Code No.: R22MA201BS | R22 | H.T.No. | | | 8 | R | | | | | |

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–II–Semester End Examinations (Regular) - September- 2023
VECTOR CALCULUS AND TRANSFORMS
(Common for all)

[Time: 3 Hours] **[Max. Marks: 60]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 10 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A **(10 Marks)**

1.a) Find $L[te^{2t}]$. [1M]
b) State First Shifting theorem. [1M]
c) Find $L^{-1}[\frac{1}{(S-4)} + \frac{5}{(S-2)^2}]$. [1M]
d) Write the formula for Laplace transform of integrals. [1M]
e) Evaluate $\int_0^2 \int_0^x y\, dxdy$. [1M]
f) Find $\beta(7,9)$. Using β-γ relation. [1M]
g) If $f=(x+3y)i+(y-2z)j+(x+pz)k$ is solenoidal, find p. [1M]
h) If $f=xy^2i+2x^2yzj-3yz^2k$ find curl f at (1,-1,1). [1M]
i) State Green's Theorem. [1M]
j) Find the work done by the force $F=3x^2i+(2xz-y)j+zk$ along the straight line joining the points (0,0,0) to (2,1,3). [1M]

PART-B **(50 Marks)**

2. Find the Laplace transform of $\{\frac{sint}{t}\}$, $\{\frac{e^{-at}-e^{-bt}}{t}\}$, {tsin2t}. [10M]

OR

3. Find the Laplace Transform of the square wave function of period 2a defined as f(t)= k when 0<t<a and f(t) =-k when a<t<2a. [10M]

4. Find the Inverse Laplace transform of $log\ \frac{(s+3)}{(s+4)}$. [10M]

OR

5. Using Laplace transform, solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}sint$, given that y(0)=0, $y^1(0)=1$. [10M]

6.a) Evaluate $\int_0^\infty x^2 e^{-x^2} dx$ using beta-gamma function. [5M+5M]
b) Show that $\gamma(n) = \int_0^1 (log\frac{1}{x})^{n-1}, n > 0$.

OR

7. Evaluate $\int\int (x^2 + y^2) dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [10M]

8.a) Find the directional derivative of $f(x,y,z)=zx^2-xyz$ at the point (1,3,1) in the direction of the vector 3i-2j+k. [5M+5M]
b) Find $div\bar{f}$ where $\bar{f}$ = grad$(x^3+y^3+z^3-3xyz)$.

OR

9. Find whether the function $\bar{F}=(x^2- yz)i+(y^2-zx)j+(z^2-xy)k$ is irrotational, hence find scalar potential function corresponding to it. [10M]

10.a) If $\bar{F}= 3xyi-y^2j$ Evaluate $\int_c \bar{F}.d\bar{r}$ where C is the curve $y=2x^2$ in the xy plane from (0,0) to (1,2). [5M+5M]

b) If $\bar{F}=4xyi-y^2j+yzk$ Evaluate $\iint \bar{F}.n\,ds$ where S is the surface of the cube bounded by x=0,X=1,y=0,y=1,z=0,z=1.

OR

11. Verify Green's theorem for $\iint_c[(3x^2-8y^2)dx+(4y-6xy)dy]$ where c is the region bounded by $y = x^2$ and $y =\sqrt{x}$. [10M]

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