

Laplace Transform

Laplace is very useful tool in circuit analysis. It transforms integro-diff eq's into linear polynomial eq's.

Laplace is a mathematical tool which transforms a function in time domain into a function in frequency domain.

If $f(t)$ is function defined over the limits $0 < t < \infty$.

The Laplace transform is given by.

$$f(t) \xrightarrow{\text{L.T.}} \int_0^{\infty} f(t) e^{-st} dt$$

For any function to apply L.T., it must satisfy Condition $\int_0^{\infty} f(t) e^{-st} dt < \infty$

A Laplace transform is said to be bilateral when it takes the limits between $-\infty$ to $+\infty$. It is said to be unilateral when it takes the limits from 0 to ∞ .

Functional Laplace:- These are Laplace transforms of the special operational Laplace. defined mathematical property of L.T. functions

i) Unit step function:- $f(t) = u(t)$

$$u(t) = 1, \quad t > 0$$

$$0, \quad t < 0$$

$$\begin{aligned} L[f(t)] &= \int_0^{\infty} f(t) e^{-st} dt = \int_0^{\infty} 1 \cdot e^{-st} dt = -\frac{1}{s} [e^{-st}]_0^{\infty} \\ &= -\frac{1}{s} [e^{\infty} - e^0] = \frac{1}{s} \end{aligned}$$

$$f(t) \rightarrow u(t) \xrightarrow{\text{L.T.}} \frac{1}{s}$$

ii) Unit impulse function:-

$$\delta(t) = 1, \quad t = 0$$

$$= 0, \quad t > 0$$

$$L\{\delta(t)\} = \int_0^{\infty} 1 \cdot e^{-s \cdot 0} dt = 1$$

$$\delta(t) \xrightarrow{\text{L.T.}} 1$$

unit Ramp function:-

$$x(t) = at, t > 0 \\ = 0, t < 0$$

$$\int u \cdot v = u \left[v - \int \frac{dv}{dt} \right] dt$$

if $a=1 \Rightarrow x(t) = t, t > 0$

$$\begin{aligned} L[x(t)] &= \int_0^{\infty} x(t) e^{-st} dt = \int_0^{\infty} t e^{-st} dt \\ &= \left. \frac{t e^{-st}}{-s} \right|_0^{\infty} - \int_0^{\infty} 1 \cdot \int_0^{\infty} e^{-st} dt dt = \left. \frac{t e^{-st}}{-s} \right|_0^{\infty} - \int_0^{\infty} \frac{e^{-st}}{-s} dt \\ &= \left[\frac{\infty e^{-\infty}}{-s} - \frac{0 \cdot e^{-0}}{s} \right] + \frac{1}{s} \left[\frac{e^{-st}}{-s} \right]_0^{\infty} = 0 - \frac{1}{s^2} [e^{-\infty} - e^{-0}] = \frac{1}{s^2} \end{aligned}$$

$$\boxed{t \xrightarrow{L.T.} \frac{1}{s^2}}$$

Exponential function:- $f(t) = e^{-at}$

$$\begin{aligned} L[e^{-at}] &= \int_0^{\infty} e^{-at} \cdot e^{-st} dt = \int_0^{\infty} e^{-(s+a)t} dt \\ &= \left. \frac{-1}{s+a} [e^{-(s+a)t}] \right|_0^{\infty} = \frac{-1}{s+a} [e^{\infty} - e^0] = \frac{1}{s+a} \end{aligned}$$

$$\boxed{e^{-at} \xrightarrow{L.T.} \frac{1}{s+a}}$$

cosine function:- $f(t) = \cos \omega t$

$$\begin{aligned} L[\cos \omega t] &= \int_0^{\infty} \cos \omega t e^{-st} dt = \int_0^{\infty} \left[\frac{e^{j\omega t} + e^{-j\omega t}}{2} \right] e^{-st} dt \\ &= \frac{1}{2} \left\{ \int_0^{\infty} e^{-(s-j\omega)t} dt + \int_0^{\infty} e^{-(s+j\omega)t} dt \right\} \\ &= \frac{1}{2} \left\{ \frac{-1}{s-j\omega} [e^{-(s-j\omega)t}]_0^{\infty} + \frac{-1}{s+j\omega} [e^{-(s+j\omega)t}]_0^{\infty} \right\} \end{aligned}$$

$$= \frac{1}{2} \left\{ \frac{-1}{s-j\omega} [e^{\alpha} - e^0] - \frac{1}{s+j\omega} [e^{\alpha} - e^0] \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{s-j\omega} - \frac{1}{s+j\omega} \right\} = \frac{1}{2} \left\{ \frac{s+j\omega}{s^2+\omega^2} - \frac{s-j\omega}{s^2+\omega^2} \right\}$$

$$= \frac{1}{2} \left\{ \frac{2s}{s^2+\omega^2} \right\} = \frac{s}{s^2+\omega^2}$$

$$\Rightarrow \boxed{\cos \omega t \xrightarrow{L.T} \frac{s}{s^2+\omega^2}}$$

sine function:- $f(t) = \sin \omega t$

$$L[\sin \omega t] = \int_0^{\infty} f(t) e^{-st} dt$$

$$= \int_0^{\infty} \sin \omega t e^{-st} dt = \int_0^{\infty} \left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j} \right) e^{-st} dt$$

$$= \frac{1}{2j} \left\{ \int_0^{\infty} e^{-(s-j\omega)t} dt - \int_0^{\infty} e^{-(s+j\omega)t} dt \right\}$$

