$

$$V_0 = \frac{V_i}{0.2}\left[1 - e^{-0.2}\right] = \frac{V_i}{0.2}\left[1 - 0.818\right]$$

$$V_0 = 0.90 V_i$$

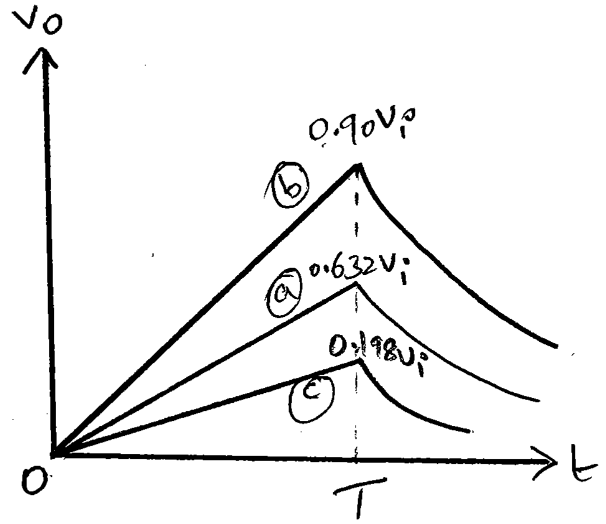
c) at $T = 5RC$

$$V_0 = \frac{V_i \cancel{RC}}{5\cancel{RC}}\left[1 - e^{-\frac{5\cancel{RC}}{\cancel{RC}}}\right]$$

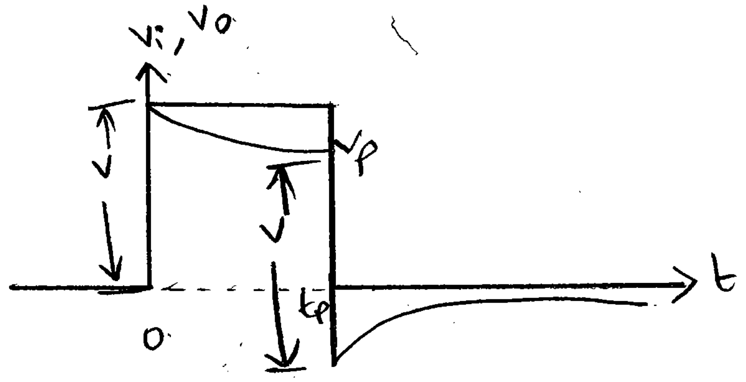
$$V_0 = \frac{V_i}{5}\left[1 - e^{-5}\right] = \frac{V_i}{5}\left[1 - 6.737 \times 10^{-3}\right]$$

$$V_0 = 0.198 V_i$$

The output waveform for different cases are shown below:



(2.1) A pulse of shown below is applied to a high-pass RC circuit. prove by direct integration that the area under the curve of fig shown below is zero.



Sol: for $t<t_p$

$$\text{Area} = \int_0^{t_p} V \cdot e^{-t/RC}\, dt$$

$$= V \int_0^{t_p} e^{-t/RC}\, dt$$

$$A_1 = V\left[\frac{e^{-t/RC}}{-1/RC}\right]_0^{t_p} = -VRC\left[e^{-t_p/RC} - e^{-0}\right]$$

$$A_1 = -VRC\left[e^{-t_p/RC} - 1\right] \quad \text{——— (1)}$$

As $t>t_p$

$$\text{Area} = \int_{t_p}^{\infty} (V_p - V) \cdot e^{-\frac{(t-t_p)}{RC}} \cdot dt$$

$$A_2 = \int_{t_p}^{\infty} \left(Ve^{-t_p/RC} - V\right) \cdot e^{-\left(\frac{t-t_p}{RC}\right)} \cdot dt$$

$$= V \int_{t_p}^{\infty} \cancel{e^{-t_p/RC}} \cdot e^{-t/RC} \cdot \cancel{e^{+\frac{t_p}{RC}}} - e^{-\left(\frac{t-t_p}{RC}\right)} \cdot dt$$

$$= V \int_{t_p}^{\infty} e^{-t/RC} - e^{+\frac{t_p}{RC}} \cdot e^{-t/RC}\, dt$$

$$= V\left[\left(\frac{e^{-t/RC}}{-1/RC}\right)_{t_p}^{\infty} - e^{\frac{t_p}{RC}} \cdot \left(\frac{e^{-t/RC}}{-1/RC}\right)_{t_p}^{\infty}\right]$$

$$= -VRC\left[e^{-\infty} - e^{-\frac{t_p}{RC}} - e^{\frac{t_p}{RC}}\left(e^{-\infty} - e^{-t_p/RC}\right)\right] \qquad e^{-\infty} = 0$$

$$= -VRC\left[-e^{-t_p/RC} - e^{t_p/RC} \times -e^{-t_p/RC}\right]$$

$$= -VRC\left[-e^{-t_p/RC} + e^{0}\right]$$

$$= -VRC\left[-e^{-t_p/RC} + 1\right]$$

$$A_2 = VRC\left[e^{-t_p/RC} - 1\right] \quad \text{—— (2)}$$

Total area $(A) = A_1 + A_2$

$$= \cancel{-VRC\left[e^{-t_p/RC} - 1\right]} + \cancel{VRC\left[e^{-t_p/RC} - 1\right]}$$

Total area $= A = 0$

(1) An oscilloscope has a coupling capacitor $C_c = 1\mu F$. When the input resistance $R_i = 1M\Omega$, determine the minimum square wave frequency that can be displayed if the tilt is not to exceed 1% ?

Sol: Given data:

$R_i = 1M\Omega$

$C_c = 1\mu f$

% tilt = 1%

$f = ?$

$$\% \text{ tilt} = \frac{T}{2RC} \times 100$$

$$T = 2RC \times \% \text{ tilt} \times 100$$

$$T = 2 \times 1 \times 10^6 \times 1 \times 10^{-6} \times \frac{1}{100} \times 100.$$

$$T = 0.01 \cancel{\times} 2 = 0.02 \text{ sec}$$

$$f = \frac{1}{T} = \frac{1}{0.02} = 50 Hz$$

(2) Calculate the lowest square wave frequency that can be passed by an amplified with a cut-off frequency of 10 Hz, if the output tilt is not to exceed 2% ?

Sol: Given data

$f_1 = 10 Hz$, % tilt = 2%

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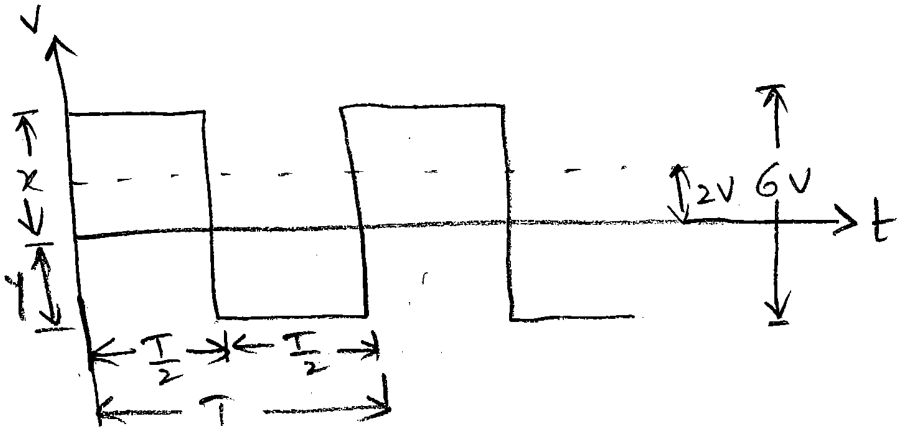
$$\% \text{ tilt} = \frac{\pi f_1}{f} \times 100$$

$$2\% = \frac{\pi \times 10}{f} \times 100$$

$$f = \frac{\pi \times 10 \times 100}{2} = 1570.8 \text{ Hz}$$

③ A DC voltmeter indicates 2V when measuring a square wave with a peak-to-peak amplitude of 6V. Calculate the positive and negative amplitudes of the square wave.



Sol: Let T = period of square wave
x = amplitude of positive peak
y = amplitude of negative peak

Given peak-to-peak amplitude $= x + y = 6V$ — (1)

Since DC voltmeter indicates average value

$$V_{dc} = \frac{\text{Positive area} - \text{negative area}}{\text{period}} = \frac{x\left(\frac{T}{2}\right) - y\left(\frac{T}{2}\right)}{\frac{T}{2} + \frac{T}{2}}$$

$$V_{dc} = \frac{x - y}{2}$$

i.e., $\frac{x-y}{2} = 2$

$x - y = 4$ —— (2)

solving (1) & (2)

$$\begin{aligned} \cancel{x} + y &= 6 \\ \cancel{x} - y &= 4 \\ \hline 2y &= 2 \end{aligned}$$

$y = \underline{1}$

sub y value in equ (1)

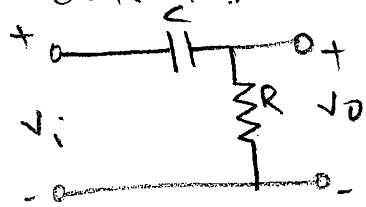
$x + y = 6$

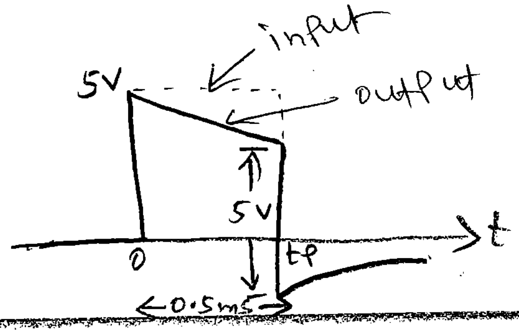
$x + 1 = 6$

$x = \underline{5}$

∴ positive peak amplitude = 5V
negative peak amplitude = 1V

(4) A pulse of 5V amplitude and 0.5 ms duration is applied to an RC high pass circuit with R = 22kΩ and C = 0.47μF. Sketch the output waveform and determine the percentage tilt in the output?





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Sol: Given Data:

$R = 22k\Omega$

$C = 0.47\mu f$

$V = 5V$

$t_p = 0.5ms$

$\tau = RC = 22\times10^3 \times 0.47\times10^{-6} = 10.34ms.$

$\therefore \quad RC >> t_p$

i) $0<t<t_p$

$$V_0 = Ve^{-t/RC} = 5e^{-t/RC}$$

ii) at $t=t_p$

$$V_p = V_0 = 5e^{-t_p/RC} = 5e^{-0.5/10.34} = 4.763 \text{ volts}$$

iii) at $t>t_p$.

$$V_0 = (V_p - V)e^{-(t-t_p)/RC}$$

$$V_0 = (4.763-5)e^{-(t-0.5\times10^{-3})/10.34\times10^{-3}}$$

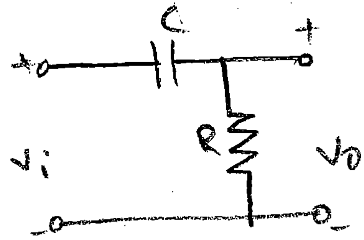
$$V_0 = -0.237e^{-(t-0.5\times10^{-3})/10.34\times10^{-3}}$$

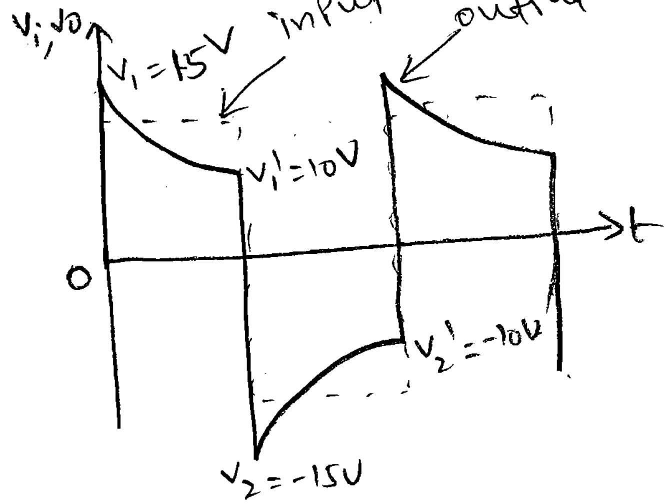
$$\%\text{ tilt }(P) = \frac{V_1 - V_1'}{V/2}\times100$$

$$= \frac{5-4.763}{5/2}\times100$$

$P = $ ___

(5) If a square wave of 5KHZ is applied to an RC high pass circuit and resultant waveform measured on a CRO was tilted from 15V to 10V, find out the lower 3-dB frequency of the high-pass circuit?





Sol: Given data:

$f = 5KHz$

$V_1 = 15V$

$V_1' = 10V$

$f_1 = ?$

Peak to peak voltage $(V) = |V_1| + |V_2| = 15 + 15 = 30$

$$\% \text{ tilt} = \frac{V_1 - V_1'}{V/2} \times 100$$

$$P = \frac{15-10}{30/2} \times 100$$

$$P = 33.33\%.$$

$$P = \frac{\pi f_1}{f} \times 100$$

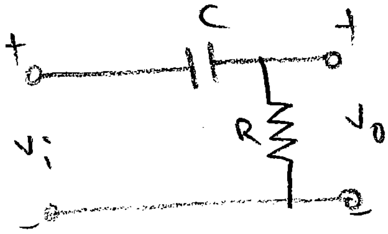
$$33.33 = \frac{3.14 \times f_1}{5 \times 10^3} \times 100$$

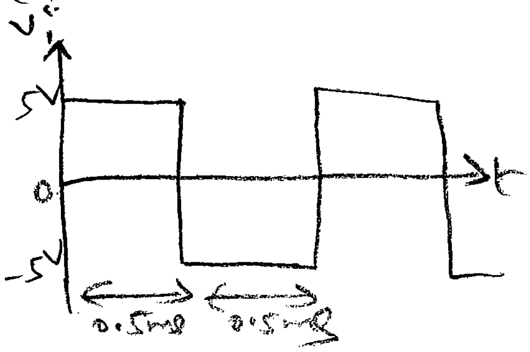
$f_1 = 530.56\,Hz$

⑥ A symmetrical square wave is applied to high-pass circuit having $R = 20k\Omega$, $C = 0.05\mu f$

a) If the frequency of input signal is 1KHz and signal swings between $\pm 5V$, draw the output waveform and indicate the voltages.

b) what happens if the frequency of the signal is reduced to 100Hz? show the output waveform?





