Code No.: MA101BS | R20 | H.T.No. | | | 8 | R | | | | | |

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–I–Semester End Examinations (Supply) - September- 2023
LINEAR ALGEBRA AND CALCULUS
(Common for all)

[Time: 3 Hours] **[Max. Marks: 70]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 20 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A **(20 Marks)**

1. a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ [2M]

b) Define Orthogonal Matrix. [2M]

c) Determine the sum and product of the Eigen values $\begin{bmatrix} 2 & 1 & -1 \\ 3 & 4 & 2 \\ 1 & 0 & 2 \end{bmatrix}$. [2M]

d) Define Nature of a Matrix. [2M]

e) Define Absolute convergence & Conditional convergence. [2M]

f) Test for convergence of $\sum \frac{1}{(\log n)^n}$ [2M]

g) State Cauchy's mean value theorem. [2M]

h) Define Beta function. [2M]

i) State Euler's theorem. [2M]

j) Define Functional Dependence. [2M]

PART-B **(50 Marks)**

2. Discuss for what values of λ, μ the simultaneous equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have (i). no solution (ii). a unique solution (iii). an infinite number of solutions. [10M]

OR

3. Solve the system of equations $x+y+z=6, x-y+2z=5, 3x+y+z=8$ [10M]

4. Verify Cayley Hamilton theorem for matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and hence find A^{-1}. [10M]

OR

5. Find the Eigen values and Eigen Vectors of the Matrix $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$ [10M]

6. Test for convergence of the series $\sum \frac{4.7 \ldots\ldots (3n+1)}{1.2.3 \ldots\ldots n} x^n$ [10M]

OR

7. Explain Conditionally convergent and Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$ is conditionally convergent. [10M]

8. Show that for 0<a<b<1, [10M]

$$\frac{1}{1+a^2} > \frac{\tan^{-1}b-\tan^{-1}a}{b-a} > \frac{1}{1+b^2}$$

OR

9. State Lagrange's mean value theorem and Find c of the Lagrange's theorem of f(x)=(x-1)(x-2)(x-3) on [0,4]. [10M]

10. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$. Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. [10M]

OR

11. Find the minimum value of $x^2 + y^2 + z^2$ given that $xyz = a^3$. [10M]

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