$$= \frac{1}{2j} \left[\frac{e^{-(s-j\omega)t}}{s-j\omega} - \frac{e^{-(s+j\omega)t}}{s+j\omega} \right]_0^{\infty}$$

$$= \frac{1}{2j} \left[\frac{(-1)}{s-j\omega} - \frac{(-1)}{s+j\omega} \right] = \frac{1}{2} \left[\frac{s+j\omega}{s^2+\omega^2} + \frac{s-j\omega}{s^2+\omega^2} \right]$$

$$= \frac{1}{2j} \left[\frac{-s+j\omega + s+j\omega}{s^2+\omega^2} \right] = \frac{2j\omega}{2j(s^2+\omega^2)} = \frac{\omega}{s^2+\omega^2}$$

$$\boxed{\sin \omega t \xrightarrow{L.T} \frac{\omega}{s^2+\omega^2}}$$

Type	$f(t)$	$F(s)$
Impulse	$\delta(t)$	1
step	$u(t)$	$1/s$
Ramp	t	$1/s^2$
Exponential	e^{-at}	$1/(s+a)$
sine	$\sin \omega t$	$\omega / (s^2 + \omega^2)$
cosine	$\cos \omega t$	$s / (s^2 + \omega^2)$
First Differentiative	$\frac{df(t)}{dt}$	$sF(s) - f(0)$
Integral	$\int_0^t f(t) dt$	$\frac{F(s)}{s}$
	$e^{-at} f(t)$	$F(s+a)$

Diff

$$\int_0^{\infty} \frac{d}{dt} f(t) e^{-st} dt = \int_0^{\infty} \underbrace{\frac{df(t)}{dt}}_v \underbrace{e^{-st}}_u dt$$

$$[f(t) e^{-st}]_0^{\infty} - \int_0^{\infty} f(t) (-s) e^{-st} dt = -f(0) + s \int_0^{\infty} f(t) e^{-st} dt$$

$$= [0 - f(0)] + s \int_0^{\infty} e^{-st} f(t) dt = -f(0) + sF(s) = sF(s) - f(0)$$

$$\mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\} = \int_0^{\infty} \underbrace{e^{-st}}_v \left(\underbrace{\int_0^t f(\tau) d\tau}_u \right) dt = \left[\int_0^t f(\tau) d\tau \cdot \frac{e^{-st}}{-s} \right]_0^{\infty} + \int_0^{\infty} f(\tau) \frac{e^{-st}}{-s} (-s) d\tau$$

$$= 0 + \frac{1}{s} \int_0^{\infty} f(\tau) e^{-st} d\tau = \frac{1}{s} F(s)$$

Resistor in the frequency domain:-

Consider the resistor connected to voltage source $V(t)$. Ohm's law,

$$V(t) = R i(t)$$

Taking Laplace transform on both sides

$$V(s) = R I(s)$$

The ratio of frequency domain representation of voltage to the frequency domain representation of current is transform impedance of resistor,

$$Z(s) = \frac{V(s)}{I(s)} = R ; \text{ units } - \underline{\Omega}$$

The impedance of a resistor does not change while going from time domain to the frequency domain, as it is independent of frequency.

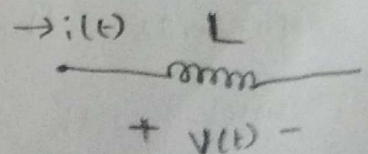
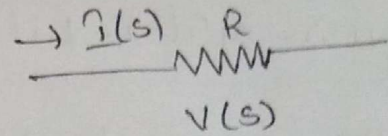
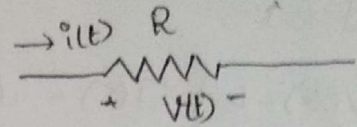
The admittance $Y(s)$ of a resistor is defined as the ratio of $I(s)$ to $V(s)$

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{R} = G \rightarrow \text{units } - \underline{\text{Siemen 'S'}}$$

Inductor in frequency domain:-

Consider an inductor connected to time varying voltage source $V(t)$,

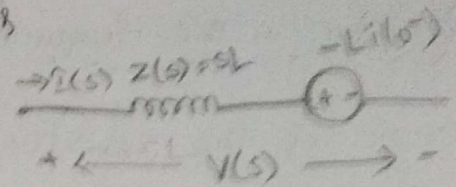
$$V(t) = L \frac{di(t)}{dt}$$



Taking Laplace transform on both sides

$$V(s) = L \left[sI(s) - i(0^-) \right]$$

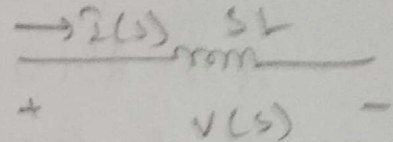
$$\Rightarrow \boxed{V(s) = sLI(s) - Li(0^-)} \rightarrow \text{①} \rightarrow$$



If the energy stored in an inductor is initially zero i.e., $i(0^-) = 0$, then

$$V(s) = sLI(s)$$

$$\Rightarrow \boxed{Z(s) = \frac{V(s)}{I(s)} = sL}, \text{ the transformed impedance of inductor.}$$



