Code No.: MA303BS

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CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
II–B.TECH–I–Semester End Examinations (Regular) - January- 2022
PROBABILITY AND STATISTICS & COMPLEX VARIABLES
(MECH)

[Time: 3 Hours]   [Max. Marks: 70]

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 20 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A   (20 Marks)

1. a) If A and B are events with $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, find $P\left(A^C \cap B^C\right)$.   [2M]

   b) A random variable X has the probability function.   [2M]

x	-2	-1	0	1
P(x)	0.4	K	0.2	0.3

   Find K and the mean value of X

   c) Write the four properties of the Normal distribution.   [2M]

   d) If X is a Poisson variate such that $P(X = 2) = 9P(X = 4) + 90P(X = 6)$ then find mean.   [2M]

   e) Define Null Hypothesis and Critical region.   [2M]

   f) A sample of 400 items is taken from a population whose standard deviation is 10. The mean of the sample is 40 and a sample is drawn from a population with mean 30. Calculate 95% confidence interval for the population.   [2M]

   g) Let $a, b,$ and c be real constants. Find a relationship among them for the function $u(x, y) = ax^2 + bxy + cy^2$ to be harmonic.   [2M]

   h) Find the real and imaginary parts of $e^{\left(z^2\right)}$.   [2M]

   i) Evaluate $\int_C \dfrac{dz}{z^2 e^z}$ where C is $|z| = 1$.   [2M]

   j) Find the invariant points of the transformation $w = \dfrac{6z - 9}{z}$.   [2M]

PART-B   (50 Marks)

2. a) A random variable has following function   [5M]

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	K²	2k²	7k² +k

   Find i) k   ii) P(x<6)   iii) p(0<x<5)   iv) P(0≤x≤4)   v) mean.

   b) Companies B_1, B_2 and B_3 produce 30%, 45% and 25% of the cars respectively. It is known that 2%, 3% and 2% of these cars produced from B_1, B_2 and B_3 are defective. If a car purchased is found to be defective, What is the probability that a car purchased is defective?   [5M]

OR

3. a) The probability that a man hit a target is 1/3. If he fires 6 times find the probability that he fires (i)At most 5 times (ii) exactly two times (iii) at least two times.   [5M]

b) Find the mean and standard deviation of a continuous random variable X, whose probability density [5M] function is given by $f(x) = \begin{cases} e^{-x}, & 0 < x < \infty \\ 0, & else\ where \end{cases}$

4. Fit a Poisson distribution to the following data [10M]

x	0	1	2	3	4	5	6	7
f	305	365	210	80	28	9	2	1

OR

5. a) If the masses of 300 students are normally distributed with mean 68 kgs and standard deviation 3 kgs, how [5M] many students have masses (i) greater than 72kg (ii) less than or equal to 64kg.

b) 20% of items produced from a factory are defective. Find the probability that in a sample of 5 chosen at [5M] random (i) none is defective (ii) One is defective (iii) $P(1 < X < 4)$.

6. Two random samples from two normal populations are given below. Do the estimates of Population variances [10M] differ significantly?

Sample-1	16	26	27	23	24	22
Sample-2	33	42	35	32	28	31

OR

7. In random samples of 600 and 1000 men from two cities 400 and 600 men are found to be literate. Do the data [10M] indicate at 5% level of significance that the populations are significantly different in the Percentage literacy?

8. If $f(z) = u + iv$ is an analytic function and $u - v = \dfrac{\sin x + \cos x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$, find $f(z)$ subject to the [10M]

condition $f\left(\dfrac{\pi}{2}\right) = 0$.

OR

9. a) Prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)|f(z)|^p = p^2|f(z)|^{p-2}|f'(z)|^2$ where $W = f(z)$ is analytic function. [5M]

b) Determine the imaginary part of an analytics function f(z) whose real part of an analytic function is [5M]
$e^x(x\cos y - y\sin y)$

10. a) Using Cauchy's integral formula evaluate $\int_C \dfrac{z^4}{(z+1)(z-i)^2}dz$ where C is ellipse $9x^2 + 4y^2 = 36$. [5M]

b) State Taylor's theorem. Expand $f(z) = \dfrac{1}{z^2 - z - 6}$ about $z = 1$. [5M]

OR

11. a) Find the poles and residues of $\dfrac{3z+1}{(z+1)(2z-1)}$. [5M]

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ to the points $w = i, 0, -i$ [5M] respectively.

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