Sol: $\tau = RC = 20\times10^3 \times 0.05\times10^{-6} = 1ms$

Given $f = 1KHz$

$$\therefore T = \frac{1}{f} = 1ms$$

$$T/2 = 0.5ms$$

$$v_1 = \frac{V}{1+e^{-T/2RC}} = \frac{10}{1+e^{-0.5\times10^{-3}/10^{-3}}} = 6.23V$$

$$v_1' = \frac{V}{1+e^{T/2RC}} = \frac{10}{1+e^{0.5}} = 3.78V$$

$$v_2 = -v_1 = -6.23V$$

$$v_2' = -v_1' = -3.78V$$

b) If f is reduced to 100 Hz

$$T = \frac{1}{f} = \frac{1}{100} = 10\,ms$$

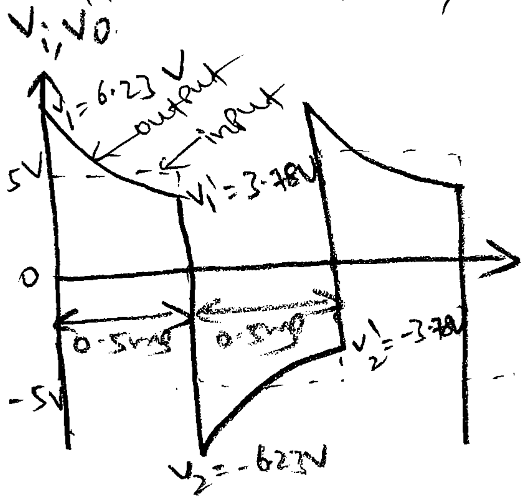
$$v_1 = \frac{V}{1+e^{-T/2RC}} = \frac{10}{1+e^{-5}} = 9.93\,V$$

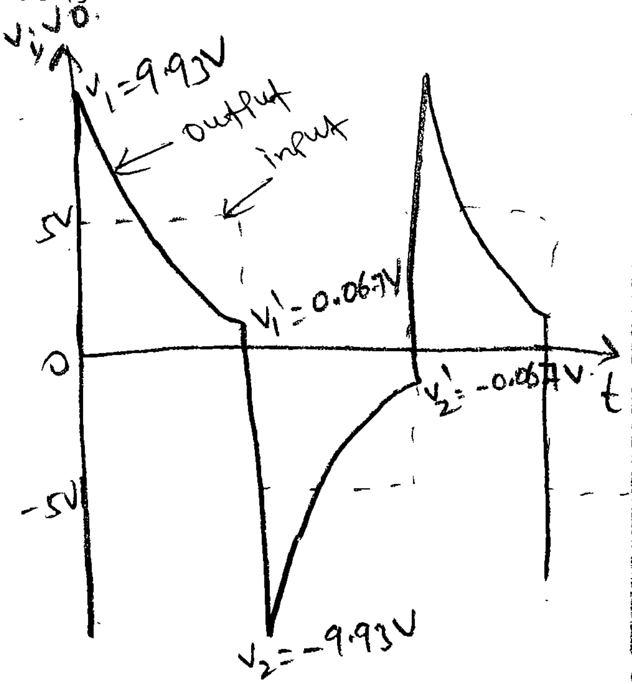
$$v_2 = -v_1 = -9.93\,V$$

$$v_1' = \frac{V}{1+e^{T/2RC}} = \frac{10}{1+e^{5}} = 0.067\,V$$

$$v_2' = -v_1' = -0.067\,V$$

∴ when i/p signal frequency is reduced from 1KHZ to 100HZ, Ckt acts as differentiator.





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(7) A 100HZ triangular wave with peak-to-peak amplitude of 9V is applied to a differentiating circuit with $R = 1M\Omega$, $C = 100pF$. Calculate and sketch the waveform of the output?

Sol: time constant $(\tau) = RC = 1\times10^{6} \times 100\times10^{-12} = 100\,\mu s$

frequency of triangular wave $(f) = 100HZ$

time period $(T) = \frac{1}{f} = \frac{1}{100} = 0.01s = 10ms$

$RC << T$, so it acts as differentiator

for positive rising

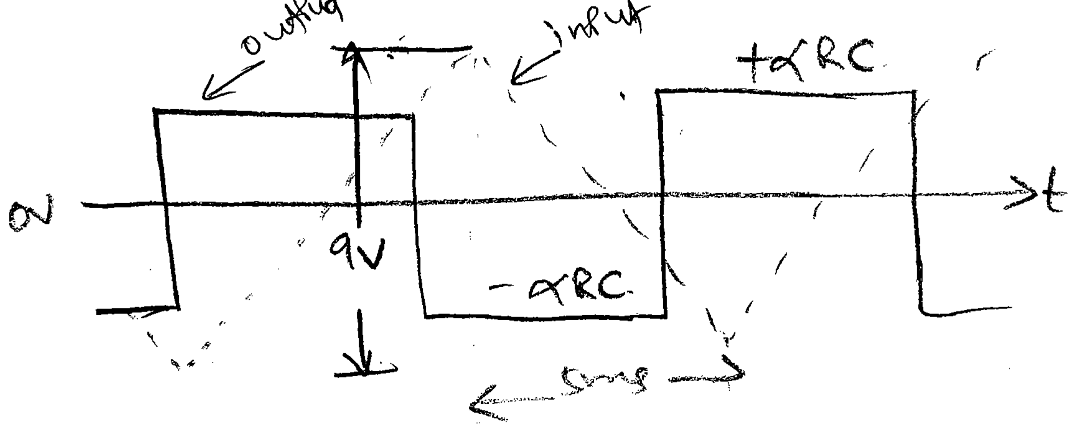
$V_0 = \alpha RC$

but $\alpha = \frac{V_i}{T} = \frac{9}{5ms}$

$V_0 = \frac{9}{5\times10^{-3}} \times 100\times10^{-6} =$

for negative rising

$V_0 = -\alpha RC = -\frac{9}{5\times10^{-3}} \times 100\times10^{-6} =$



Low pass RC circuit

(2-16) An ideal 1 μ-sec pulse is fed to an amplifier. Calculate and Plot the output waveform under the following conditions. The upper 3-dB frequency is

a) 10 MHZ b) 1 MHZ c) 0.1 MHZ

Sol: Given data:

$t_p = 1\ \mu\text{-sec}$

given upper 3-dB frequency, Low pass RC circuit.

a) $f_2 = 10\text{MHZ}$

$$f_2 = \frac{1}{2\pi RC}$$

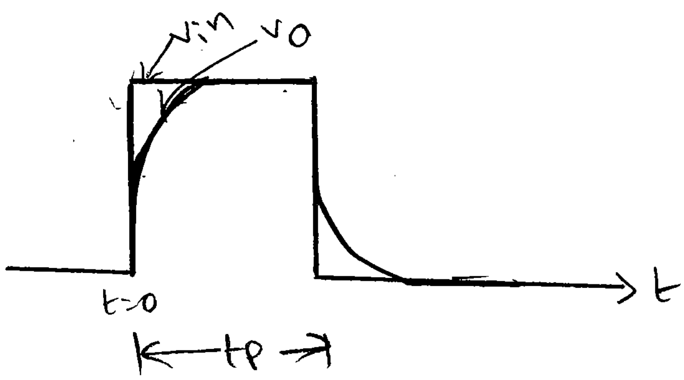
$$RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 10 \times 10^6} = 0.015\ \mu\,\text{sec.}$$

$RC << t_p$

$$V_0 = V_{in}\left[1 - e^{-t_p/RC}\right]$$

$$V_0 = V\left\{1 - e^{-1/0.015}\right]$$

$$V_0 \simeq V_{in}\ \text{volts}$$



b) $f_2 = 1\ MHz$

$$f_2 = \frac{1}{2\pi RC}$$

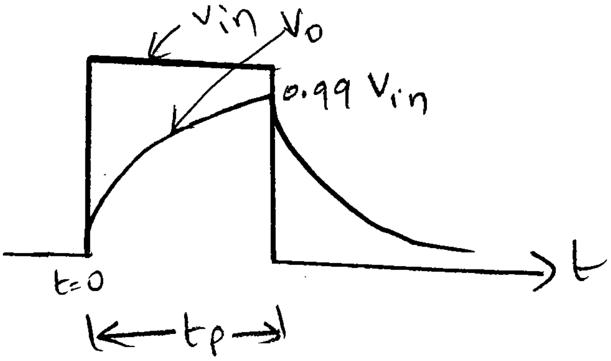
$$RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 10^6} = 0.154\ \mu sec$$

$RC < t_p$

$$V_0 = V_{in}\left[1 - e^{-t_p/RC}\right]$$

$$V_0 = V_{in}\left[1 - e^{-1/0.15}\right]$$

$$V_0 \simeq 0.99\, V_{in}$$



c) $f_2 = 0.1\ MHz$

$$f_2 = \frac{1}{2\pi RC}$$

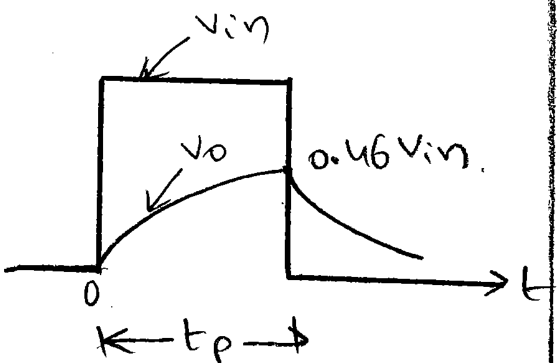
$$RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times 0.1 \times 10^6} = 1.59\ \mu sec$$

$RC > t_p$

$$V_0 = V_{in}\left[1 - e^{-t_p/RC}\right]$$

$$V_0 = V_{in}\left[1 - e^{-1/1.59}\right]$$

$$V_0 = 0.46\, V_{in}$$



(2-19) A symmetrical square wave whose peak-to-peak amplitude is 2V and whose average value is zero is applied to an RC integrating circuit. The time constant equals the half period of the square wave. Find the peak-to-peak value of the output amplitude.

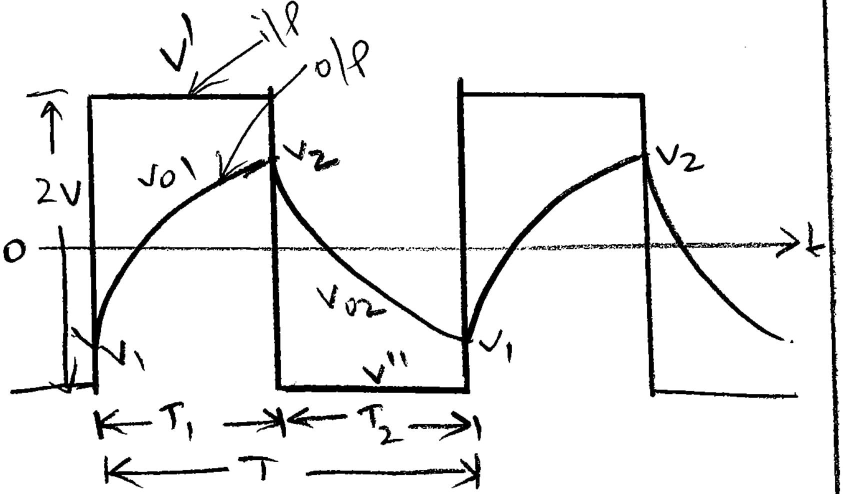
Sol: Given Data:

$V_{i(p-p)} = 2V$

$V_{d.c} = 0V$

$RC = \frac{T}{2}$

i.e; $RC < T$



During positive half cycle

$$V_0 = V_f + (V_i - V_f)\, e^{-t/\tau}$$

$$V_{01} = V' + (V_1 - V')\, e^{-t/RC}$$

$$V' = -V'' = 1V \qquad \text{at } t = T_1,\ V_0 = V_{01} = V_2$$

$$V_2 = 1 + (V_1 - 1)\, e^{-T_1/RC}$$

$$V_2 = 1 + (V_1 - 1)\, e^{-\frac{T/2}{T/2}} \qquad \text{, Symmetrical wave so } T_1 = T_2 = \frac{T}{2}$$

$$V_2 = 1 + (V_1 - 1)\, e^{-1}$$

$$V_2 = 1 + V_1 e^{-1} - e^{-1}$$

$$V_2 = V_1 e^{-1} + 0.632$$

$$V_1 e^{-1} = V_2 - 0.632 \quad \text{—— (1)}$$

During negative half cycle

$$V_0 = V_f + (V_i - V_f)\, e^{-t/RC}$$

$$V_{o2} = V'' + (V_2 - V'')\, e^{-t/RC}$$

at $t = T_2 = \frac{T}{2}$, $V_0 = V_{o2} = V_1$, $V'' = -1$

$$V_1 = -1 + (V_2 + 1)\, e^{-T_2/RC}$$

$$V_1 = -1 + (V_2 + 1)\, e^{-T/2 / T/2}$$

$$V_1 = -1 + (V_2 + 1)\, e^{-1}$$

$$V_1 = -1 + V_2 e^{-1} + e^{-1}$$

$$V_1 = V_2 e^{-1} - 0.632 \quad \text{— (2)}$$

$$V_2 e^{-1} = V_1 + 0.632 \quad \text{— (2)}$$

solving eqn (1) and eqn (2)

$$V_1 e^{-1} = V_2 - 0.632$$

$$V_2 e^{-1} = V_1 + 0.632$$

Solving (1) & (2)

$$V_1 e^{-1} = V_2 - 0.632$$

$$V_1 = V_2 e^{-1} - 0.632$$

$$V_1 e^{-1} = V_2 - 0.632$$

$$V_1 e^{-1} = V_2 e^{-2} - 0.632 e^{-1}$$

$$-\qquad -\qquad +$$

$$V_2 (1 - e^{-2}) + 0.632 (e^{-1} - 1)$$

$$V_2(1-e^{-2}) = 0.632 \times 0.367 - 0.632$$

$$V_2(1-0.135) = 0.23 - 0.632$$

$$V_2 \times 0.865 = -0.4$$

$$V_2 = -0.46 \text{ Volts}$$

Solving ① & ②

$$V_1 = V_2 e^{-1} - 0.632$$

$$V_1 e^{-1} = V_2 - 0.632$$

$$\cancel{V_1 e^{-1}} = V_2 e^{-2} - 0.632 e^{-1}$$

$$\cancel{V_1 e^{-1}} = V_2 - 0.632$$

$$0 = V_2(e^{-2}-1) + 0.632(1-e^{-1})$$

$$V_2(e^{-2}-1) = -0.632(1-e^{-1})$$

$$V_2 = \frac{-0.399}{-0.864} = 0.461 \text{ Volts}$$

sub V_2 in equ ②

$$V_1 = 0.461 \times e^{-1} - 0.632$$

$$V_1 = -0.462 \text{ Volts}$$

∴ Peak-to-Peak output voltage $= |V_1| + |V_2|$

$$V_{o\,P-P} = 0.461 + 0.462$$

$$V_{oP-P} = 0.922 \text{ Volts}$$

(or)

$$V_2 = \frac{V}{2} \tanh x$$

where $x = \frac{T}{4RC}$

$$V_2 = -V_1 = \frac{V}{2}\left[\frac{1-e^{-T/2RC}}{1+e^{-T/2RC}}\right]$$

45

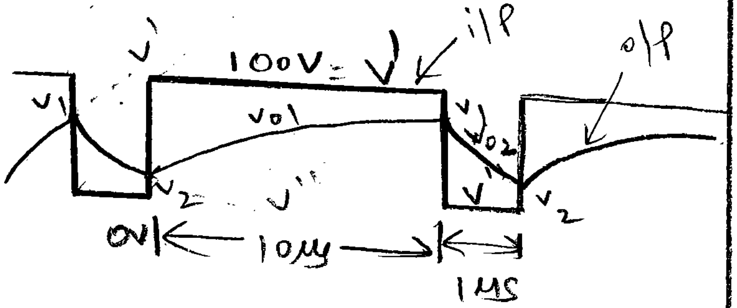
(2.20) The periodic waveform shown below is applied to an RC integrating network whose time constant is 10 μs. Sketch the output waveform. Calculate the maximum and minimum values of the output voltage with respect to ground under steady state conditions. Also, calculate and plot the output for the first two cycles of the input.