If $s = j\omega \Rightarrow \boxed{Z(j\omega) = j\omega L}$

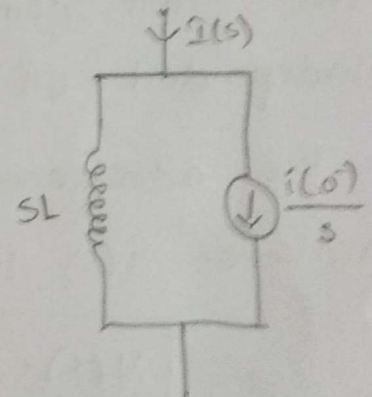
This equation represents that an impedance of sL ohms is connected in series with an independent voltage source of $Li(0^-)$

from eq ①, we can write as

$$sLI(s) = V(s) + Li(0^-)$$

$$\Rightarrow I(s) = \frac{V(s)}{sL} + \frac{Li(0^-)}{sL}$$

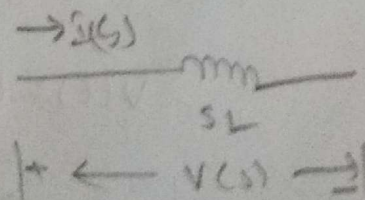
$$\Rightarrow \boxed{I(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}} \rightarrow \text{②}$$



It represents that circuit contains an impedance sL in parallel with the current source $\frac{i(0^-)}{s}$.

If $i(0^-) = 0$, then

$$\boxed{I(s) = \frac{V(s)}{sL}}$$



Capacitor in s domain:-

Consider an initially charged capacitor & the voltage-current relation is given as

$$i = C \frac{dv}{dt}$$

Taking Laplace transform on both sides

$$I(s) = C [sV(s) - v(0^-)]$$

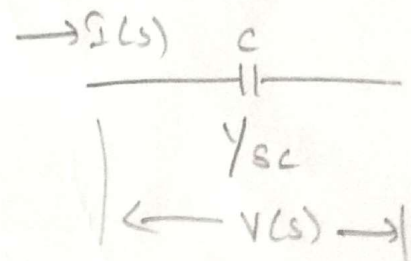
$$\Rightarrow \boxed{I(s) = sCV(s) - C v(0^-)} \rightarrow \textcircled{1} \quad I = V / \frac{1}{sC} (VY)$$

Eq ① represents that the ckt contains an admittance of capacitor sC in parallel with current source $C v(0^-)$.

* If capacitor is initially stored no energy i.e., $v(0^-) = 0$

$$\Rightarrow I(s) = sC V(s)$$

$$\Rightarrow \boxed{Z(s) = \frac{V(s)}{I(s)} = \frac{1}{sC}}$$

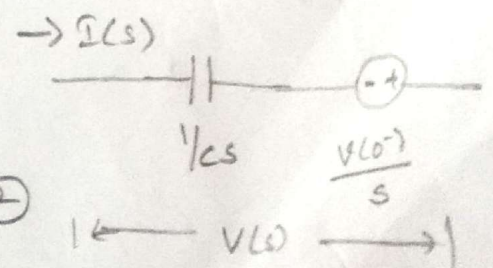


From Eq ①, we can also write

$$sC V(s) = I(s) + C v(0^-)$$

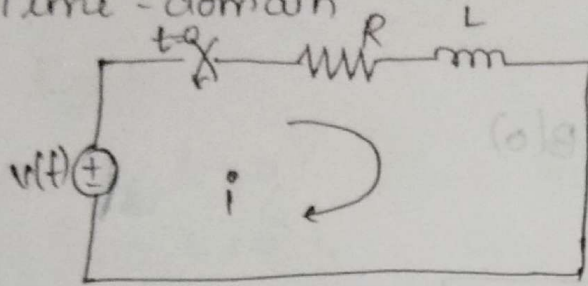
$$\Rightarrow V(s) = \frac{I(s) + C v(0^-)}{sC}$$

$$\Rightarrow \boxed{V(s) = \frac{I(s)}{sC} + \frac{v(0^-)}{s}} \rightarrow \textcircled{2}$$

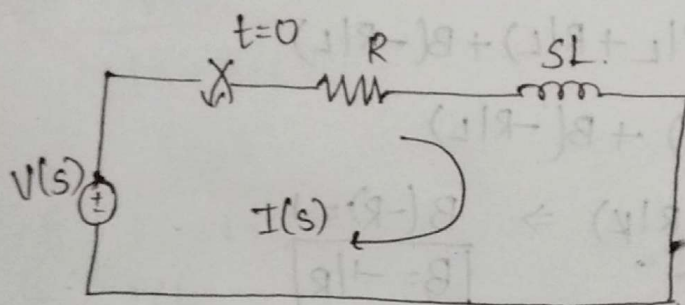


Analysis of R-L series circuit for various input.

Time-domain



In s-domain



Applying KVL for the above circuit

$$V(s) = I(s)R + sLI(s)$$

$$V(s) = I(s)[R + sL]$$

$$\frac{V(s)}{I(s)} = R + sL \Rightarrow \frac{I(s)}{V(s)} = \frac{1}{R + sL}$$

$$\frac{I(s)}{V(s)} = \frac{1}{L(s + R/L)}$$

Unit step

$$u(t) = 1, t \geq 0$$

$$0, t < 0$$

$$V(s) = U(s) = 1/s$$

$$I(s) = \frac{V(s)}{L(s + R/L)} = \frac{1/s}{L(s + R/L)}$$

$$= \frac{1/L}{s(s + R/L)}$$

Consider

$$\frac{1/L}{s(s+R/L)} = \frac{A}{s} + \frac{B}{s+R/L} \rightarrow \textcircled{I}$$

$$1/L = A(s+R/L) + B(s) \rightarrow \textcircled{1}$$

Let $s=0$.

