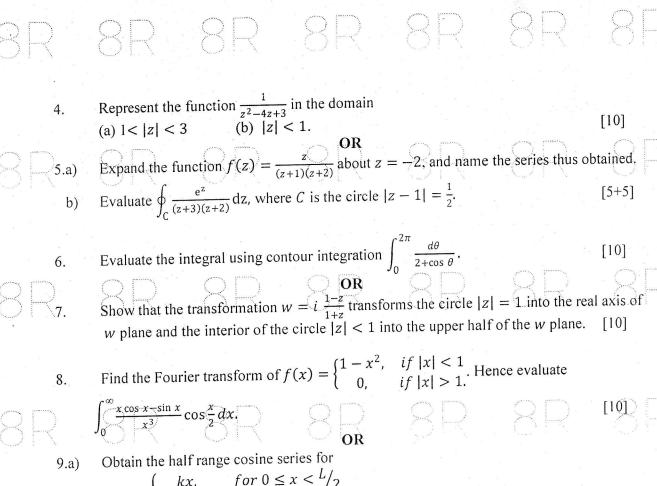
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R16 Code No: 133BD JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD B.Tech II Year I Semester Examinations, May/June - 2019 MATHEMATICS - IV (Common to CE, EEE, ME, ECE, CSE, EIE, IT MCT, ETM MMT, AE, MIE, PTM, CEE, MSNT) Max. Marks: 75 Time: 3 Hours Note: This question paper contains two parts A and B. Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions. PART-A (25 Marks) State the necessary and sufficient conditions for a function f(z) = u + iv to be analytic. 1.a) [3] Show that $f(z) = |z|^2$ is not analytic at any point. b) [2] State Cauchy's integral theorem. c) Find the poles and the residues at the poles of the function $f(z) = \frac{e^z}{\cos \pi z}$. [3] d) [2] Define bilinear transformation and cross ratio. e) Find the image of the circle |z| = 2, under the transformation w = z + 3 + 2i. [3] f) [2] State Fourier integral theorem. g) Expand $f(x) = \pi x - x^2$ in a half range sine series in $(0, \pi)$. [3] h) Classify the partial differential equation $u_{xx} + 6u_{xy} + 2u_{yy} + 2u_x - 2u_y + u = x^2y$. i) Write the three possible solutions of the heat equation. [3] **PART-B** (50 Marks) If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$ Let $f(z) = u(r, \theta) + iv(r, \theta)$ be an analytic function. If $u = -r^3 \sin 3\theta$, then construct b) the corresponding analytic function f(z) in terms of z. Show that the function f(z) defined by 3.a) $\frac{x^2y^3(x+iy)}{x^6+y^{10}}$ for $z \neq 0$, is not analytic at the origin, even though it satisfies the f(0) = 0Cauchy-Riemann equations at the origin. Determine the analytic function whose real part is $\log \sqrt{x^2 + y^2}$. [5+5]



- Obtain the half range cosine series for $f(x) = \begin{cases} kx, & \text{for } 0 \le x < \frac{L}{2} \\ k(L-x), & \text{for } \frac{L}{2} \le x \le L \end{cases}$ b) Find the Fourier sine transform of $f(x) = e^{-|x|}$. Hence show that $\int_0^\infty \frac{x \sin mx}{x^2 + 1} dx = \frac{\pi}{2} e^{-m}.$
- 10. A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{L}$ from which it is released at time t = 0. Find the displacement of any point at a distance x from one end at time t. [10]
- Write down the one dimensional heat equation. Find the temperature u(x,t) in a slab whose ends x = 0 and x = L are kept at zero temperature and whose initial temperature f(x) is given by $f(x) = \begin{cases} k, & \text{when } 0 < x < \frac{1}{2}L \\ 0, & \text{when } \frac{1}{2}L < x < L \end{cases}$ [10]

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