Sol: Given data:

$RC = 10\,\mu s$

$T_1 = 10\,\mu s$

$T_2 = 1\,\mu s$



i) between $0 < t < 10\,\mu s$: $V' = 100V$

for rising portion

$$V_0 = V_f + (V_i - V_f)\, e^{-t/RC}$$

$$V_{01} = V' + (V_2 - V')\, e^{-t/10}$$

$$V_{01} = 100 + (V_2 - 100)\, e^{-t/10}$$

ii) At $t_1 = 10\,\mu s$, $V_{01} = V_1$

$$V_1 = V' + (V_2 - V')\, e^{-t_1/RC}$$

$$V_1 = 100 + (V_2 - 100)\, e^{-10/10}$$

$$V_1 = 63.21 + 0.368 V_2 \quad \text{--- (1)}$$

iii) between $10\,\mu s < t < 11\,\mu s$ $V'' = 0V$

$$V_{02} = V'' + (V_1 - V'')\, e^{-(t - T_1)/RC}$$

$$V_{02} = 0 + (V_1 - 0)\, e^{-(t-10)/10}$$

iv) at $t = 11\mu s$, $v_{o2} = V_2$

$$V_2 = 0 + V_1 e^{-1/10} = 0.905 V_1 \quad \text{--- (2)}$$

substitute eqn (2) in eq (4)

$$V_1 = 63.22 + 0.368 V_2$$

$$V_1 = 63.22 + 0.368 \times 0.905 V_1$$

$$V_1 = 63.22 + 0.33 V_1$$

$$V_1 - 0.33 V_1 = 63.22$$

$$V_1 (1 - 0.33) = 63.22$$

$$V_1 = \frac{63.22}{0.67} = 94.350$$

$$\underline{V_1 = 94.35\,V}$$

substitute V_1 in eqn (2)

$$V_2 = 0.905 V_1 = 0.905 \times 94.35$$

$$V_2 = \underline{85.39}\,V$$

(2-21) A symmetrical square wave whose average value is zero has a peak-to-peak amplitude of 20V and a period of 2 μ sec. This waveform is applied to a Low pass RC circuit whose upper 3-dB frequency is $\frac{1}{2\pi}$ MHz. Calculate and sketch the steady-state output waveform. In particular, what is the peak-to-peak amplitude?

1.6

Sol: Given data:

$V_{i(P-P)} = 20V$

$T = 2\mu sec$

$f_2 = \frac{1}{2\pi} MHz$

$T_1 = T_2 = \frac{T}{2} = 1\mu sec.$

$f_2 = \frac{1}{2\pi RC}$

$RC = \frac{1}{2\pi f_2} = \frac{1}{2\pi \times \frac{1}{2\pi} \times 10^6} = 10^{-6} sec = 1\mu sec.$

$V_0 = V_f + (V_i - V_f) e^{-t/RC}$

$V_{01} = V' + (V_1 - V') e^{-t/RC}$

at $t = T_1$, $V_{01} = V_2$

$V_2 = 10 + (V_1 - 10) e^{-\frac{1\mu sec}{1\mu sec}}$

$V_2 = 10 + (V_1 - 10) e^{-1}$

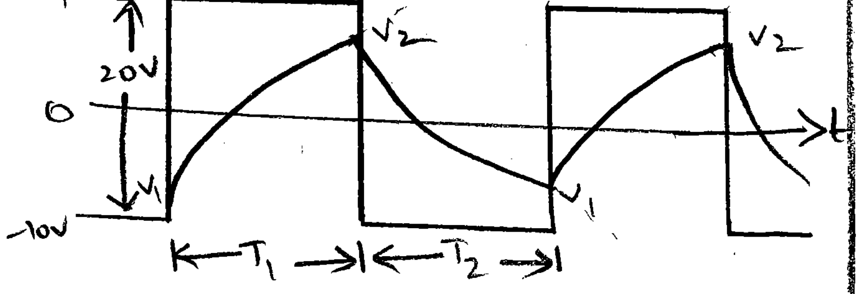
Due to symmetry, $V_1 = -V_2$

$V_2 = 10 + (-V_2 - 10) e^{-1}$

$V_2 = 4.619V$

$\therefore V_1 = -4.619V$

$V_{O(P-P)} = |V_1| + |V_2| = 2 \times 4.619 = \underline{9.23} \text{ volts}$



(2-22) A square wave whose peak-to-peak amplitude is 2V extends ±1V with respect to ground. The duration of positive section is 0.1 sec and that of the negative section is 0.2 sec. If this waveform is impressed upon an RC integrating circuit whose time constant is 0.2 sec. What are the steady-state maximum and minimum values of the output waveform?

sol: Given data:

$V_{p-p} = 2V$

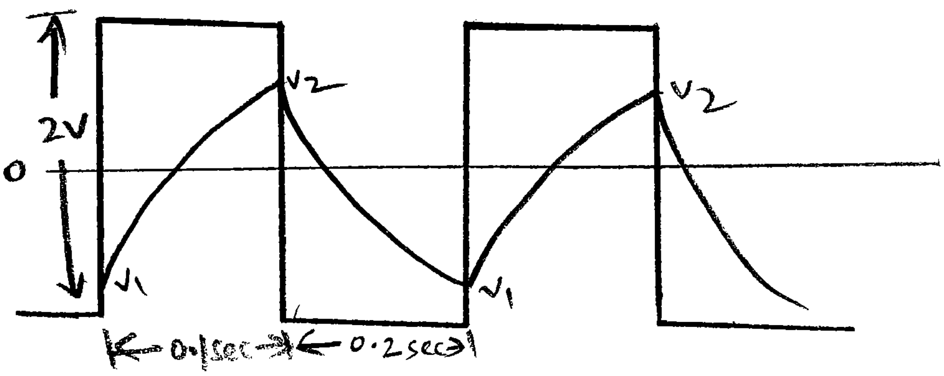
$T_1 = 0.1\,sec$

$T_2 = 0.2\,sec$

$RC\,(\tau) = 0.2\,sec$

$T = T_1 + T_2 = 0.3\,sec$

$RC > T_1,\ RC = T_2$



$V_0 = V_f + (V_i - V_f)\,e^{-t/RC}$

for $t = T_1$: $V_0 = V_2$, $V_f = V' = 1V$, $V_{in} = V_1$

$V_2 = 1 + (V_1 - 1)\,e^{-T_1/RC}$

$$V_2 = 1 + (V_1 - 1)\, e^{-\frac{0.1}{0.2}}$$

$$V_2 = 1 + (V_1 - 1)\, e^{-\frac{1}{2}}$$

$$V_2 = 1 + 0.606 V_1 - 0.606.$$

$$0.606 V_1 - V_2 = -0.393 \quad \text{—— } (1)$$

for $\underline{t = T_2}$, $V_0 = V_1$, $V_f = V'' = -1$, $V_{in} = V_2$

$$V_1 = -1 + (V_2 + 1)\, e^{-\frac{0.2}{0.2}}$$

$$V_1 = -1 + 0.368 V_2 + 0.368$$

$$V_1 - 0.368 V_2 = -0.632 \quad \text{—— } (2)$$

solving (1) & (2)

$$0.606 V_1 - V_2 = -0.393$$

$$V_1 - 0.368 V_2 = -0.632$$

$$0.606 V_1 - V_2 = -0.393$$

$$0.606 V_1 - 0.223 V_2 = -0.3829$$

$$\quad - \quad\quad + \quad\quad\quad +$$

$$-0.777 V_2 = -0.01$$

$$\therefore V_2 = \underline{0.012} \text{ volt}$$

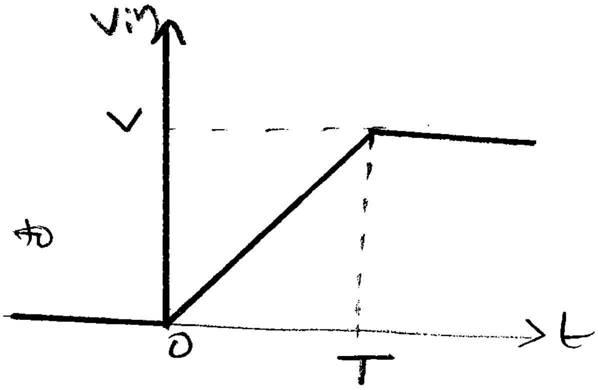
Sub V_2 in eqn (1)

$$0.606 V_1 - 0.012 = -0.393$$

$$\therefore V_1 = \underline{-0.628} \text{ volt}$$

(2-26) The Limited ramp of below fig is applied to a Low-pass RC integrating circuit. Draw to scale the output waveforms for the cases:

a) $T = RC$ b) $T = 0.2RC$ c) $T = 5RC$



sol:

a) $T = RC$

By applying ramp input to high-pass RC circuit

output voltage

$$V_o = \alpha RC\left[1 - e^{-t/RC}\right]$$

By applying to Low-pass RC circuit

output voltage

$$v_o = \alpha t - \alpha RC\left(1 - e^{-t/RC}\right)$$

if $T = RC$

$$v_o = \alpha RC - \alpha RC\left(1 - e^{-RC/RC}\right)$$

$$v_o = \cancel{\alpha RC} - \cancel{\alpha RC} + \alpha RC e^{-1}$$

$$v_o = \alpha RC \times 0.367 \text{ volts}$$

$$v_i = \alpha t \Rightarrow \alpha = \frac{v_i}{t} = \frac{V}{RC}$$

$$v_o = \frac{V}{\cancel{RC}} \cdot \cancel{RC} \times 0.367$$

$$v_o = 0.367\,V \text{ volts}$$

b) $T = \underline{0.2RC}$

$$V_0 = \alpha t - \alpha RC\left(1 - e^{-T/RC}\right)$$

$$V_0 = \alpha RC(0.2) - \alpha RC + \alpha RC\, e^{-\frac{0.2RC}{RC}}$$

$$V_0 = \alpha RC\left(0.2 - 1 + e^{-0.2}\right)$$

$$V_0 = \alpha RC \times 0.018$$

$$V_0 = \frac{V}{0.2\cancel{RC}} \cdot \cancel{RC} \times 0.018$$

$$V_0 = 0.093V$$

$$V_0 \simeq \underline{0.1V} \text{ volts}$$

c) $T = \underline{5RC}$

$$V_0 = \alpha T - \alpha RC\left(1 - e^{-T/RC}\right)$$

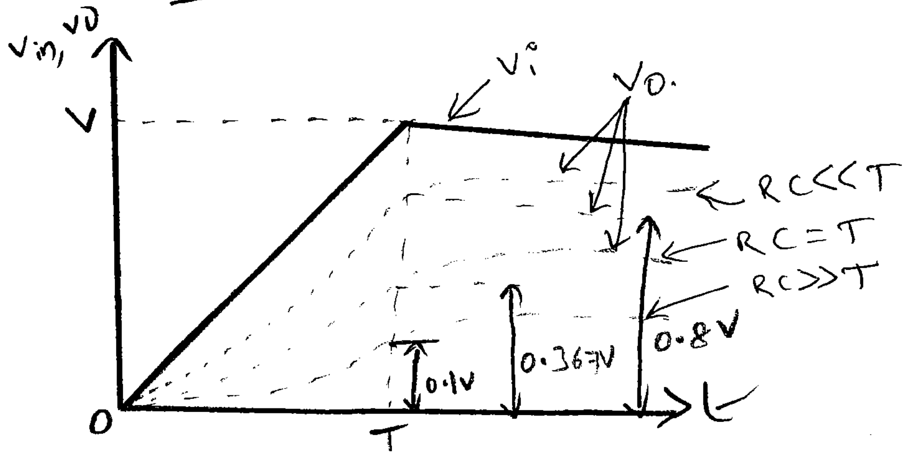
$$V_0 = 5\alpha RC - \alpha RC\left(1 - e^{-5RC/RC}\right)$$

$$V_0 = \alpha RC\left(5 - 1 + e^{-5}\right)$$

$$V_0 = \alpha RC \times 4.0067$$

$$V_0 = \frac{V}{5\cancel{RC}} \cdot \cancel{RC} \times 4.0067$$

$$V_0 = \underline{0.8V} \text{ volts}$$



problem :

The output wave-form from an amplifier under pulse test has a rise time of 1 μ sec. Determine the upper 3-dB frequency of the amplifier?

sol : Given data:

$t_r = 1\,\mu sec$

$f_2 = ?$

$$f_2 = \frac{0.35}{t_r} = \frac{0.35}{1\times10^{-6}} = 350\,KHZ$$

" For two stages whose individual rise times are t_{r_1} and t_{r_2} respectively. The resultant rise time t_r is given by.

$$\boxed{t_r = 1.05\sqrt{t_{r_1}^2 + t_{r_2}^2}}$$ "

Problem :

A pulse wave with a rise time of 10 μ sec is applied to an amplifier which has an upper cut-off frequency of 350 KHZ. Calculate the rise time in the output wave form.

sol : $t_{rs} = 10\,\mu sec$ ← i/p rise time

$$t_{rc} = \frac{0.35}{f_h} = \frac{0.35}{350k} = 1\,\mu sec$$

output rise time is

$$t_r = 1.05\sqrt{(t_{rs})^2 + (t_{rc})^2}$$

$$t_r = 1.05\sqrt{(10\times10^{-6})^2 + (1\times10^{-6})^2}$$

$$t_r = 1.05\sqrt{10^{-10} + 10^{-12}}$$

$$t_r = 1.055\times10^{-5}$$

$$t_r \simeq 10.55\,\mu sec$$

→ Let the time of individual stages be $t_{r1}, t_{r2}, \cdots t_{rn}$ and let the input waveform rise time be t_{ro}, then the output signal rise time is given by

$$\boxed{t_r = 1.1\sqrt{t_{ro}^2 + t_{r1}^2 + t_{r2}^2 + \cdots}}$$

2-28 a) prove that an RC circuit behaves as a reasonably good integrator if $RC \gg 15T$, where T is the period of an input sinusoid $E_m \sin\omega t$.