$$1/L = A(0+R/L) + B(0)$$

$$1/L = A(R/L)$$

$$AR = 1 \Rightarrow \boxed{A = 1/R}$$

Substitute $s = -R/L$ in eqn $\textcircled{1}$.

$$1/L = A(-R/L + R/L) + B(-R/L)$$

$$1/L = A(0) + B(-R/L)$$

$$1/L = B(-R/L) \Rightarrow \boxed{B = -1/R}$$

Substitute A, B in $\textcircled{I}$

$$\frac{1/L}{s(s+R/L)} = \frac{1/R}{s} + \frac{-1/R}{s+R/L}$$

$$I(s) = \frac{1/R}{s} + \frac{-1/R}{s+R/L} \rightarrow \textcircled{2}$$

Apply Inverse Laplace transform for the eqn $\textcircled{2}$

$$i(t) = \frac{1}{R}(1) + -1/R(e^{-Rt/L})$$

$$\boxed{i(t) = \frac{1}{R}(1 - e^{-Rt/L})}$$

Unit Impulse:-

$$\delta(t) = 1,$$

$$V(s) = 1$$

$$I(s) = \frac{V(s)}{L(s+R/L)} = \frac{1}{L(s+R/L)}$$

$$I(s) = \frac{1}{L(s + R/L)} \rightarrow (1)$$

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Apply Inverse Laplace transform for equation (1)

$$i(t) = \frac{1}{L} e^{-Rt/L}$$

Unit Ramp:

$$v(t) = t, t \geq 0$$

$$0, t < 0$$

$$V(s) = 1/s^2$$

$$I(s) = \frac{V(s)}{L(s + R/L)}$$

$$= \frac{1/s^2}{L(s + R/L)}$$

$$I(s) = \frac{1/L}{s^2(s + R/L)}$$

Consider $\frac{1/L}{s^2(s + R/L)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s + R/L} \rightarrow (2)$

$$1/L = A(s)(s + R/L) + B(s + R/L) + C(s^2)$$

$$\text{Let } s = 0$$

$$1/L = A(0)(0 + R/L) + B(0 + R/L) + C(0)$$

$$1/L = B(R/L)$$

$$B = 1/R$$

$$\text{Let } s = -R/L$$

$$1/L = A(-R/L)(-R/L + R/L) + B(-R/L + R/L) + C\left(\left(-\frac{R}{L}\right)^2\right)$$

$$1/L = C\left(\frac{R^2}{L^2}\right)$$

$$\frac{R^2 C}{L} = 1 \Rightarrow C = \frac{L}{R^2}$$

$$\frac{1}{L} = A(s^2 + R/Ls) + B(s + R/L) + Cs^2$$

$$\frac{1}{L} = s^2(A+C) + s(AR/L+B) + BR/L$$

Comparing s^2 coefficients on both sides

$$A+C=0$$

$$A + \frac{L}{R^2} = 0$$

$$A = -L/R^2$$

Sub A, B, C values $\rightarrow$ ①

$$\frac{1/L}{s^2(s + R/L)} = \frac{-L/R^2}{s} + \frac{1/R}{s^2} + \frac{L/R^2}{s + R/L}$$

$$I(s) = \frac{-L/R^2}{s} + \frac{1/R}{s^2} + \frac{L/R^2}{s + R/L}$$

Apply Inverse Laplace Transform for eq ③

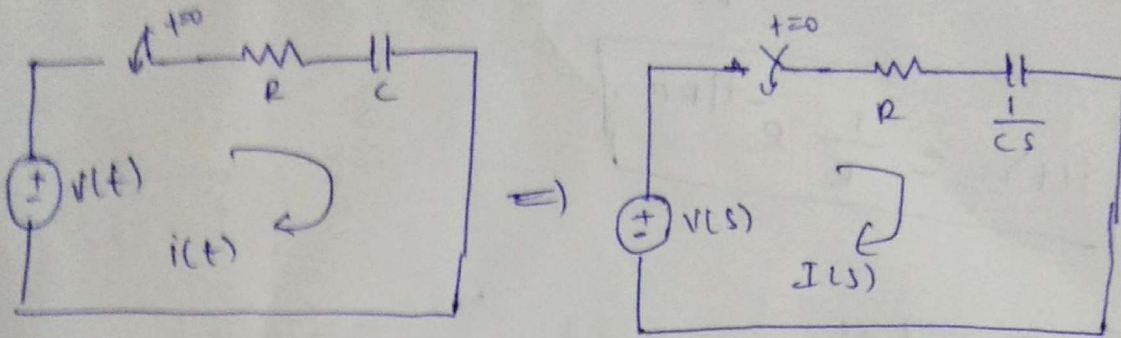
$$i(t) = -\frac{L}{R^2}(1) + \frac{1}{R}(t) + \frac{L}{R^2}(e^{-Rt/L})$$

$$i(t) = \frac{L}{R^2}(e^{-Rt/L} - 1) + \frac{1}{R}(t)$$

Analysis of series R-C circuit

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$$V(s) = I(s)R + \frac{I(s)}{Cs}$$

