Code No: 132AB **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year II Semester Examinations, May/June - 2017
MATHEMATICS-II
(Common to EEE, ECE, CSE, EIE, IT)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find the Laplace transform of $f(t) = \begin{cases} K, & 0 < t < 2 \\ 0, & 2 < t < 4 \end{cases}$, $f(t+4) = f(t), \forall t > 0$. [2]

b) Find the Laplace transform of $f(t) = \frac{1-e^t}{t}$. [3]

c) Evaluate $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$. [2]

d) Evaluate $\int_0^\infty e^{-x^2} dx$. [3]

e) Evaluate $\int_0^{\frac{\pi}{2}} \sin^2\theta \cos^2\theta \, d\theta$ using beta and gamma functions. [2]

f) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. [3]

g) Find a vector normal to the surface $xyz^2 = 20$ at the point $(1, 1, 2)$. [2]

h) If $u\bar{F} = \nabla u$, where u, v are scalar fields and $\bar{F}$ is a vector field, show that $\bar{F} \cdot curl\,\bar{F} = 0$. [3]

i) State Green's theorem. [2]

j) Find the work done by a force $y\bar{i} + x\bar{j}$ which displays a particle from origin to a point $(\bar{i} + \bar{j})$ along the line $y = x$. [3]

PART-B

(50 Marks)

2.a) Express the function $f(t)$ in terms of unit step function, where $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$.
Hence find its Laplace transform.

b) Find the Laplace transform of $\int_0^\infty te^{-3t}\sin t \; dt$. [5+5]

OR

3.a) State the convolution theorem on Laplace transforms. Using it find the inverse Laplace transform of $\frac{1}{s(s^2+a^2)}$.

b) Solve $y'' + 2y' + 5y = e^{-t}\sin t$, $y(0) = 0$ and $y'(0) = 1$ using Laplace transforms. [5+5]

4.a) Evaluate $\int_0^1 x^{5/2}(1-x)^{3/2} dx$ using Beta, Gamma functions.

b) Evaluate $\int_0^1 \frac{dx}{(1-x^n)^{\frac{1}{n}}}$. [5+5]

OR

5.a) Show that $\int_0^{\infty} \frac{t^{m-1}}{(a+bt)^{m+n}} dt = \frac{\beta(m,n)}{a^n b^m}$, where m, n, a, b are positive integers.

b) Evaluate $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx$. [5+5]

6.a) Evaluate $\int_0^1 \int_0^x \frac{x^3 \, dx \, dy}{\sqrt{x^2+y^2}}$ by changing into polar coordinates.

b) By double integration, calculate the area bounded by the curve $a^2x^2 = y^3(2a-y)$. [5+5]

OR

7.a) Find the area enclosed in the first quadrant by the curve $\left(\frac{x}{a}\right)^{\alpha} + \left(\frac{y}{b}\right)^{\beta} = 1$, $\alpha > 0, \beta > 0$, using beta gamma functions.

b) Find the center of gravity of the area of the cardioid $r = a(1+\cos\theta)$. [5+5]

8.a) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$.

b) If $f = (x^2+y^2+z^2)^{-n}$, find $div\ grad\ f$ and determine n if $div\ grad\ f = 0$. [5+5]

OR

9.a) Show that the vector $\bar{F} = (x+3y)\bar{i} + (y-3z)\bar{j} + (x-2z)\bar{k}$ is solenoidal and also find $\bar{F} \cdot curl\ \bar{F}$.

b) In what direction from $(3,1,-2)$ is the directional derivative of $\phi = x^3y^2 + yz$ maximum? Find also the magnitude of this maximum. [5+5]

10. State Stokes theorem. Verify it for the vector field $\bar{F}(2x-y)\bar{i} - yz^2\bar{j} - y^2z\bar{k}$ over the upper half surface of the sphere $x^2+y^2+z^2 = 1$, bounded by its projection on the xy −plane. [10]

OR

11.a) Using Green's theorem, find the area of the region in the first quadrant bounded by the curves $y = x, y = \frac{1}{x}, \ y = \frac{x}{4}$.

b) Evaluate $\iiint_V div\ \bar{F}\ dV$, where $\bar{F} = y\bar{i} + x\bar{j} + z^2\bar{k}$ over the surface of the cylinder $x^2+y^2 = a^2, \ z = 0, \ z = h$. [5+5]

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