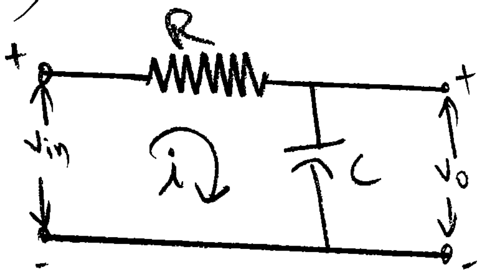
[or]

The condition for good integration is $RC \gg 15T$

b) show that the output is approximately

$$-\left[\frac{E_m}{\omega RC}\right]\cos\omega t.$$

Sol: a)



Apply KVL at output side

$$V_0 = \frac{X_C}{j} \times I$$

where $I = \dfrac{V_{in}}{R + \frac{X_C}{j}}$, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega C}$

$$V_0 = \frac{1/j\omega C}{R + 1/j\omega C} \times V_{in}$$

$$V_0 = \frac{V_{in}}{1 + j\omega RC}$$

if $V_i = E_m \sin\omega t$

$$V_0 = \frac{E_m \sin\omega t}{1 + j\omega RC}$$

$$\therefore |V_0| = \frac{V_m}{\sqrt{1 + \omega^2 R^2 C^2}} \sin(\omega t - \theta)$$

where $\theta = \tan^{-1}(\omega RC)$

$\theta = \tan^{-1}(2\pi f RC)$

$\theta = \tan^{-1}\left(\frac{2\pi}{T} RC\right)$

Low pass RC circuit acts as an integrator if θ reaches near to 90°.

$$\theta = \tan^{-1}\left(\frac{2\pi}{T} \times RC\right)$$

if $RC >> 15T$ or at $RC = 15T$

$$\theta = \tan^{-1}\left(\frac{2\pi}{\cancel{T}} \times 15\cancel{T}\right)$$

$$\theta = \tan^{-1}(30\pi)$$

$$\theta = \underline{\underline{89.40^\circ}}$$

If we define satisfactory integration, means that input sinusoid has been shifted atleast 89.4° or 90°, then it is necessary that $RC > 15T$.

where T = Time period of the sinewave.

b) $$V_0 = \frac{V_i}{1 + j\omega RC}$$

$$V_0 = \frac{V_i}{1 + j(\text{large } \omega RC)}$$, since $\omega RC >> 1$, we can neglect 1

$$V_0 = \frac{V_i}{j(\text{large } \omega RC)}$$

$$V_0 = \frac{-V_i\, j}{\omega RC}$$

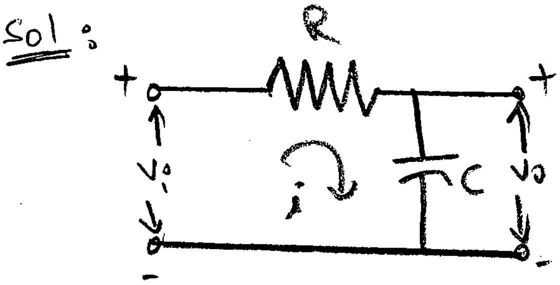
but $V_i = E_m \sin \omega t$

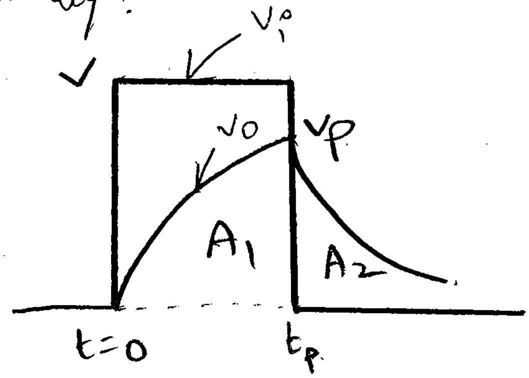
$$V_0 = \frac{-j E_m \sin \omega t}{\omega RC} = -\frac{E_m}{\omega RC} \cos \omega t$$, since $j \sin \omega t = \cos \omega t$

$$\therefore \boxed{V_0 = -\frac{E_m}{\omega RC} \cos \omega t}$$

(2-17) A pulse is applied to a Low pass RC circuit. prove by direct integration that the area under the pulse is the same as the area under the output waveform across the capacitor. Explain the result physically?

Sol:





i) During pulse duration, $t<t_p$

Area under output waveform is A_1

$$A_1 = \int_0^{t_p} V_p \cdot dt$$

$$A_1 = \int_0^{t_p} V\left[1-e^{-t/RC}\right] dt$$

$$A_1 = V\left[\int_0^{t_p} 1\, dt - \int_0^{t_p} e^{-t/RC}\, dt\right]$$

$$A_1 = V\left[t_p - \left(\frac{e^{-t/RC}}{-1/RC}\right)_0^{t_p}\right]$$

$$A_1 = V\left\{t_p + RC\left(e^{-t_p/RC} - e^{-0}\right)\right\}$$

$$A_1 = Vt_p + VRC\left[e^{-t_p/RC} - 1\right] \quad \text{--- (1)}$$

ii) At $t>t_p$:

area under the output waveform is

$$A_2 = \int_{t_p}^{\infty} V_p \cdot e^{-\frac{(t-t_p)}{RC}}\, dt$$

$$A_2 = \int_{tp}^{\infty} V(1-e^{-tp/RC}) \cdot e^{-\frac{(t-tp)}{RC}} \, dt$$

$$A_2 = V \int_{tp}^{\infty} e^{-\left(\frac{t-tp}{RC}\right)} - e^{-\frac{tp}{RC}} \cdot e^{-\left(\frac{t-tp}{RC}\right)} \, dt$$

$$A_2 = V \cdot \int_{tp}^{\infty} e^{-\frac{t}{RC}} \cdot e^{\frac{tp}{RC}} - e^{-\frac{tp}{RC}} \cdot e^{-\frac{t}{RC}} \cdot e^{\frac{tp}{RC}} \, dt$$

$$\cancel{A_2 = V e^{\frac{tp}{RC}} \int_{tp}^{\infty} e^{-\frac{t}{RC}} -}$$

$$A_2 = V \int_{tp}^{\infty} e^{-\frac{t}{RC}} \cdot e^{\frac{tp}{RC}} - e^{-\frac{t}{RC}} \cdot 1 \, dt$$

$$= V e^{\frac{tp}{RC}} \int_{tp}^{\infty} e^{-\frac{t}{RC}} - V \int_{tp}^{\infty} e^{-\frac{t}{RC}} \, dt$$

$$= V e^{\frac{tp}{RC}} \left[\frac{e^{-\frac{t}{RC}}}{-1/RC} \right]_{tp}^{\infty} - V \left[\frac{e^{-t/RC}}{-1/RC} \right]_{tp}^{\infty}$$

$$= -VRC\, e^{\frac{tp}{RC}} \left(e^{-\infty} - e^{-\frac{tp}{RC}} \right) + VRC \left(e^{-\infty} - e^{-tp/RC} \right)$$

$$= +VRC\, e^{\frac{tp}{RC}} e^{-\frac{tp}{RC}} - VRC\, e^{-tp/RC}$$

$$A_2 = VRC - VRC\, e^{-tp/RC} \quad \text{— (2)}$$

$$\text{Total Area} = A_1 + A_2 = Vtp + VRC(e^{-tp/RC} - 1) + VRC - VRC\, e^{-tp/RC}$$

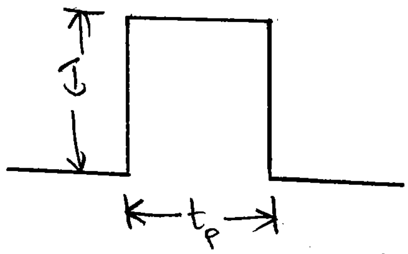
$$A = Vtp.$$

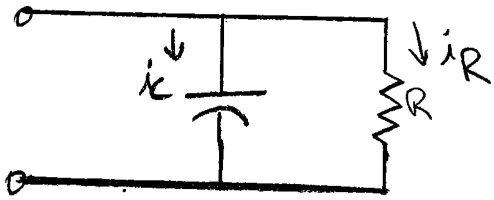
$$A = Vt_p + VRC\cancel{e^{-t_p/RC}} - V\cancel{RC} + \cancel{VRC} - VRC\cancel{e^{-t_p/RC}}$$

$$\therefore \boxed{A = V t_p}$$

which is equal to the area under the pulse.

(2.6) A current pulse of amplitude I is applied to a parallel RC combination. Plot to scale the waveforms of the current i_c ~~for~~ the cases. a) $t_p < RC$ b) $t_p = RC$ c) $t_p > RC$.

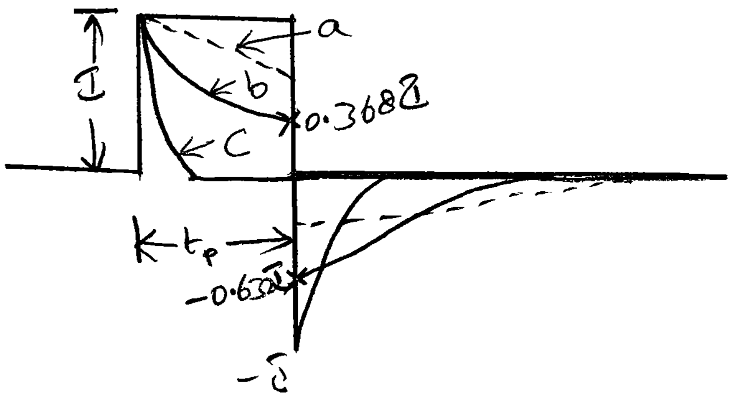




Sol:

$$I_c = I \cdot e^{-t_p/RC}, \quad t < t_p$$

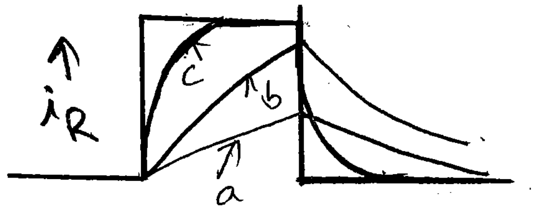
$$I_c = I\left(e^{-t_p/RC} - 1\right) \cdot e^{-\frac{(t-t_p)}{RC}}, \quad t > t_p$$



Repeat the above problem to sketch i_R waveforms for different cases.

$$i_R = I\left(1 - e^{-t/RC}\right), \quad 0 < t < t_p$$

$$i_R = I\left(1 - e^{-t_p/RC}\right) \cdot e^{-\frac{(t-t_p)}{RC}}, \quad t > t_p.$$



① A pulse generator with an output resistance $R_s = 500\Omega$ is connected to an oscilloscope with an input capacitance $C_i = 30pF$. Determine the fastest rise time that can be displayed?

Sol:

$$t_r = 2.2RC = 2.2 \times 500 \times 30 \times 10^{-12} = \underline{33ns}$$

② A 10V step is switched on to a $50k\Omega$ resistor in series with a 500pF capacitor. Calculate a) the rise time of the capacitor voltage b) The time for the capacitor to charge to 63.2% of its maximum voltage and c) The time for the capacitor to be completely charged?

Sol: Given data:

$R = 50k\Omega$

$C = 500pf$

a) $t_r = 2.2RC = 2.2 \times 50k \times 500p = \underline{55\mu s}$

b) $\tau = RC = 50k \times 500p = 25\mu s$

c) $5\tau = 5RC = 5 \times 25\mu s = 125\mu s$

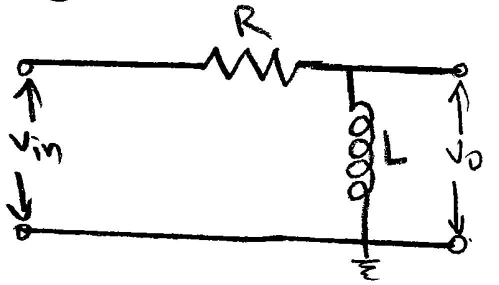
RL circuits

→ Suppose the capacitor C and resistor R of the preceding sections are replaced by a resistor R' and an inductor L respectively, then if the time constant $\frac{L}{R'}$ equals the time constant RC, all the preceding results remain unchanged.

Disadvantage of Inductor at large time constant:

1) A large value of inductance is obtained only with an iron-core inductor which is physically large, heavy and expensive relative to the cost of a capacitor for a similar application.
2) The non-linear properties of the iron cause distortion, which may be undesirable.

High-pass RL circuit



we know $X_L = 2\pi f L$

i) At low frequencies, the inductance reactance decreases and the inductor acts as short-circuit. so, for lower frequency signals, the output voltage across inductor is zero.

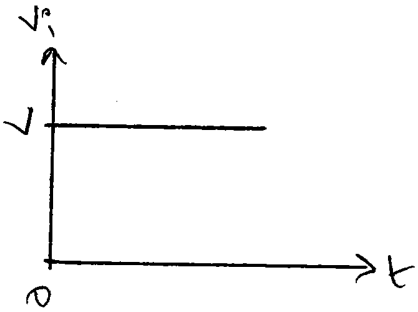
$$V_0 = 0$$

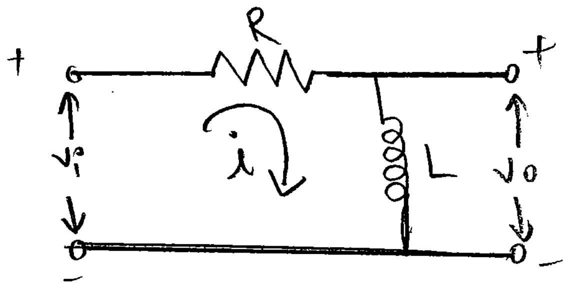
ii) At higher frequencies, the inductive reactance increases and the inductor acts as open circuit.

∴ All high frequency signals are blocked and the signal appears at the output.

"As the higher frequency signals appear at the output, it is called High pass RL circuit".

Step response of High pass RL circuit:





By applying KVL to the input Loop circuit & output

$$V_i = iR + L\frac{di}{dt} \quad \text{——— (1)}$$

$$V_0 = L\frac{di}{dt} \quad \text{——— (2)}$$

divide equation (1) with L, we get

$$\frac{V_i}{L} = \frac{Ri}{L} + \frac{di}{dt}$$

$$\left(D + \frac{R}{L}\right) i = \frac{V_i}{L}$$

The solution of above Differential equation is

$$\boxed{\begin{array}{l} \text{formula} \quad i \times I.F = \int R.H.S \times I.F \, dt + K \\ \qquad I.F = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t} \end{array}}$$

$$i \times e^{\frac{R}{L}t} = \int \frac{V_i}{L} e^{\frac{R}{L}t} dt + K$$

$$i e^{\frac{R}{L}t} = \frac{V_i}{L} \frac{e^{R/Lt}}{R/L} + K$$

$$i e^{R/Lt} = \frac{V_i}{\cancel{L}} \times \frac{\cancel{L}}{R} e^{R/Lt} + K$$

$$i e^{R/Lt} = \frac{V_i}{R} e^{R/Lt} + K$$

$$i = \frac{V_i}{R} e^{\frac{R}{L}t} \times e^{-\frac{R}{L}t} + K e^{-\frac{R}{L}t}$$

$$i = \frac{V_i}{R} + K \cdot e^{-\frac{R}{L}t}.$$

to find the value of K, apply initial conditions

at $t=0$, the current in the inductor is zero, since the inductor does not allow sudden changes in currents.

$\therefore$ at $t=0$, $i=0$

$$0 = \frac{V_i}{R} + K e^{-\frac{R}{L}t}$$

$$0 = \frac{V_i}{R} + K$$

$$K = -\frac{V_i}{R}$$

$$i = \frac{V_i}{R} - \frac{V_i}{R} e^{-\frac{R}{L}t}$$

$$i = \frac{V_i}{R}\left[1 - e^{-\frac{R}{L}t}\right] \quad \text{——— (3)}$$

substitute eqn ③ in eqn ②

$$v_0 = L\frac{di}{dt}$$

$$v_0 = L \times \frac{d}{dt}\left(\frac{V_i}{R}\left[1-e^{-R/L\,t}\right]\right)$$

$$v_0 = L\times\frac{V_i}{R}\left(-e^{-R/L\,t}\times -\frac{R}{L}\right)$$

$$v_0 = \cancel{L}\times\frac{V_i}{\cancel{R}}\times\frac{\cancel{R}}{\cancel{L}}\cdot e^{-R/L\,t}$$

$$\therefore v_0 = V_i\, e^{-R/L\,t}$$

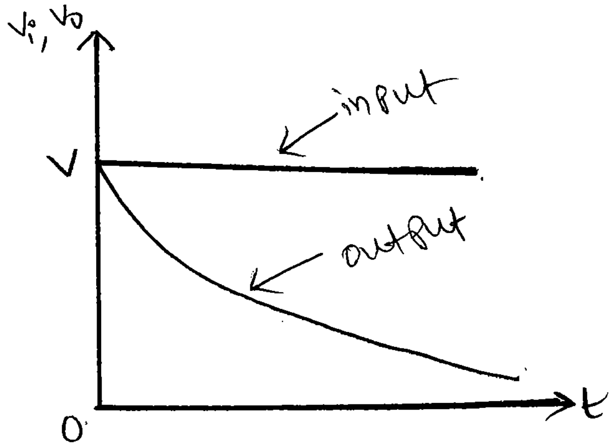
where $V_i = V$, at $t>0$

$$\therefore \boxed{v_0 = V e^{-R/L\,t}}$$

Comparing this equation with output response of an High pass RC circuit.

$$v_0 = V_{in}\, e^{-\frac{t}{RC}}$$

$\therefore RC = \frac{L}{R}$ where $\frac{L}{R}$ = time constant of an High pass RL circuit.