$$V(s) = I(s) \left(R + \frac{1}{Cs} \right)$$

$$\frac{V(s)}{I(s)} = R + \frac{1}{Cs}$$

$$\frac{V(s)}{I(s)} = \frac{CsR + 1}{Cs}$$

$$\frac{I(s)}{V(s)} = \frac{Cs}{RCs + 1}$$

$$\frac{I(s)}{V(s)} = \frac{s}{R(s + \frac{1}{RC})}$$

$$\frac{I(s)}{V(s)} = \frac{s}{R(s + \frac{1}{RC})}$$

for unit step funⁿ

$$V(s) = \frac{1}{s}$$

$$I(s) = \frac{V(s) \times s}{R(s + \frac{1}{RC})}$$

$$= \frac{\frac{1}{s} \times s}{R(s + \frac{1}{RC})}$$

$$\Rightarrow I(s) = \frac{1}{R(s + \frac{1}{RC})}$$

$$i(t) = \frac{1}{R} e^{-t/RC}$$

unit Impulse

$$V(s) = 1$$

$$I(s) = \frac{s}{R(s + \frac{1}{RC})}$$

$$\frac{s}{R(s + \frac{1}{RC})} = \frac{1}{R} \left[\frac{s + \frac{1}{RC} - \frac{1}{RC}}{(s + \frac{1}{RC})} \right]$$

$$= \frac{1}{R} \left[\frac{s + \cancel{\frac{1}{RC}}}{s + \cancel{\frac{1}{RC}}} - \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \right]$$

$$= \frac{1}{R} \left[1 - \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \right]$$

$$i(t) = \frac{1}{R} \left[s(t) - \frac{1}{RC} e^{-t/RC} \right]$$

gamp function

$$V(s) = \frac{1}{s^2}$$

$$I(s) = \frac{\frac{1}{s^2} \times A}{R(s + \frac{1}{RC})}$$

$$= \frac{1}{R s(s + \frac{1}{RC})}$$

$$= \frac{\frac{1}{R}}{s(s + 1/Rc)}$$

$$\frac{\frac{1}{R}}{s(s + \frac{1}{Rc})} = \frac{A}{s} + \frac{B}{s + \frac{1}{Rc}}$$

$$A + s = 0$$

$$A + s = -1/Rc$$

$$\frac{1}{R} = A(0 + \frac{1}{Rc})$$

$$\frac{1}{R} = B \times (-\frac{1}{Rc})$$

$$\frac{1}{R} = A(\frac{1}{Rc})$$

$$\frac{1}{R} = \frac{-B}{Rc}$$

$$A = C$$

$$B = -C$$

$$= \frac{C}{s} - \frac{C}{s + 1/Rc}$$

$$= C \left[\frac{1}{s} - \frac{1}{s + 1/Rc} \right]$$

$$i(t) = C \left[1 - e^{-t/Rc} \right]$$

1) In series RL circuit, an exponential voltage $V = 50e^{-100t}$ V is applied by closing switch at $t=0$. Find the current.

Sol:- Applying KVL for the circuit

$$V = iR + L \frac{di}{dt}$$

& $i(0^-) = 0$ in the circuit

In s-domain

$$\Rightarrow V(s) = R I(s) + sL I(s) - Li(0^-)$$

$$\Rightarrow \frac{50}{s+100} = 10 I(s) + 0.2s I(s)$$

$$\Rightarrow I(s) = \frac{50}{(s+100)(0.2s+10)} = \frac{50/0.2}{(s+100)(s+\frac{10}{0.2})}$$

$$\Rightarrow I(s) = \frac{250}{(s+100)(s+50)} = \frac{A}{s+100} + \frac{B}{s+50}$$

$$\Rightarrow 250 = A(s+50) + B(s+100)$$

$$\text{At } s = -50 \Rightarrow 250 = B(-50+100)$$

$$\Rightarrow B = \frac{250}{50} = 5$$

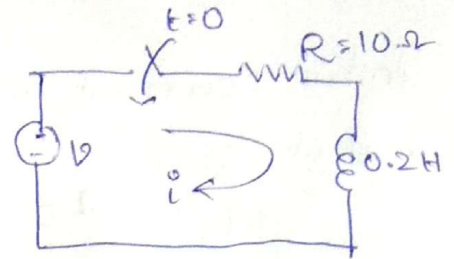
$$\text{At } s = -100 \Rightarrow 250 = A(-100+50)$$

$$\Rightarrow A = \frac{-250}{50} = -5$$

$$\Rightarrow I(s) = -\frac{5}{s+100} + \frac{5}{s+50} = 5 \left(\frac{1}{s+50} - \frac{1}{s+100} \right)$$

Inverse Laplace,

$$i(t) = 5 \left(e^{-50t} - e^{-100t} \right) \text{ A}$$



$$V(t) = 50e^{-100t}$$

$$\Rightarrow V(s) = \frac{50}{s+100}$$

✓

$$\frac{50/0.2}{(s+100)(s+\frac{10}{0.2})}$$

✓

$$\frac{250}{(s+100)(s+50)}$$

$$\frac{A}{s+100} + \frac{B}{s+50}$$

$$250 = A(s+50) + B(s+100)$$

$$\text{At } s = -50 \Rightarrow 250 = B(-50+100)$$

$$\Rightarrow B = \frac{250}{50} = 5$$

$$\text{At } s = -100 \Rightarrow 250 = A(-100+50)$$

$$\Rightarrow A = \frac{-250}{50} = -5$$

$$\Rightarrow I(s) = -\frac{5}{s+100} + \frac{5}{s+50}$$

$$= 5 \left(\frac{1}{s+50} - \frac{1}{s+100} \right)$$

Inverse Laplace,

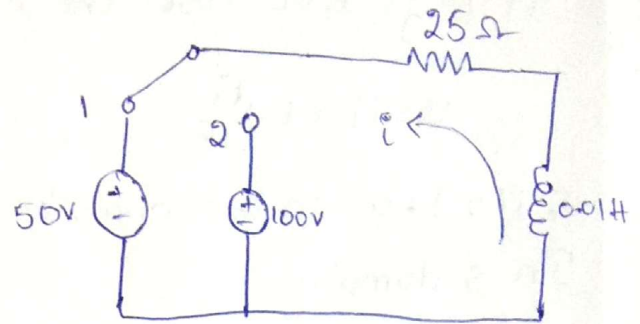
$$i(t) = 5 \left(e^{-50t} - e^{-100t} \right) \text{ A}$$

