Code No.: MA101BS

R20	H.T.No.		8	R				

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–I–Semester End Examinations (Supply) - March- 2023
LINEAR ALGEBRA AND CALCULUS
(Common for all)

[Time: 3 Hours]   [Max. Marks: 70]

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 20 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question
carries 10 marks and may have a, b, c as sub questions.

PART-A   (20 Marks)

1. a) Define Rank of a Matrix?   [2M]

   b) Find the rank of the Matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ [2M]

   c) Explain Diagonalization of a matrix?   [2M]
   d) Write any two Eigen Value Properties?   [2M]
   e) Explain Comparison test?   [2M]
   f) State Raabe's test?   [2M]
   g) State Rolle's theorem?   [2M]
   h) Explain improper integrals.   [2M]
   i) Obtain the total derivative of $z = \tan^{-1}\left(\frac{x}{y}\right)$ [2M]

   j) Explain Maxima and Minima functions of two variables.   [2M]

PART-B   (50 Marks)

2. For what values of $'k'$ the equations   [10M]
   $x + y + z = 1$, $x + 4y + 10z = k^2$, $x + 2y + 4z = k$ have a solution and solve
   them in each case.

OR

3. P.T the following equations are Consistent and solve them   [10M]
   $3x + 3y + 2z = 1, x + 2y = 4, 10y + 3z = -2, 2x - 3y - z = 5$

4. Verify Cayley Hamilton theorem for matrix   [10M]
   $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix}$ and hence find A^{-1}.

OR

5. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ into canonical form   [10M]
   using orthogonal transformation and find the nature of quadratic form.

6. Test whether the following series is Absolutly convergent / Conditionally convergent   [10M]

$$\frac{1}{5\sqrt{2}} - \frac{1}{5\sqrt{3}} + \frac{1}{5\sqrt{4}} - \cdots + (-1^n)\frac{1}{5\sqrt{n}} + \ldots\ldots$$

OR

[10M]

7. Test for convergence of

   i. $\sum \frac{1}{(\log n)^n}$

   ii. $\sum (\frac{n+2}{n+3})^n x^n$

8. State Lagrange's mean value theorem and Show that for 0<a<b<1,   [10M]

$$\frac{1}{1+a^2} > \frac{\tan^{-1} b - \tan^{-1} a}{b-a} > \frac{1}{1+b^2}$$

OR

[10M]

9. Find the relation between Beta and Gamma functions.

10. If $u = log(x^3 + y^3 + z^3 - 3xyz)$. Show that $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 u = \frac{-9}{(x+y+z)^2}$.   [10M]

OR

11. If $x = r\cos\theta, y = r\sin\theta$. Verify that $\frac{\partial(x,y)}{\partial(r,\theta)} \cdot \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.   [10M]

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