High pass RL circuit acts as Differentiator

$$V_i = iR + L\frac{di}{dt}$$

but $V_0 = L\frac{di}{dt}$

$$V_i = iR + V_0$$

but $i = \frac{V_{in}}{R}\left[1 - e^{-R/L\,t}\right]$

if $\frac{L}{R} << t$

$$i = \frac{V_{in}}{R}$$

$$V_0 = L\frac{d}{dt}\frac{V_{in}}{R}$$

$$\boxed{V_0 = \frac{L}{R}\frac{dV_{in}}{dt}}$$

(or)

$$V_i = iR + L\frac{di}{dt}$$

$$V_0 = L\frac{di}{dt} \Rightarrow i = \int \frac{V_0}{L}dt$$

$$V_i = R\int \frac{V_0}{L}dt + V_0$$

Differentiating both sides

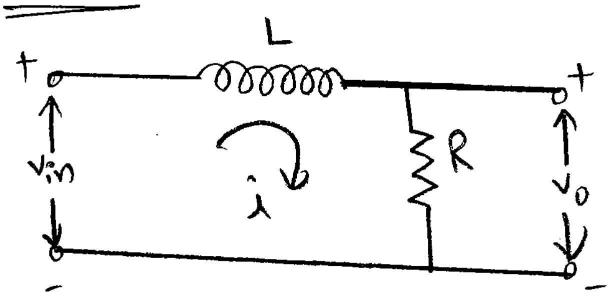
$$\frac{dV_i}{dt} = \underbrace{\frac{R}{L}V_0}_{\text{steady state response}} + \underbrace{\frac{dV_0}{dt}}_{\text{Transient response}}$$

If $\frac{L}{R} << t$, transient response is negligible.

$$\therefore \boxed{V_0 = \frac{L}{R}\frac{dV_i}{dt}}$$

$\therefore$ output voltage is directly proportional to the derivative of the input.

Low-pass RL circuit:



$$X_L = 2\pi f L$$

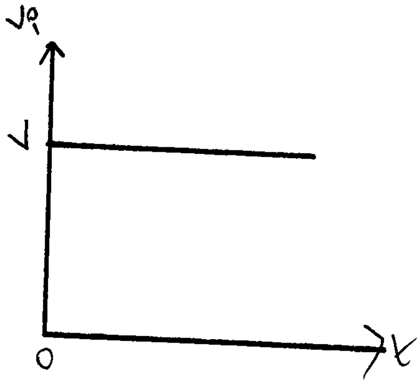
At Low frequencies (or D.C), the inductive reactance decreases and inductor acts as short circuit, so the low frequency signals will appear at the output.

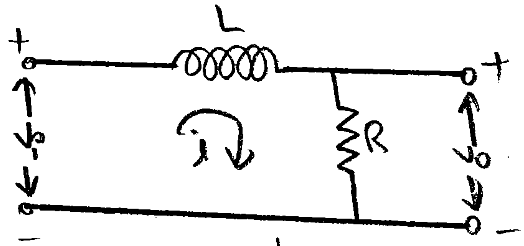
At high frequencies, the inductive reactance increases and it opposes the signal to pass through it. Therefore output is zero.

$$V_0 = 0$$

"As lower frequency signals appears at the output, it is called as Low pass RL circuit".

Step response of Low pass RL circuit:





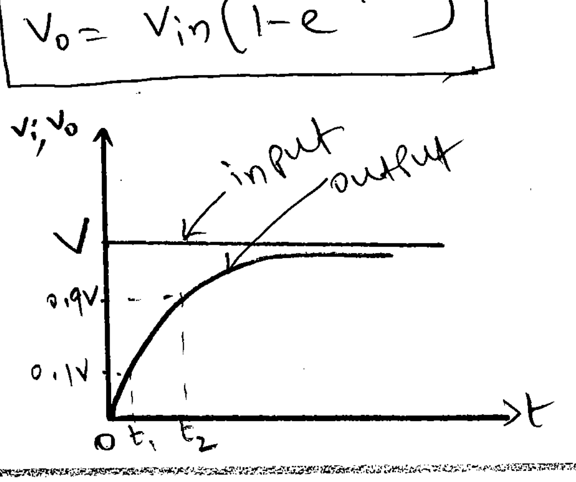
$$V_i = L\frac{di}{dt} + iR$$

but $V_0 = iR$

we know $i = \frac{V_{in}}{R}\left[1 - e^{-R/Lt}\right]$

$$V_0 = \frac{V_{in}}{\cancel{R}}\left[1 - e^{-R/Lt}\right] \times \cancel{R}$$

$$\boxed{V_0 = V_{in}\left(1 - e^{-R/Lt}\right)}$$



Rise time $(t_r) = 2.2 \times \tau$

$$t_r = 2.2 \times \frac{L}{R}$$

$$V_0 = V_{in}\left(1 - e^{-R/Lt}\right)$$

at $t = t_1$, $V_0 = 0.1V$

$$0.1\cancel{V} = \cancel{V}\left(1 - e^{-\frac{R}{L}t_1}\right)$$

$$e^{-\frac{R}{L}t_1} = 0.9$$

$$t_1 = 0.1\frac{L}{R}$$

at $t = t_2$, $V_0 = 0.9V$

$$0.9\cancel{V} = \cancel{V}\left(1 - e^{-\frac{R}{L}t_2}\right)$$

$$e^{-\frac{R}{L}t_2} = 0.1$$

$$t_2 = 2.3\frac{L}{R}$$

$$t_r = t_2 - t_1$$

$$t_r = 2.3\frac{L}{R} - 0.1\frac{L}{R}$$

$$\therefore \boxed{t_r = 2.2\frac{L}{R}}$$

Low pass RL circuit acts as an Integrator:

$$V_i = L\frac{di}{dt} + V_0$$

$$V_0 = iR$$

$$i = \frac{V_0}{R}$$

$$V_i = L\frac{d}{dt}\left(\frac{V_0}{R}\right) + iR$$

$$V_i = \underbrace{\frac{L}{R}\frac{dV_0}{dt}}_{\text{Transient response}} + \underbrace{iR}_{\text{steady state response}}$$

If $\frac{L}{R} \gg t$, transient response is predominant

$$V_i = \frac{L}{R}\frac{dV_0}{dt}$$

$$\boxed{V_0 = \frac{R}{L}\int V_i\,dt}$$

∴ The output voltage is directly proportional to the integral of the input.

Why RC differentiator is superior to RL differentiator:

1) Capacitor C is relatively cheaper than inductor L.

2) The size of capacitor is small when compared to inductor L, so space occupied by RC circuit is less than RL circuit.

3) $$\text{Inductor } (L) = \frac{N^2}{S} = \frac{N^2}{l/\mu_0 \mu_r A} = \frac{N^2 A \mu_0 \mu_r}{l}$$

L = Directly proportional to the number of turns N, so to have large inductance, number of turns must be increased and the inductance will be bulky.

L = Directly proportional to μ_r, so to have large inductance, relative permeability (μ_r) of the material should be more.

4) Inductance (L) is effected easily by rise in temperature.

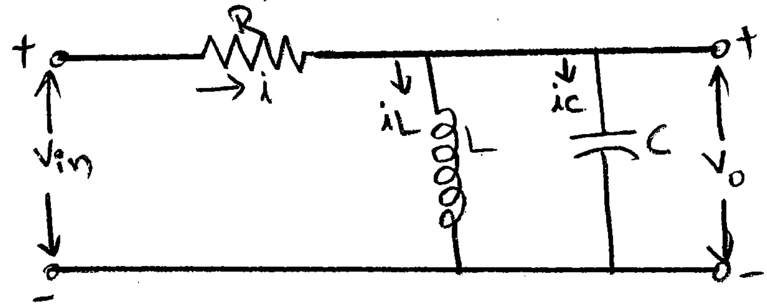
As temperature increases, μ_r decreases, intrun L decreases.

As $\tau = \frac{L}{R}$, if L decreases, time constant (τ) decreases.

∴ Because of all the above reasons, RC differentiator is mostly preferred over RL differentiator circuit.

RLC circuits

→ If capacitance C is the effect of coil winding capacitance, output capacitance and stray capacitance to ground. By including the effect of this capacitance into consideration, the circuit is shown below:



The above fig. shows a signal V_i applied through a resistor R to a parallel LC circuit.

If "i" is the current flowing through R and "i_L" is the current flowing through L and "i_C" is the current flowing through C.

then. $i = i_L + i_C$

but $i_L = \frac{1}{L}\int V_0\, dt$

$i_C = C\frac{dV_0}{dt}$

$$i = \frac{1}{L}\int V_0\, dt + C\cdot\frac{dV_0}{dt} \quad \text{—— (1)}$$

$$V_{in} = iR + V_0.$$

$$V_{in} = \left[\frac{1}{L}\int V_0\, dt + C\frac{dV_0}{dt}\right]R + V_0$$

$$V_{in} = \frac{R}{L}\int V_0\, dt + RC\frac{dV_0}{dt} + V_0 \quad \text{—— (2)}$$

→ By applying step input to RLC circuit and differentiate above eqn (2) with respect to 't', we get

$$\frac{dV_{in}}{dt} = \frac{R}{L}V_0 + RC\frac{d^2V_0}{dt^2} + \frac{dV_0}{dt}$$

but $\frac{dV_{in}}{dt} = \frac{dV}{dt} = 0$

$$0 = \frac{R}{L}V_0 + RC\frac{d^2V_0}{dt^2} + \frac{dV_0}{dt}$$

$$\left(D^2RC + D + \frac{R}{L}\right)V_0 = 0$$

where $D^2RC + D + \frac{R}{L}$ is called as characteristic equation.

The roots of the characteristic equation are.

$$\text{roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = RC$, $b = 1$, $c = \frac{R}{L}$

$$S_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \times RC \times \frac{R}{L}}}{2RC}$$

$$S_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{4RC \times \frac{R}{L}}{(2RC)^2}}$$

$$S_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{\cancel{4}\cancel{RC} \times \frac{\cancel{R}}{L}}{\cancel{4}\cancel{R^2}C^{\cancel{2}}}}$$

$$S_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

if K = Damping constant $= \frac{1}{2R}\sqrt{\frac{L}{C}}$

f_0 = Natural frequency or frequency of undamped oscillations $= \frac{1}{2\pi\sqrt{LC}}$

T_0 = resonant or undamped time period $= 2\pi\sqrt{LC}$

$K = \frac{1}{2R}\sqrt{\frac{L}{C}}$, $T_0 = 2\pi\sqrt{LC}$

$$\frac{K}{T_0} = \frac{1}{2R}\sqrt{\frac{L}{C}} \times \frac{1}{2\pi\sqrt{LC}}$$

$$\frac{K}{T_0} = \frac{1}{4\pi R}\sqrt{\frac{\cancel{L}}{C}} \times \frac{1}{\sqrt{\cancel{L}C}}$$

$$\frac{K}{T_0} = \frac{1}{4\pi RC}$$

$$S_{1,2} = -\frac{1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} - \frac{1}{LC}}$$

$$S_{1,2} = -\frac{2\pi K}{T_0} \pm \sqrt{\frac{4\pi^2 K^2}{T_0^2} - \frac{4\pi^2}{T_0^2}}$$

$$S_{1,2} = \frac{-2\pi K}{T_0} \pm \frac{2\pi}{T_0}\sqrt{K^2 - 1}$$

$$S_{1,2} = -\frac{2\pi K}{T_0} \pm \frac{2\pi}{T_0}\sqrt{-(1-K^2)}$$

$$\boxed{S_{1,2} = -\frac{2\pi K}{T_0} \pm j\frac{2\pi}{T_0}\sqrt{1-K^2}}$$

$$\frac{2\pi K}{T_0} = \cancel{2\pi} \times \frac{1}{\cancel{4\pi}_2 RC}$$

$$\frac{2\pi K}{T_0} = \frac{1}{2RC}$$

$$\frac{4\pi^2 K^2}{T_0^2} = \frac{\cancel{4\pi^2} \times 1}{4^2\cancel{\pi^2} R^2C^2}$$

$$\frac{4\pi^2 K^2}{T_0^2} = \frac{1}{4R^2C^2}$$

$$\frac{4\pi^2}{T_0^2} = \frac{\cancel{4\pi^2}}{\cancel{4\pi^2}(\sqrt{LC})^2}$$

$$\frac{4\pi^2}{T_0^2} = \frac{1}{LC}$$

$S_1 = \frac{-2\pi K}{T_0} + j\frac{2\pi}{T_0}\sqrt{1-K^2}$, $S_2 = \frac{-2\pi K}{T_0} - j\frac{2\pi}{T_0}\sqrt{1-K^2}$

1) If $K=0$, $S_1 = +j\frac{2\pi}{T_0}$ and $S_2 = -j\frac{2\pi}{T_0}$, both roots S_1 and S_2 are imaginary and the response is an "undamped sinusoid of time period T_0".

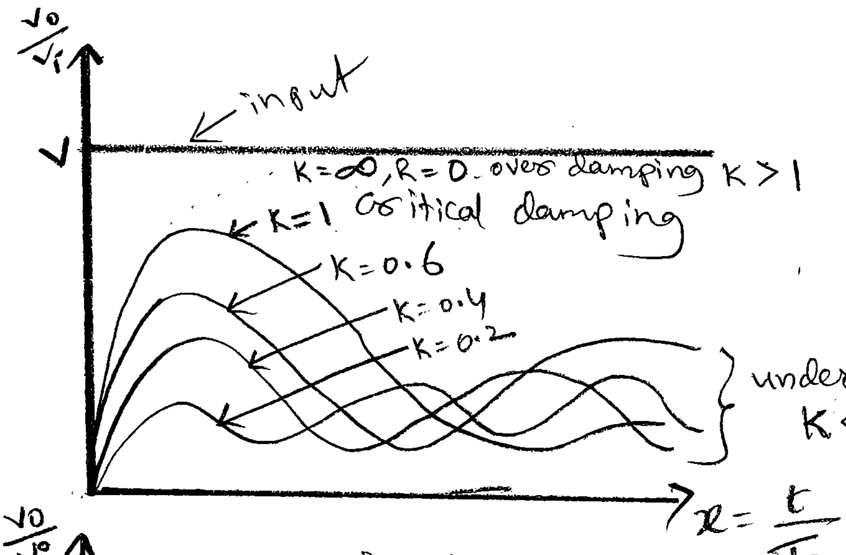
2) If $K=1$, $S_1 = -\frac{2\pi}{T_0}$ and $S_2 = -\frac{2\pi}{T_0}$, the two roots are real and equal which represents "critical damping".

