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Code No.: R22MA201BS | R22 | H.T.No. | | | 8 | R | | | | | | |

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–II–Semester End Examinations (Supply) - February - 2024
VECTOR CALCULUS AND TRANSFORMS
(Common for all)

[Time: 3 Hours] **[Max. Marks: 60]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 10 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A **(10 Marks)**

1. a) Define Laplace Transformation. [1M]
   b) Define Laplace Transformation of Unit Step function. [1M]
   c) Find $L^{-1}\{\frac{1}{s^{n+1}}\}$ [1M]
   d) Define Convolution. [1M]
   e) Evaluate $\beta(2,2)$. [1M]
   f) Define the Gamma function and find the value $\Gamma(2)$. [1M]
   g) Define the Divergence of a vector. [1M]
   h) Define the Irrotational vector. [1M]
   i) State physical Interpretation of Surface integral. [1M]
   j) State Green's Theorem. [1M]

PART-B **(50 Marks)**

2. a) Evaluate L{tsin3tcos2t} [5M]
   b) Evaluate $L\{e^{-4t}\int_0^t \frac{\sin 3t}{t}\,dt\}$ [5M]

OR

3. Find L{f(t)} where f(t) is given by f(t)=t; 0<t<b, and f(t)=2b-t; b<t<2b, 2b being the period of f(t). [10M]

4. a) Find the inverse Laplace transform of $\frac{4}{(s+1)(s+2)}$ [5M]
   b) solve the integral equation $y(t) = 1 + \int_0^t y(u)\sin(t-u)\,du$, Using Laplace Transform [5M]

OR

5. Solve the differential equation using Laplace transform [10M]

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = e^{-t}, x(0) = 0, x'(0) = 1.$$

6. a) Show that $\beta\left(m,\frac{1}{2}\right) = 2^{2m-1}\,\beta(m,m)$. [5M]
   b) Evaluate $\int_0^2 (8-x^3)^{\frac{1}{3}}dx$ by using β –Γ function. [5M]

OR

7. a) Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)dxdy$. [5M]
   b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy$. By changing to polar coordinates. [5M]

8. a) Find the directional derivate of $f = xyz$ along the direction of the normal to the surface $x^2z + y^2x + yz^2$ at the point (1,1,1). [5M]

b) Show that $\frac{\bar{r}}{r^3}$ is Solenoidal. [5M]

OR

9. If $\bar{A}$ is a constant vector and $\bar{r} = xi + yj + zk$. Prove that $\nabla \times \left(\frac{\bar{A} \times \bar{r}}{r^n}\right) = \frac{(2-n)\bar{A}}{r^n} + \frac{n(\bar{r}.\bar{A})}{r^{n+2}}$ [10M]

10. Find the work done by F=(2x-y-z)i+(x+y-z)j+(3x-2y-5z)k along a curve 'C' in the XY-Plane given by $x^2+y^2=9$,Z=0. [10M]

OR

11. Verify Green Theorem in the plane for $\oint(x^2 - xy^3)dx + (y^2 - 2xy)dy$, Where C is Square with vertices (0,0), (2,0), (2,2) &(0,2). [10M]

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