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2) In RL circuit shown in fig. the switch is in position 1. At $t=0$ it is switched to position 2. find the current.

Sol: Initial current in the circuit is

$$i(0^-) = \frac{-50}{25} = -2A$$



At $t=0^+$, switch is at position 2.

$$\Rightarrow 100 = 25i + 0.01 \frac{di}{dt}$$

In s domain

$$\frac{100}{s} = 25I(s) + s \cdot 0.01I(s) - 0.01(-2)$$

$$\Rightarrow I(s) [25 + 0.01s] = \frac{100}{s} - 0.02$$

$$\Rightarrow I(s) = \frac{100}{s(25 + 0.01s)} - \frac{0.02}{(25 + 0.01s)}$$

$$\Rightarrow I(s) = \frac{10^4}{s(s + 2500)} - \frac{2}{(2500 + s)}$$

$$\frac{10^4}{s(s + 2500)} = \frac{A}{s} + \frac{B}{s + 2500}$$

$$\Rightarrow 10^4 = A(s + 2500) + Bs$$

$$s=0, \Rightarrow 10^4 = A \cdot 2500 \Rightarrow A = \frac{100}{25} = 4$$

$$s = -2500 \Rightarrow 10^4 = B(-2500) \Rightarrow B = \frac{-100}{25} = -4$$

$$\Rightarrow \frac{10^4}{s(s + 2500)} = \frac{4}{s} - \frac{4}{s + 2500}$$

$$\Rightarrow \hat{I}(s) = \frac{4}{s} - \frac{4}{s+2500} - \frac{2}{s+2500} = \frac{4}{s} - \frac{6}{s+2500}$$

Inverse Laplace

$$i(t) = 4 - 6e^{-2500t} \text{ A}$$

3. In series RL circuit, the source is $v = 100 \sin(500t)$. Determine the current.

Sol:- $V(s) = 5\hat{I}(s) + 0.01s\hat{I}(s) - 0.01i(0)$

$$\Rightarrow V(s) = 5\hat{I}(s) + 0.01s\hat{I}(s) \quad \because i(0) = 0$$

$$\Rightarrow \hat{I}(s) [5 + 0.01s] = \frac{5 \times 10^4}{s^2 + 500^2}$$

$$\Rightarrow \hat{I}(s) = \frac{5 \times 10^4 \times 10^2}{(s^2 + 500^2)(s + 500)}$$

$$\Rightarrow \hat{I}(s) = \frac{5 \times 10^6}{(s + j500)(s - j500)(s + 500)} = \frac{A}{s + j500} + \frac{B}{s - j500} + \frac{C}{s + 500}$$

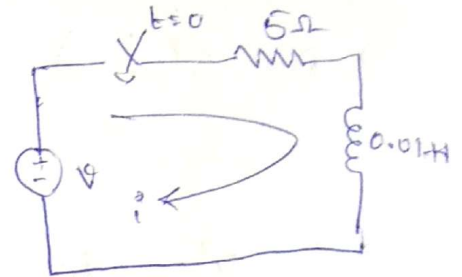
$$\Rightarrow 5 \times 10^6 = A(s - j500)(s + 500) + B(s + j500)(s + 500) + C(s + j500)(s - j500)$$

At $s = +j500$

$$\Rightarrow 5 \times 10^6 = B(j500 + j500)(j500 + 500)$$

$$\Rightarrow 5 \times 10^6 = B \cdot j \cdot 10^3 \cdot \cancel{j} \cdot 10^3 (1 + j) \Rightarrow B = \frac{10}{j(1+j)} = \frac{10}{(j-1)} \times \frac{j+1}{j+1}$$

$$\Rightarrow B = \frac{10(j+1)}{-1-1} = -5(j+1)$$



$$V(t) = 100 \sin 500t$$

$$\Rightarrow V(s) = \frac{100 \cdot 500}{s^2 + 500^2}$$

At $s = -j500$

$$\Rightarrow 5 \times 10^6 = A(-j500 - j500)(-j500 + 500)$$

$$\Rightarrow 5 \times 10^6 = A \cdot j \cdot 10^3 \cdot (1-j)$$

$$\Rightarrow A = \frac{10}{-j(1-j)} = \frac{10}{1+j} \times \frac{1+j}{1+j} = \frac{10(1+j)}{-2} = -5(1+j)$$

$$\Rightarrow A = 5(j-1)$$

At $s = -500$

$$\Rightarrow 5 \times 10^6 = C((-500)^2 + (500)^2) = C(250000 + 250000)$$

$$\Rightarrow 5 \times 10^6 = C \cdot 500000 \Rightarrow C = 10$$

$$\Rightarrow \mathcal{I}(s) = \frac{5(j-1)}{s+j500} - \frac{5(j+1)}{s-j500} + \frac{10}{s+500}$$