3) if $K>1$, the circuit becomes "over damped" and there are no oscillations of the output.

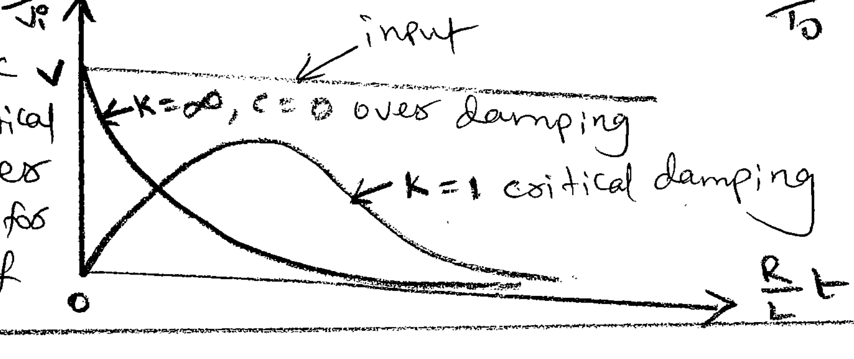
4) if $K<1$, the output is a sinusoid whose amplitude progressively decays with time. This represents "under damped response".

→ Let the forcing function (input) be a step response of magnitude V. Let it be assumed that the circuit is relaxed at $t=0^-$ i.e; there is no initial inductor current or initial capacitor voltage.

1) Response of RLC circuit for critical damping, over damping under damping cases for a fixed value of L and C are shown below when the input is a step signal:



2) Response of RLC circuit for critical damped & over damped cases for fixed values of R and L.



1) **Undamped**

If $K=0$, roots are purely imaginary $s_1 = +j\frac{2\pi}{T_0}$, $s_2 = -j\frac{2\pi}{T_0}$

then $$v_0 = c_1 \cos\frac{2\pi}{T_0}t + c_2 \sin\frac{2\pi}{T_0}.t \quad \text{— (1)}$$

To find c_1 and c_2 values we have to apply initial conditions.

i.e; at $t=0$, $v_0=0$

$$0 = c_1 \times 1 + c_2 \times 0 \quad \left(\text{since } \begin{matrix} \cos 2\pi t = 1 \\ \sin 2\pi t = 0 \end{matrix}\right)$$

$$\boxed{c_1 = 0}$$

differentiate eqn (1) with respect to 't'

$$\frac{dv_0}{dt} = -c_1 \sin\frac{2\pi}{t_0}t \times \frac{2\pi}{T_0} + c_2 \cos\frac{2\pi}{t_0}t \times \frac{2\pi}{T_0}$$

Since differentiation	of Cos is	− sin
	sin is	+ Cos
Integration	of Cos is	+ sin
	sin is	− Cos

at $t=0$, c_1 is 0 and acts as short circuit

$$\frac{dv_0}{dt} = c_2 \cos\frac{2\pi}{t_0}t \times \frac{2\pi}{T_0}$$

$$V_{in} = iR$$

$$i = \frac{V_{in}}{R}$$

but $$i = C\frac{dv_0}{dt}$$

$$\frac{dv_0}{dt} = \frac{i}{C} = \frac{V_{in}}{RC}$$

By substituting in the above equation, we get

$$\frac{V_{in}}{RC} = C_2 \times 1 \times \frac{2\pi}{T_0} \quad , \text{ since } \cos\frac{2\pi t}{t_0} = 1$$

$$\boxed{C_2 = \frac{V_{in} T_0}{2\pi RC}}$$

By substituting C_1 and C_2 values in equation ①, we get

$$\boxed{V_0 = \frac{V_{in}}{2\pi RC} T_0 \cdot \sin \frac{2\pi}{T_0} t}$$

2) <u>critical damping</u>

If $K=1$, $S_1 = S_2 = -\frac{2\pi}{T_0}$ the roots are real and equal

$$V_0 = (C_1 + C_2 t) e^{-\frac{2\pi}{T_0} t} \quad — ①$$

at $t=0$, $V_0 = 0$

$$\therefore \boxed{C_1 = 0}$$

formula:

$$\boxed{uv = u\,dv + v\,du}$$

let $C_1 + C_2 t = v$

$e^{-\frac{2\pi}{T_0} t} = u$

differentiate eqn ① with respect to 't' on both sides

$$\frac{dV_0}{dt} = e^{-\frac{2\pi}{T_0} t} \frac{d}{dt}(C_1 + C_2 t) + (C_1 + C_2 t) \frac{d}{dt} e^{-\frac{2\pi}{T_0} t}$$

$$\frac{dV_0}{dt} = C_2 e^{-\frac{2\pi}{T_0} t} + (C_1 + C_2 t) e^{-\frac{2\pi}{T_0} t} \times -\frac{2\pi}{T_0}$$

Applying initial conditions

at $t=0$, $\frac{dV_0}{dt} = \frac{V_{in}}{RC}$

$$\therefore \frac{V_{in}}{RC} = C_2 e^{-0} + (0+0)e^{-0}$$

$$\frac{V_{in}}{RC} = C_2$$

Substituting C_1 and C_2 values in equ (1)

$$V_0 = \left(0 + \frac{V_{in}}{RC}t\right) e^{-\frac{2\pi}{T_0}t}$$

$$V_0 = \frac{V_{in}}{RC} \cdot t \cdot e^{-\frac{2\pi}{T_0}t} \quad \text{--- (2)}$$

we know $\frac{K}{T_0} = \frac{1}{4\pi RC}$

$$\cancel{4\pi RCK = \frac{1}{T_0}}$$

$$\cancel{\pi RC =}$$

$$\frac{1}{RC} = \frac{4\pi K}{T_0}$$

but $K = 1$

$$\therefore \frac{1}{RC} = \frac{4\pi}{T_0}$$

$$V_0 = V_{in} \times \frac{4\pi}{T_0} t \cdot e^{-\frac{2\pi}{T_0}t}, \quad V_{in} = V$$

let $\boxed{x = \frac{t}{T_0}}$

$$\boxed{\frac{V_0}{V} = 4\pi x \cdot e^{-2\pi x}}$$

we know

$$K = \frac{1}{2R}\sqrt{\frac{L}{C}}$$ and $$T_0 = 2\pi\sqrt{LC}$$

squaring on both sides

$$\frac{K}{T_0} = \frac{1}{4\pi RC}$$

$$K^2 = \frac{1}{4R^2}\cdot\frac{L}{C}$$

if $K = 1$

if $K=1$

$$T_0 = 4\pi RC$$

$$1 = \frac{L}{4R^2C}$$

$$T_0 = \cancel{4}\pi \times \frac{L}{\cancel{4}R}$$

$$RC = \frac{L}{4R}$$

$$T_0 = \frac{\pi L}{R}$$

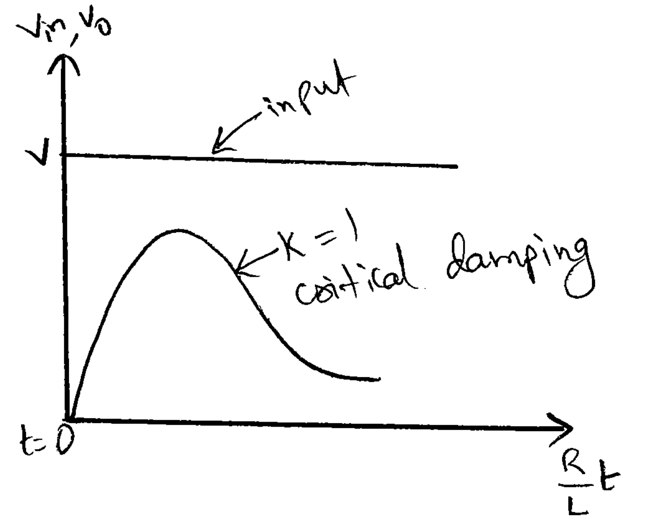
$$\frac{1}{RC} = \frac{4R}{L}$$

Sub these values in equation (2)

$$V_0 = \frac{V_{in}}{RC} t e^{-\frac{2\pi}{T_0}t}$$

$$V_0 = \frac{4R}{L}\cdot V_{in}\, t\, e^{-\frac{2\cancel{\pi}R}{\cancel{\pi}L}t}$$

$$\boxed{V_0 = \left(\frac{4R}{L}\right) V_{in}\, t\, e^{-\frac{2R}{L}t}}$$



3) <u>over damping</u>

if $k>1$, we get over damping

$$S_{1,2} = -\frac{2\pi}{T_0}k \pm \frac{2\pi}{T_0}\sqrt{k^2-1}$$

$$S_{1,2} = -\frac{2\pi}{T_0}k \pm \frac{2\pi}{T_0}k\sqrt{1-\frac{1}{k^2}}$$

$$S_{1,2} = \frac{-2\pi}{T_0}k \pm \frac{2\pi}{T_0}k\left(1-\frac{1}{k^2}\right)^{1/2}$$

<u>formula</u>: Using Binomial expansion let $\frac{1}{k^2}=x$, $\frac{1}{2}=n$. $(1-x)^n = 1 - \frac{nx}{1!} + \frac{n(n-1)}{2!}x^2 - \cdots$

$$S_{1,2} = \frac{-2\pi}{T_0}k \pm \frac{2\pi k}{T_0}\left[1 - \frac{\frac{1}{2}\cdot\frac{1}{k^2}}{1!} + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{1}{k^2}\right)^2 - \cdots\right]$$

if $k>1$, $\frac{1}{k^2} <<< 1$, so we can ignore

$$S_{1,2} = -\frac{2\pi k}{T_0} \pm \frac{2\pi k}{T_0}\left[1-\frac{1}{2k^2}\right]$$

let us solve the two roots

$$S_1 = \frac{-2\pi k}{T_0} + \frac{2\pi k}{T_0}\left(1-\frac{1}{2k^2}\right)$$

$$S_1 = \cancel{-\frac{2\pi k}{T_0}} + \cancel{\frac{2\pi k}{T_0}} - \frac{\cancel{2}\pi \cancel{k}}{T_0} \times \frac{1}{\cancel{2}k^{\cancel{2}}}$$

$$\therefore S_1 = \frac{-\pi}{kT_0}$$

$$S_2 = \frac{-2\pi k}{T_0} - \frac{2\pi k}{T_0}\left[1 - \frac{1}{2k^2}\right]$$

$$S_2 = \frac{-2\pi k}{T_0}\left[1 + 1 - \frac{1}{2k^2}\right]$$

$$S_2 = -\frac{2\pi k}{T_0}\left[2 - \frac{1}{2k^2}\right]$$

$$\cancel{S_2 = -\frac{4\pi k}{T_0} + \frac{2\pi k}{T_0} \times \frac{1}{2k^2}}$$

if $k > 1$, $\frac{1}{k^2} \ll 1$ so when compared to 2, $\frac{1}{2k^2}$ can be ignored.

$$\therefore S_2 = -\frac{4\pi k}{T_0}$$

Two roots are real and unequal.

$$\therefore v_0 = c_1 e^{\frac{-\pi}{kT_0}t} + c_2 e^{\frac{-4\pi k}{T_0}t} \quad \text{——} \ (1)$$

at $t = 0$, $v_0 = 0$

$$0 = c_1 e^0 + c_2 e^{-0}$$

$$0 = c_1 + c_2$$

$$c_1 = -c_2 \quad \text{——} \ (2)$$

Differentiate equation (1) with respect to 't'.

$$\frac{dv_0}{dt} = c_1 e^{\frac{-\pi}{kT_0}t} \times \frac{-\pi}{kT_0} + c_2 e^{\frac{-4\pi k}{T_0}t} \times \frac{-4\pi k}{T_0}$$

$$\cancel{\frac{dv_0}{dt} = -\pi q_1 0}$$

$$\frac{dV_0}{dt} = \frac{-\pi}{KT_0} \cdot c_1 e^{\frac{-\pi}{KT_0}t} - \frac{4\pi}{T_0} k c_2 e^{-\frac{4\pi k}{T_0}t}$$

at $t=0$, $\frac{dV_0}{dt} = \frac{V_{in}}{RC}$

$$\frac{V_{in}}{RC} = \frac{-\pi}{KT_0} c_1 e^{-0} - \frac{4\pi}{T_0} k c_2 e^{-0}$$

$$\frac{V_{in}}{RC} = \frac{-\pi}{KT_0} c_1 - \frac{4\pi}{T_0} k c_2$$

sub $c_1 = -c_2$

$$\frac{V_{in}}{RC} = \frac{\pi c_2}{kT_0} - \frac{4\pi}{T_0} k c_2$$

$$\frac{V_{in}}{RC} = \frac{\pi}{T_0} c_2 \left[\frac{1}{k} - 4k\right]$$

$$c_2 = \frac{V_{in} T_0}{RC\pi\left(\frac{1}{K} - 4K\right)}$$

but $\frac{K}{T_0} = \frac{1}{4\pi RC}$

$$4\pi RCK = T_0$$

$$\pi RC = \frac{T_0}{4K}$$

$$\frac{1}{\pi RC} = \frac{4K}{T_0}$$

$$c_2 = \frac{V_{in} 4K \cancel{T_0}}{\cancel{T_0}\left(\frac{1}{K} - 4K\right)}$$

$$c_2 = \frac{V_{in} \cancel{4K}}{-\cancel{4K}\left(1 - \frac{1}{4K^2}\right)}$$

$$C_2 = \frac{-V_{in}}{1-\frac{1}{4K^2}}$$

if $K>1$, $\frac{1}{4K^2} << 1$ so when compared to 1, $\frac{1}{4K^2}$ we can neglect.

