Code No.: MA201BS | R20 | H.T.No. | | | 8 | R | | | | | |

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–II–Semester End Examinations (Supply) - January- 2022
DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS
(Common to CSC, CSD, CSE, CSM, ECE, IT, MECH)

[Time: 3 Hours] **[Max. Marks: 70]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 20 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A (20 Marks)

1. a) The number N of bacteria in a culture grew at a rate proportional to N .The value of N was initially 100 and increased to 332 in one hour. What was the value of N after $1\frac{1}{2}\,hrs$ [2M]

b) Write the statement of newton's law of cooling. [2M]

c) Solve $(D^2 - 3D + 4)\,y = 0$ [2M]

d) Find the Particular Integral of $(D^2+5D+6)y=e^x$ [2M]

e) Evaluate $\int_0^5\int_0^{x^2} x\left(x^2+y^2\right)dx\,dy$ [2M]

f) Evaluate $\int_0^1\int_0^{\pi/2} r\sin\theta\, d\theta\, dr$ [2M]

g) Find a unit normal vector to the given surface $x^2y+2xz = 4$ at the point (2,-2,3). [2M]

h) If $\bar{f} = (x+3y)\bar{i} + (y-2z)\bar{j} + (x+pz)\bar{k}$ is solenoidal, find p. [2M]

i) If $\bar{F} = (x^2-27)\bar{i} - 6yz\,\bar{j} + 8xz^2\,\bar{k}$, evaluate $\int \bar{F}\cdot d\bar{r}$ from the point (0,0,0) to the point (1,1,1) along the Straight line from (0,0,0) to (1,0,0), (1,0,0) to (1,1,0) and (1,1,0) to (1,1,1). [2M]

j) If $\bar{F} = axi + byj + czk$ where a, b, c are constants then evaluate $\iint \bar{F}\cdot\bar{n}\,dS$ where S is the surface of the unit sphere is $x^2+y^2+z^2=1$ [2M]

PART-B (50 Marks)

2. a) Solve $y = 2px + y^2p^3$ [5M]

b) Solve $(1+xy)\,y\,dx + (1-xy)\,x\,dy = 0$ [5M]

OR

3. a) Solve $x\frac{dy}{dx} + y = \log x$ [5M]

b) A body kept in air with temperature 25^0C cools from 140^0C to 80^0C in 20 min's. Find when the body cools down to 35^0C. [5M]

4. a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + y = \operatorname{cosec}x$ [7M]

b) Find the PI of $(D^2+9)y = \cos 3x$ [3M]

OR

5. Solve $(D^2 - 4D + 4)y = x^2 \sin x + e^{2x} + \cos x$ [10M]

6. Change the order of Integration and evaluate $\int_{x=0}^{4a}\int_{y=x^2/4a}^{2\sqrt{ax}} dy\,dx$ [10M]

OR

7. Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ [10M]

8. a) Find the directional derivative of $\phi(x,y,z) = x^2yz + 4xz^2$ at the point(1,-2,1) in the direction of the normal to the surface $f(x,y,z) = x\log z - y^2$ at (-1,2,1) [5M]

b) Find curl $\bar{f}$ where $\bar{f} = \text{grad}(x^3+y^3+z^3-3xyz)$ [5M]

OR

9. a) Prove that $\text{div.}(\text{grad } r^m) = m(m+1)r^{m-2}$ [7M]

b) Find curl $\bar{f}$ where $\bar{f} = \text{grad}(x^3+y^3+z^3-3xyz)$ [3M]

10. Verify Gauss Divergence theorem for $\bar{F} = (x^3 - yz)\bar{i} - 2x^2y\bar{j} + z\bar{k}$ taken over the surface of the cube bounded by the planes x = y = z = a and coordinate planes. [10M]

OR

11. Verify Green's theorem for $\int_c [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where c is the region bounded by x=0, y=0 and x+y=1 [10M]

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