Inverse Laplace

$$\Rightarrow \mathcal{I}(s) = \frac{(5j-5)(s-j500) - (5j+5)(s+j500)}{(s+j500)(s-j500)} + \frac{10}{s+500}$$

$$= \frac{5j/s - 5j \cdot j500 - 5s + j2500 - 5j/s - 5j \cdot j500 - 5s - j2500}{s^2 + 500^2}$$

$$= \frac{5(500) - 10s + 5(500)}{s^2 + 500^2} = \frac{10(500 - s)}{s^2 + 500^2}$$

$$= 10 \left[\frac{500}{s^2 + 500^2} - \frac{s}{s^2 + 500^2} \right] + \frac{10}{s+500}$$

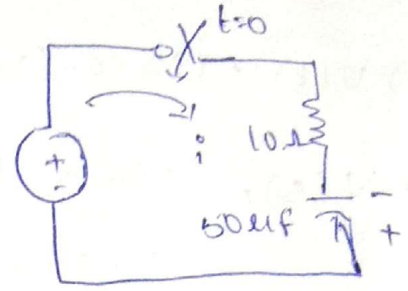
Inverse Laplace

$$\Rightarrow i(t) = 10 \sin 500t - 10 \cos 500t + 10 e^{-500t} \quad A$$

4. In series RC circuit, the capacitor has an initial charge of 2.5 mC . At $t=0$, switch is closed. Determine the current.

Sol:- In time domain eq is

$$100 = 10i + \frac{1}{50 \times 10^{-6}} \int i dt + \frac{1}{50 \times 10^{-6}} (-2.5 \times 10^{-3})$$



$$\Rightarrow 100 = 10i + \frac{1}{50 \times 10^{-6}} \int i dt - \frac{25 \times 10^{-4}}{50 \times 10^{-6}}$$

In s domain

$$\Rightarrow \frac{100}{s} = 10I(s) + \frac{I(s)}{50 \times 10^{-6} \cdot s} - \frac{1}{2 \times 10^{-2} \cdot s}$$

$$\Rightarrow I(s) \left[10 + \frac{1}{s \cdot 50 \times 10^{-6}} \right] = \frac{100}{s} + \frac{100}{2s} = \frac{100}{s} + \frac{50}{s} = \frac{150}{s}$$

$$\Rightarrow I(s) = \frac{150}{s \left[10 + \frac{1}{s \cdot 50 \times 10^{-6}} \right]} = \frac{150}{10s + \frac{1}{50 \times 10^{-6}}} = \frac{150}{10s + 2000}$$

$$\Rightarrow I(s) = \frac{15}{s + \frac{200 \times 10^3}{10}} = \frac{15}{s + 2000}$$

Inverse Laplace

$$\Rightarrow i(t) = 15 e^{-2000t} \text{ A}$$

5) The series RC circuit has a sinusoidal voltage source $v = 180 \sin(2000t + \phi) \text{ V}$ and an initial charge of 1.25 mC . Determine current if switch is closed at $t=0$ & $\phi = 90^\circ$.

6. An series RLC circuit, there is no initial charge in capacitor.
If switch is closed at $t=0$, calculate current.

Sol:- Time domain equation

$$50 = R i(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int i(t) dt$$

$$\Rightarrow 50 = 2i + 1 \frac{di}{dt} + \frac{1}{0.5} \int i dt$$

S-Domain

$$\Rightarrow \frac{50}{s} = 2I(s) + I(s) \cdot s + \frac{1}{0.5} \frac{I(s)}{s}$$

$$\Rightarrow I(s) \left[2 + s + \frac{1}{0.5s} \right] = \frac{50}{s}$$

$$\Rightarrow I(s) \left[\frac{2 \times 0.5s + 0.5s^2 + 1}{0.5s} \right] = \frac{50}{s}$$

$$\Rightarrow I(s) = \frac{50 \times 0.5}{0.5s^2 + 2 \times 0.5s + 1} = \frac{50}{s^2 + 2s + 2}$$
$$= \frac{50}{(s - (-1+j))(s - (-1-j))} = \frac{50}{(s+1-j)(s+1+j)}$$

$$\Rightarrow \frac{50}{(s+1-j)(s+1+j)} = \frac{A}{s+1-j} + \frac{B}{s+1+j}$$

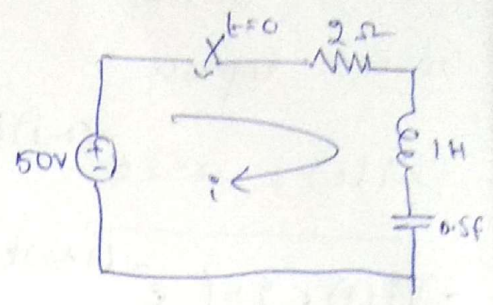
$$\Rightarrow 50 = A(s+1+j) + B(s+1-j)$$

$$\text{At } s = -1-j \Rightarrow 50 = B(-j-j+1-j) = B(-2j+1-j)$$

$$\Rightarrow B = \frac{50}{-2j} = \frac{-25}{j} = 25j$$

$$\text{At } s = -1+j \Rightarrow 50 = A(-1+j+1+j) = 2jA$$

$$\Rightarrow A = \frac{50}{2j} = -25j$$



$$\frac{-2 \pm \sqrt{4 - 4 \times 2}}{2}$$
$$= \frac{-2 \pm \sqrt{-4}}{2}$$
$$= \frac{-2 \pm j2}{2}$$
$$= -1 \pm j$$
$$\frac{1}{0.5} = \frac{10}{s}$$