$\therefore C_2 = -V_{in}$, $C_1 = V_{in}$

Substitute C_1 and C_2 values in equation (1)

$$V_0 = C_1 e^{-\frac{\pi}{KT_0}t} + C_2 e^{-\frac{4\pi}{T_0}Kt}$$

$$V_0 = V_{in} e^{-\frac{\pi}{KT_0}t} - V_{in} e^{-\frac{4\pi K}{T_0}t}$$

$$V_0 = V_{in} \cdot e^{-\frac{\pi}{KT_0}t} - V_{in} e^{-\frac{\pi}{KT_0} \cdot t(4K^2)}$$

$K \gg 1$ means $\frac{1}{K} < 1$

so the first term is less than 1 except at $t=0$. The second term is equal to the first term raised to the power $4K^2$. Hence, the second term is negligible compared with the first term except at near the origin.

$$\therefore \boxed{V_0 \simeq V_{in} e^{-\frac{\pi}{KT_0} \times t}} \quad \text{—— (3)}$$

but $KT_0 = \frac{1}{\cancel{2}R}\sqrt{\frac{L}{\cancel{C}}} \times \cancel{2}\pi\sqrt{L\cancel{C}}$

$$KT_0 = \frac{\pi L}{R}$$

$$V_0 \simeq V_{in} e^{-\frac{\cancel{\pi} R}{\cancel{\pi} L}t}$$

$$\boxed{V_0 \simeq V_{in} e^{-\frac{R}{L}t}}$$

from equation (3)

$$V_0 \simeq V_{in}\, e^{-\frac{\pi}{KT_0} \times t}$$

we know $\frac{t}{T_0} = x$

$$\therefore \boxed{V_0 \simeq V_{in}\, e^{-\frac{\pi x}{K}}}$$

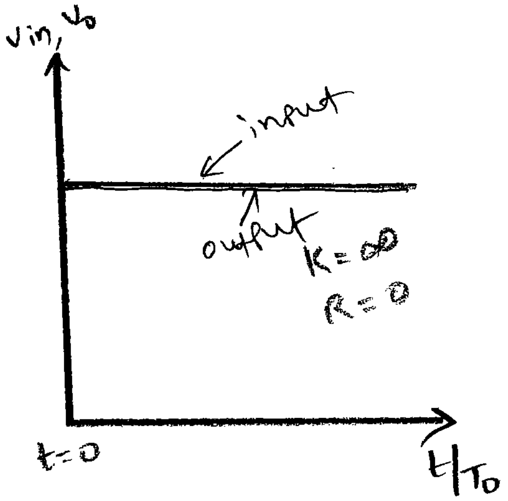
input and output waveforms are shown below:

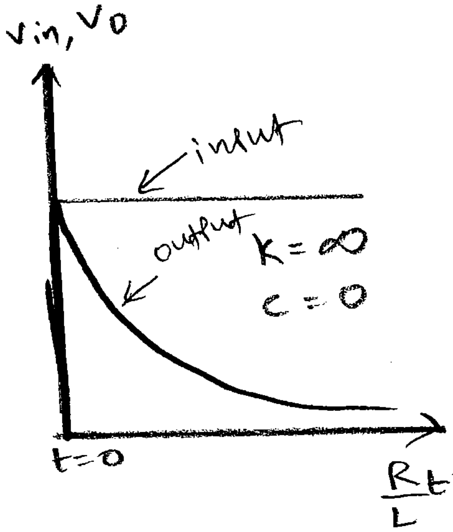
$K = \frac{1}{2R}\sqrt{\frac{L}{C}}$

for $\underline{K>1}$

if $R=0$ or $C=0$

then $K = \infty$





4) <u>Under damping.</u>

$$S_{1,2} = -\frac{2\pi K}{T_0} \pm j\,\frac{2\pi}{T_0}\sqrt{1-K^2}$$

since the roots are complex, so

$$V_0 = e^{-\frac{2\pi K}{T_0}t}\left[C_1 \cos \frac{2\pi}{T_0}\sqrt{1-K^2}\cdot t + C_2 \sin \frac{2\pi}{T_0}\sqrt{1-K^2}\cdot t\right] \quad —(1)$$

Applying initial conditions at $t=0$, $V_0 = 0$.

$$0 = e^{-0}\left[C_1 \times \cos 0 + C_2 \sin 0\right]$$

$$0 = e^{-0}\left[C_1 + 0\right]$$

$$\boxed{C_1 = 0}$$

Differentiating equation (1) with respect to 't'.

$$\frac{dV_0}{dt} = e^{-\frac{2\pi}{T_0}k \cdot t}\left[-c_1 \sin \frac{2\pi}{T_0}\sqrt{1-k^2}\cdot t \times \frac{2\pi}{T_0} \times \sqrt{1-k^2} + c_2 \cos \frac{2\pi}{T_0}\sqrt{1-k^2}\cdot t \times \frac{2\pi}{T_0}\sqrt{1-k^2}\right]$$

$$+ e^{-\frac{2\pi}{T_0}k \cdot t} \times -\frac{2\pi}{T_0}k\left[c_1 \cos \frac{2\pi}{T_0}\sqrt{1-k^2}\cdot t + c_2 \sin \frac{2\pi}{T_0}\sqrt{1-k^2}\cdot t\right]$$

<u>formula</u>:

$$\frac{d}{dt}[uv] = u\,dv + v\,du$$

$$\text{let } u = e^{-\frac{2\pi}{T_0}k \cdot t}, \quad v = c_1 \cos \frac{2\pi}{T_0}\sqrt{1-k^2}\cdot t + c_2 \sin \frac{2\pi}{T_0}\sqrt{1-k^2}\cdot t$$

$$\text{at } t=0, \quad \frac{dV_0}{dt} = \frac{V_{in}}{RC}$$

$$\frac{V_{in}}{RC} = e^{-0}\left[-c_1 \sin 0 + c_2 \cos 0\right] + e^{-0} \times -\frac{2\pi}{T_0}k\left[c_1 \cos 0 + c_2 \sin 0\right]$$

$$\frac{V_{in}}{RC} = 1\left[0 + c_2\right] + 1 \times -\frac{2\pi}{T_0}k\left[c_1 + 0\right]$$

$$\frac{V_{in}}{RC} = e^{-0}\left[-c_1 \sin 0 \times \frac{2\pi}{T_0}\sqrt{1-k^2} + c_2 \cos 0 \times \frac{2\pi}{T_0}\sqrt{1-k^2}\right]$$

$$+ e^{-0} \times -\frac{2\pi}{T_0}k\left[c_1 \cos 0 + c_2 \sin 0\right]$$

$$\frac{V_{in}}{RC} = 1 \times \left[0 + c_2 \frac{2\pi}{T_0}\sqrt{1-k^2}\right] + 1 \times -\frac{2\pi}{T_0}k(c_1)$$

$$\text{but } c_1 = 0$$

$$\frac{V_{in}}{RC} = c_2 \times \frac{2\pi}{T_0}\sqrt{1-k^2}$$

$$C_2 = \frac{T_0 \times V_{in}}{2\pi\sqrt{1-k^2} \times RC}$$

$$\frac{K}{T_0} = \frac{1}{4\pi RC}$$

$$\frac{1}{RC} = \frac{4\pi K}{T_0}$$

but $\frac{1}{RC} = \frac{4\pi k}{T_0}$

$$C_2 = \frac{\cancel{T_0}}{\cancel{2\pi}\sqrt{1-k^2}} \times V_{in} \times \frac{\cancel{4\pi}^{2} k}{\cancel{T_0}}$$

$$\boxed{C_2 = \frac{2 V_{in} k}{\sqrt{1-k^2}}}$$

Substitute the values of c_1 and c_2 in equation (1)

$$V_0 = e^{-\frac{2\pi}{T_0}kt}\left[C_1 \cos\frac{2\pi}{T_0}\sqrt{1-k^2}\,t + C_2 \sin\frac{2\pi}{T_0}\sqrt{1-k^2}\,t\right]$$

$$V_0 = e^{-\frac{2\pi}{T_0}kt} \cdot \left[0 + \frac{2k}{\sqrt{1-k^2}} V_{in} \sin\frac{2\pi}{T_0}\sqrt{1-k^2}\,t\right]$$

$$V_0 = e^{-\frac{2\pi}{T_0}kt} \cdot \frac{2k}{\sqrt{1-k^2}} \cdot V_{in} \cdot \sin\frac{2\pi}{T_0}\sqrt{1-k^2}\,t$$

$$\boxed{\frac{V_0}{V_{in}} = \frac{2k}{\sqrt{1-k^2}} \cdot e^{-2\pi k x} \sin 2\pi\sqrt{1-k^2} \cdot x}$$

where $x = \frac{t}{T_0}$

$\therefore$ for $k<1$, the output is a sinusoidal whose amplitude progressively decays with time. This represents under damped response.

Ringing circuit:

→ **Uses:**

1) To generate a sequence of pulses regularly spaced in time.
2) It is used in many timing operations.

→ **Def:**

"The circuit which gives nearly undamped oscillation ($K<1$ or $K=0$) for a given step input voltage is called as Ringing circuit."

→ If K is small the circuit will ring for many cycles.

→ If Q is the Quality factor of a parallel RLC circuit and K is the damping factor.

we know

$$\text{Quality factor } (Q) = \omega_0 RC$$

$$Q = \frac{2\pi}{T_0} RC, \quad \text{where } \omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

$$\text{but } \frac{K}{T_0} = \frac{1}{4\pi RC} \Rightarrow RC = \frac{T_0}{4\pi K}$$

$$Q = \frac{\cancel{2\pi}}{\cancel{T_0}} \times \frac{\cancel{T_0}}{\cancel{4\pi} K}$$

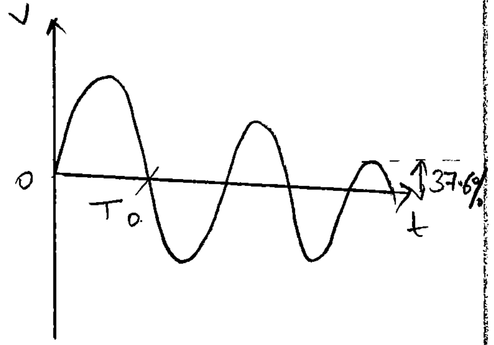
$$Q = \frac{1}{2K} \quad \text{—— } (1) \Rightarrow 2KQ = 1$$

If K is small, Q is large and the circuit will oscillate (ring) for more number of cycles.

for $K<1$

$$V_0 = \frac{2K}{\sqrt{1-K^2}} \cdot V_{in} \cdot e^{-\frac{2\pi}{T_0} \cdot t \cdot K} \cdot \sin\sqrt{1-K^2} \cdot \frac{2\pi}{T_0} \cdot t$$

∴ "It is required to find Q value of a circuit which is to ring for a given number N cycles before the amplitude reduces to $1/e$ ($e^{-1} = 37\%$) of its initial value".



$$\frac{2\pi}{T_0} K \cdot t = 1$$

given N no. of cycles

$$t = NT_0$$

$$\frac{2\pi}{\cancel{T_0}} \cdot K \cdot N\cancel{T_0} = 1$$

$$2\pi K N = 1 \quad \text{—— (2)}$$

equate (1) and (2)

$$\cancel{2}\pi \cancel{K} N = \cancel{2}\cancel{K} Q$$

$$\therefore \boxed{Q = \pi N, \quad N = \frac{Q}{\pi}}$$

where N = number of cycles (ringing)

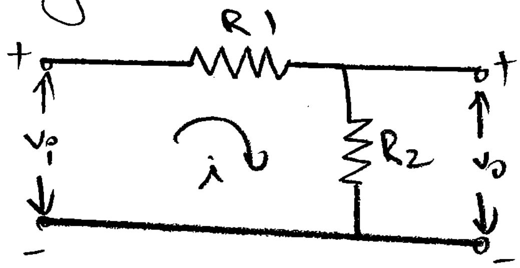
Ex : If $Q = 12$, then $N = \frac{12}{3.14} \simeq 4$ cycles on oscillations

$\therefore$ The circuit will ring for 4 cycles before the amplitude reduces to 37% $\left(\frac{1}{e} = e^{-1}\right)$ of its initial value.

Attenuators

→ "Attenuator is one which is used to reduce the amplitude of the signal waveform".

→ An attenuator is basically a resistance potential divider arrangement as shown below:



$$V_0 = iR_2$$

but $i = \frac{V_i}{R_1+R_2}$

$$V_0 = \frac{R_2 \cdot V_i}{R_1+R_2}$$

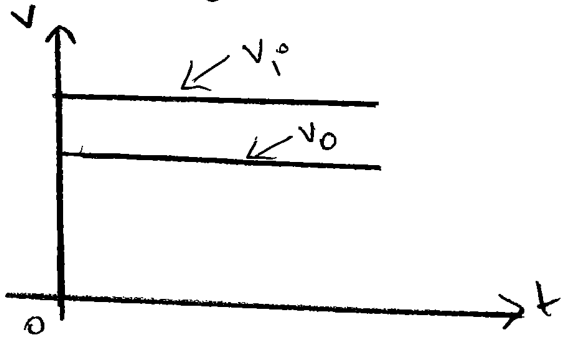
$$V_0 = aV_i$$

where $a = \frac{R_2}{R_1+R_2}$

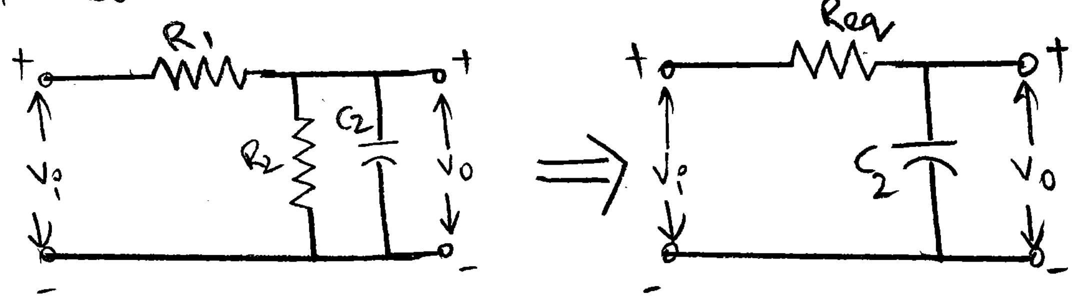
→ The simple resistance combination would multiply the input signal by the ratio $a = \frac{R_2}{R_1+R_2}$ which is independent of frequency.

→ Theoritically the waveform of the input signal is preserved, although its amplitude has changed. Thus there is no distortion produced due to the transmission of the signal through the attenuator.

The input and output waveforms are shown. Assuming the input signal to be a step voltage.



→ The attenuator circuit is considered to be a purely resistive circuit. But in practice it is not so. A distributed capacitance C_2 shunting the resistance R_2 must also be taken into consideration.



The attenuation now is not independent of frequency.

R_{eq} = Parallel combination of R_1 and R_2.

→ R_1 and R_2 values to be large because the input impedance of the attenuator may be large enough to prevent Loading down the input signal.

So, if R_1 and R_2 are large, the rise time

$$t_r = \left(\frac{R_1 R_2}{R_1 + R_2}\right) C_2 \times 2.2 \text{ will be large}$$

and a large rise time is normally unacceptable, and also the effect of the capacitance C_2 is to distort the waveshape of the input signal.

ex: If $R_1 = R_2 = 1M\Omega$, $C_2 = 15pF$

the $t_r = 2.2 \times 0.5M \times 15P = 16.5\mu sec$

∴ The undesirable effect of this is normally overcome by providing necessary and adequate compensation.

→ The attenuator may be compensated, so that its attenuation is once again independent of the frequency by shunting R_1 by a capacitor C_1 as shown below:

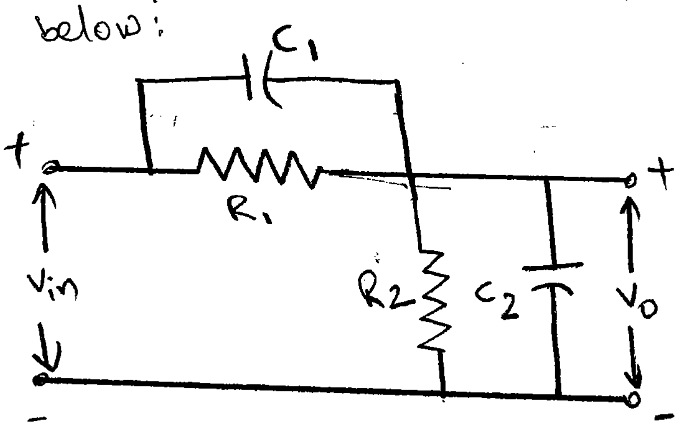
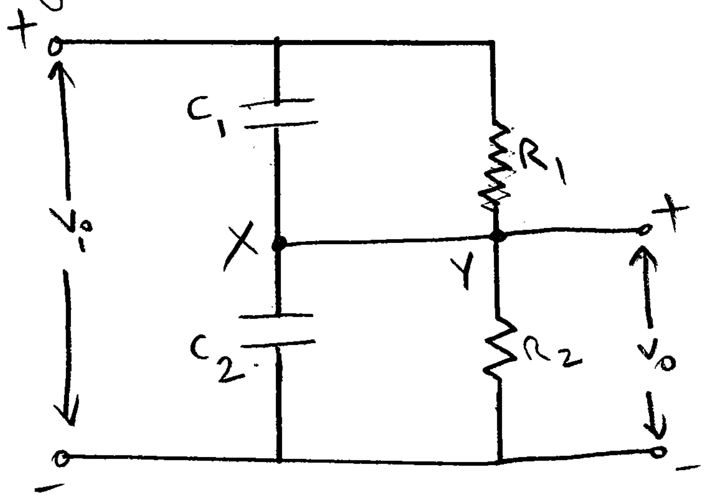


fig: compensated attenuator.

