JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD B.Tech I Year II Semester Examinations, May - 2019 MATHEMATICS-II

(Common to EEE, ECE, CSE, EIE, IT, ETM)

Max. Marks: 75 Time: 3 hours Note: This question paper contains two parts A and B. Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions. (25 Marks) [2] [3] Define Unit step function and find its Laplace transform. b) Evaluate $\Gamma\left(-\frac{3}{2}\right)$. [2] d) Evaluate $\int x^5 (1-x)^6 dx$ Using triple integral, find the volume of a rectangular box whose length is 6 ft, [2] breadth is 5 ft and height is 4 ft. Evaluate $\iint_{\mathbb{R}^{n}} (x + y^{2}) dy dx$ [3] g) Define solenoidal vector.

h) Prove that \vec{r} is an irrotational where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ [2] State stokes theorem. Evaluate $\iiint div f dx dy dz$ where v is the volume of the sphere whose radius is 'a' units and j) $\bar{f} = x\bar{i} + y\bar{j} + z\bar{k} \ .$ PART-B (50 Marks) Find the Laplace transform of $(\sin t + \cos t)^2$ 2.a) Find the inverse Laplace transform of $\frac{1}{(s^2+1)(s+1)}$. [5+5] Solve $y'' + 2y' + 5y = e^{-t}$, y(0) = 1, y'(0) = 1 using Laplace transform. 4.a) Evaluate $\int_{0}^{\infty} e^{-x/3} x^3 dx$.

b) Evaluate $\int_0^1 \frac{x dx}{\sqrt{1-x^4}}$

- 5.a) Evaluate $\int_0^\infty e^{-x^3} x^7 dx$.
 - b) Evaluate $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}}.$

[5+5]

- 6.a) Evaluate $\int_{0}^{2} \int_{0}^{\sqrt{2x^{2}x^{2}}} (x^{2} + y^{2}) dxdy$ by changing to polar coordinates.
 - b) Evaluate $\iint_R y dx dy$ where R is the region bounded by the parabola $y^2 = 4x$ and [5+5]

OR

- 7.a) Evaluate $\iiint xy^2z dx dy dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.
 - b) Evaluate $\iint_{0}^{a} \int_{0}^{x} \int_{0}^{x+y+z} dx dy dz$.

[5+5]

- 8.a) Find the directional derivative to the surface $f(x,y,z) = xy^2z 4$, at the point (1, -1, 2) along i+j+k.
 - b) A butterfly is located at (2, -1, 3) and desires to fly towards fragrance surface $f(x,y,z) = x^2 + yz^2$. Along which direction should it fly to get fragrance at the earliest? [5+5]

OR

- 9.a) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $|\bar{r}|^2 = r$.
 - b) Prove that $\nabla \left(\frac{1}{r}\right) = -\frac{\overline{r}}{r^3}$ where $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ and $|\overline{r}|^2 = r$.

[5+5]

10. Verify Greens theorem for $\oint_C (y-\sin x)dx + \cos xdy$ where C is the triangle

enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $\pi y = 2x$.

[10]

OR

11. Verify stokes theorem for a vector field defined by $\overline{F} = -y^3 \overline{i} + x^3 \overline{j}$ in the region $x^2 + y^2 \le 1$, z = 0. [10]