$$\Rightarrow \hat{I}(s) = \frac{-25j}{s+1-j} + \frac{25j}{s+1+j}$$

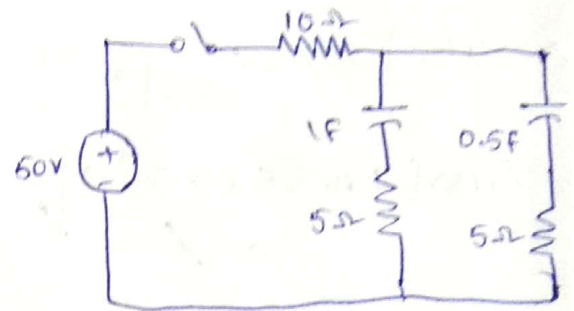
Inverse Laplace

$$\Rightarrow i(t) = -25j e^{-(1-j)t} + 25j e^{-(1+j)t}$$

$$\Rightarrow i(t) = 25 \left(e^{-(1+j)t} - e^{-(1-j)t} \right) A$$

7. In the network, the switch is closed at $t=0$. Find current?

$$\begin{aligned} \text{sol: } Z(s) &= 10 + \frac{(5 + \frac{1}{s})(5 + \frac{1}{0.5s})}{5 + \frac{1}{s} + 5 + \frac{1}{0.5s}} \\ &= 10 + \frac{25 + \frac{5}{0.8s} + \frac{5}{s} + \frac{1}{0.5s^2}}{10 + \frac{1}{s}(1 + \frac{1}{0.5})} \end{aligned}$$



$$\begin{aligned} &= 10 + \frac{25 + \frac{10}{s} + \frac{5}{s} + \frac{2}{s^2}}{10 + \frac{1}{s}(3)} = 10 + \frac{25 + \frac{15}{s} + \frac{2}{s^2}}{10 + \frac{3}{s}} \\ &= 10 + \frac{25s^2 + 15s + 2}{10s^2 + 3s} = \frac{100s^2 + 30s + 25s^2 + 15s + 2}{s(10s + 3)} \\ &= \frac{125s^2 + 45s + 2}{5(10s + 3)} \end{aligned}$$

$$\begin{aligned} \hat{I}(s) &= \frac{V(s)}{Z(s)} = \frac{50/s \cdot s(10s+3)}{125s^2 + 45s + 2} = \frac{500(s+0.3)}{125s^2 + 45s + 2} \\ &= \frac{4(s+0.3)}{125(s^2 + \frac{45s}{125} + \frac{2}{125})} = \frac{4(s+0.3)}{(s+0.308)(s+0.052)} \end{aligned}$$

on solving, we get

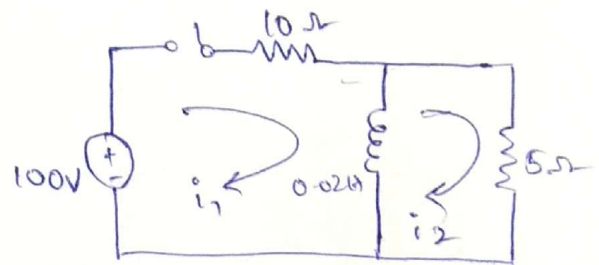
$$\underline{i}(s) = \frac{1/8}{s+0.308} + \frac{3/8}{s+0.052}$$

$$\Rightarrow i(t) = \frac{1}{8} e^{-0.308t} + \frac{3}{8} e^{-0.052t} \text{ A}$$

8. find the currents when switch is closed in both loops

KVL for loop ①

$$100 = 10i_1 + 0.02 \frac{di_1}{dt} - 0.02 \frac{di_2}{dt}$$



$$\Rightarrow \frac{100}{s} = 10\underline{i}_1(s) + 0.02s(\underline{i}_1(s) - \underline{i}_2(s))$$

$$\Rightarrow [10 + 0.02s]\underline{i}_1(s) - 0.02s\underline{i}_2(s) = \frac{100}{s} \rightarrow \textcircled{1}$$

KVL for loop ②

$$5\underline{i}_2(s) + 0.02s(\underline{i}_2(s) - \underline{i}_1(s)) = 0$$

$$\Rightarrow \underline{i}_2(s) [5 + 0.02s] - 0.02s\underline{i}_1(s) = 0$$

$$\Rightarrow \underline{i}_2(s) = \frac{0.02s\underline{i}_1(s)}{5 + 0.02s} \rightarrow \textcircled{2} = \underline{i}_1(s) \left[\frac{s}{s + 250} \right]$$

② in ①

$$\Rightarrow [10 + 0.02s]\underline{i}_1(s) - 0.02s \left[\frac{0.02s\underline{i}_1(s)}{5 + 0.02s} \right] = \frac{100}{s}$$

on solving

$$\underline{i}_1(s) = 6.67 \left[\frac{s + 250}{s(s + 166.7)} \right] = \frac{10}{s} - \frac{3.33}{s + 166.7}$$

$$\Rightarrow i_1 = 10 - 3.33 e^{-166.7t}$$

from ②

$$I_2(s) = 6.67 \int \frac{1}{s + 166.7} ds$$

$$\Rightarrow i_2 = 6.67 \cdot e^{-166.7t}$$