→ The above compensated attenuator is redrawn as a bridge which is shown below:



→ In the above fig. the two resistors and two capacitors may be viewed as four arms of the bridge.

→ If $R_1C_1 = R_2C_2$, the bridge will be balanced and no current will flow in the branch connecting the point X to the point Y.

→ For the purpose computing the output, the branch X–Y may be omitted and the output across R_2 is

$$\frac{R_2 V_i}{R_1+R_2} = aV_i$$

independent of the frequency. The capacitor C_1 is the adjustable capacitor.

→ For final adjustment for compensation is made experimentally by the method of square-wave testing.

Response of attenuator to a step input:

Let us consider a step voltage input of amplitude V which changes abruptly by V at t=0.

As input changes abruptly, the voltage across C_1 and C_2 also changes abruptly, this happens because at $t=0$, capacitors acts as short-circuit and a very large (ideally infinite) current flows through the capacitor for an infinitesimally small time. so, a finite charge $q = \int_{0^-}^{0^+} i(t)\,dt$ is delivered to each capacitor.

The voltage across $C_1 = \frac{q}{C_1}$

The voltage across $C_2 = \frac{q}{C_2}$

i) By applying KVL at $t=0^+$ ie; immediatly after the input signal is applied, the capacitors acts as short-circuit and hence resistors go out of the circuit.

input signal $V_i(0^+) = V_1 + V_2$

$$V_i(0^+) = \frac{q}{C_1} + \frac{q}{C_2}$$

$$V_i(0^+) = q\left(\frac{C_1+C_2}{C_1C_2}\right)$$

$$q = V_i(0^+)\left(\frac{C_1C_2}{C_1+C_2}\right)$$

The output voltage is

$$V_0(0^+) = \frac{q}{C_2} = \frac{VC_1\cancel{C_2}}{(C_1+C_2)\cancel{C_2}}, \text{ since } V_i(0^+) = V$$

$$\boxed{V_0(0^+) = \frac{VC_1}{C_1+C_2}} \quad \text{———} \quad (1)$$

ii) at $t=\infty$

at this condition, steady state has been reached and the capacitor acts as open-circuit and final output voltage is determined by the resistors R_1 & R_2

$$\therefore\ V_0(\infty) = i R_2$$

but $i = \dfrac{V_i(\infty)}{R_1+R_2}$

$$\boxed{V_0(\infty) = \frac{R_2 V}{R_1+R_2}} \quad \text{——— (2)}$$

perfect compensation is obtained if

$$V_0(0^+) = V_0(\infty)$$

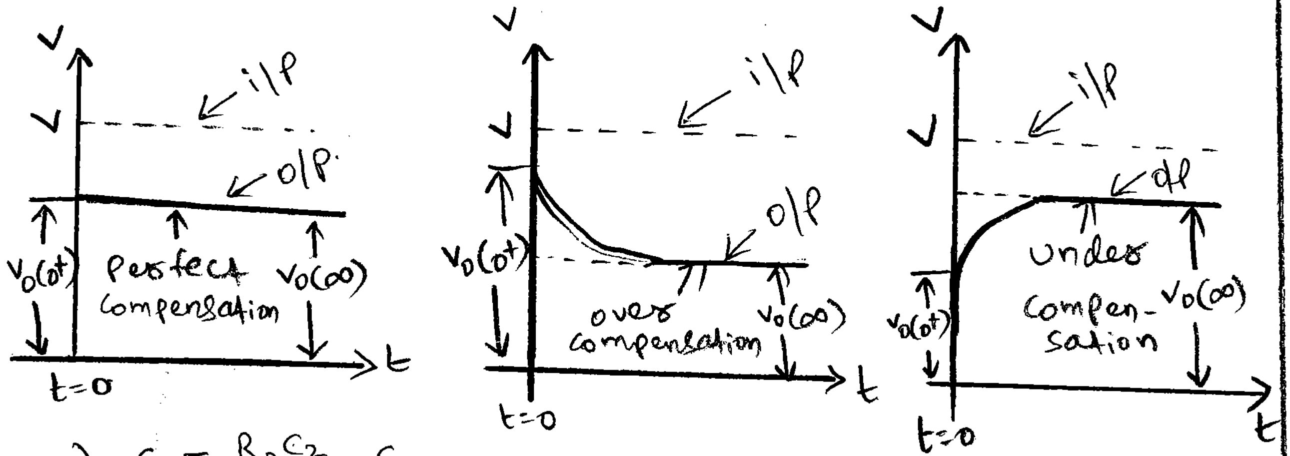
$$\frac{\cancel{V} C_1}{C_1+C_2} = \frac{\cancel{V} R_2}{R_1+R_2}$$

$$R_1C_1 + \cancel{R_2C_1} = \cancel{R_2C_1} + R_2C_2$$

$$\therefore\ \boxed{\begin{array}{c} R_1C_1 = R_2C_2 \\ \text{or} \\ C_1 = \dfrac{R_2C_2}{R_1} = C_p \end{array}}$$

which is the balanced bridge condition is obtained for perfect compensation.

The response of an attenuator to a step input is shown below:



a) $C_1 = \frac{R_2 C_2}{R_1} = C_p$ b) $C_1 > C_p$ c) $C_1 < C_p$

Application of Attenuator as a cathode ray oscilloscope probe:

To observe the waveform, the signal point of a circuit is connected to the input terminals of an oscilloscope. If the signal point is at far distance from CRO terminals and if the signal appears at a high impedance level, then shielded cable is used to connect the signal to the oscilloscope.

Shielding of the cable is necessary to isolate the input lead from stray fields. The capacitance looking into several feet of cable may be as high as 100 to 150 pF, this combination of high input capacitance together with the high output impedance of the signal source will make it impossible to make faithful observations of fast waveforms.

A probe assembly which uses shielded cable keeps the capacitance low as shown below.

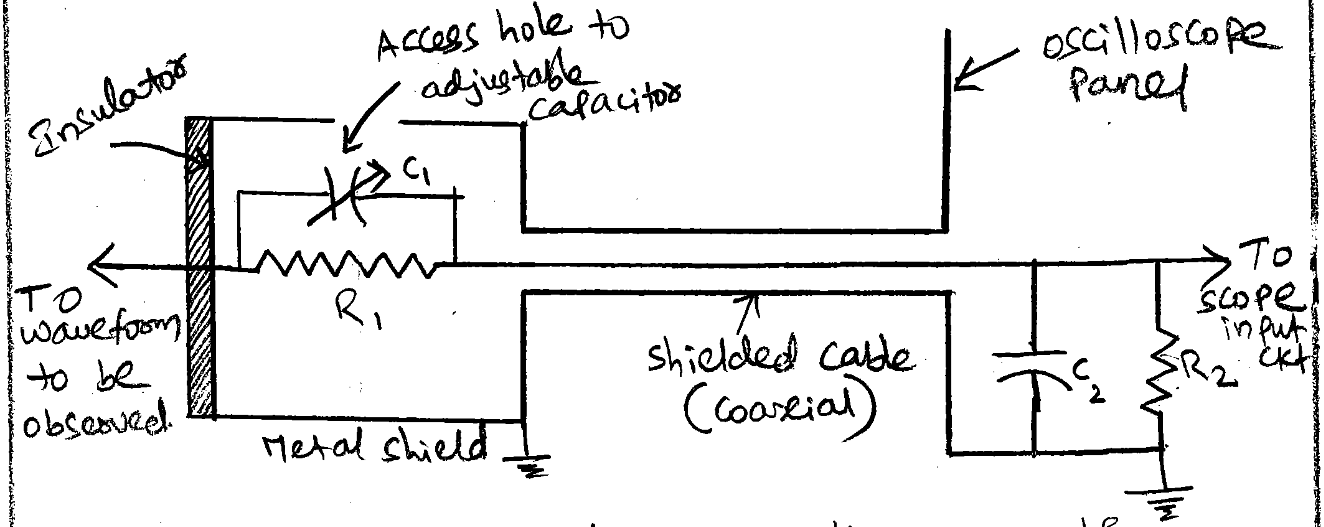
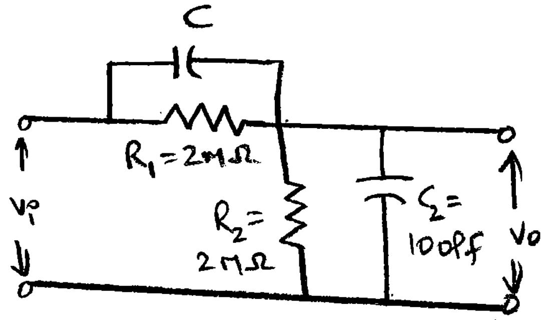


fig: A cathode-Ray oscilloscope probe

The attenuation introduced through use of the probe assembly is 10 (or) 20 and the input capacitance to the probe assembly is about 20 (or) 10 pf respectively.

Problems – Attenuators

(2-3) Compute and draw to scale the output waveform for C = 100 pF, C = 150 pF and C = 50 pF and the input is a 25V step.



Sol: Given data:

$R_1 = 2M\Omega$

$R_2 = 2M\Omega$

$C_2 = 100pF$

$V_i = 25V$ step

$C_1 = ?$

For perfect compensation

$$R_1C_1 = R_2C_2$$

$$C_1 = \frac{R_2C_2}{R_1}$$

$$C_1 = \frac{2\times10^6 \times 100\times10^{-12}}{2\times10^6}$$

$$C_1 = 100pF$$

a) $C_1 = 100pF$

$$V_o(0^+) = \frac{VC_1}{C_1+C_2} = \frac{25\times100\times10^{-12}}{100\times10^{-12}+100\times10^{-12}} = 12.5V$$

$$V_o(\infty) = \frac{VR_2}{R_1+R_2} = \frac{25\times2\times10^6}{2\times10^6+2\times10^6} = 12.5V$$

b) $C_1 = 150pF$

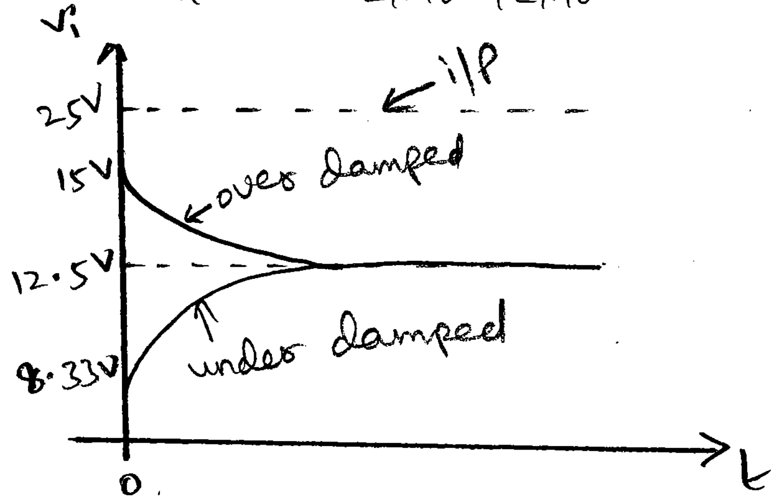
$$V_o(0^+) = \frac{VC_1}{C_1+C_2} = \frac{25\times150\times10^{-12}}{150\times10^{-12}+100\times10^{-12}} = 15V$$

$$V_o(\infty) = \frac{VR_2}{R_1+R_2} = \frac{25\times2\times10^6}{2\times10^6+2\times10^6} = 12.5V$$

c) $C_1 = 50\,pf$

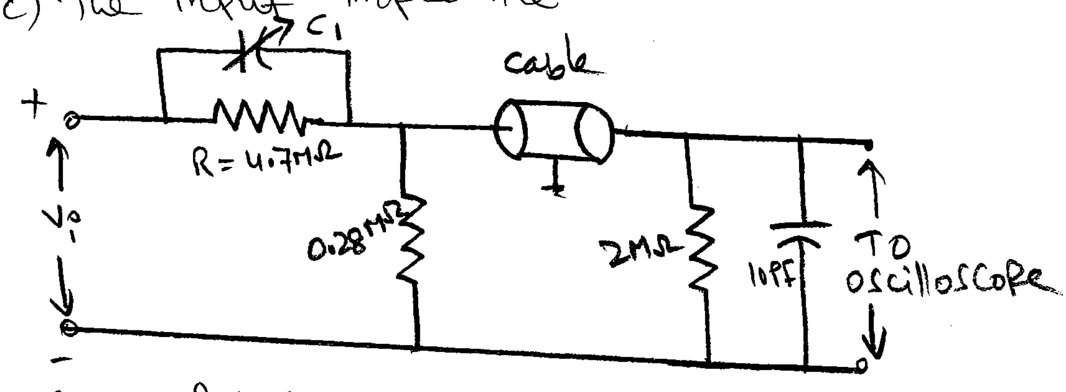
$$V_o(0+) = \frac{V C_1}{C_1 + C_2} = \frac{25 \times 50 \times 10^{-12}}{50\times10^{-12} + 100\times10^{-12}} = 8.33V$$

$$V_o(\infty) = \frac{V R_2}{R_1 + R_2} = \frac{25 \times 2 \times 10^{6}}{2\times10^{6} + 2\times10^{6}} = 12.5V$$



(2-29) An oscilloscope test probe is indicated. Assume that the cable capacitance is 100 pf. The input impedance of the scope is 2 MΩ in parallel with 10pf. what is the

a) Attenuation of the probe.
b) C_1 for best response.
c) The input impedance of the compensated probe.



Sol: Given data:

cable capacitance = 100pf

$R_i = 2\,M\Omega$

$C = 10\,pf$

Net Capacitance = Cable capacitance + 10 pF = 100 + 10 = 110 pF = C_2

Net resistance = $2M\Omega \,||\, 0.28M\Omega = \frac{2 \times 0.28}{2 + 0.28} = 0.245 M\Omega = R_2$

a) Attenuation of the probe

$$a = \frac{R_2}{R_1 + R_2}$$

$$a = \frac{0.245}{4.7 + 0.245}$$

$$a = 0.05$$

b) For perfect compensation

$$R_1 C_1 = R_2 C_2$$

$$C_1 = \frac{R_2 C_2}{R_1}$$

$$C_1 = \frac{0.245 \times 110 \times 10^{-12} \times 10^6}{4.7 \times 10^6}$$

$$C_1 = 5.74 \text{ pF}$$

c) Input impedance

$$Z_i = R_1 + R_2$$

$$Z_i = 4.7 + 0.245$$

$$Z_i = 4.95 M\Omega$$