KEY PAPER.

PART – A

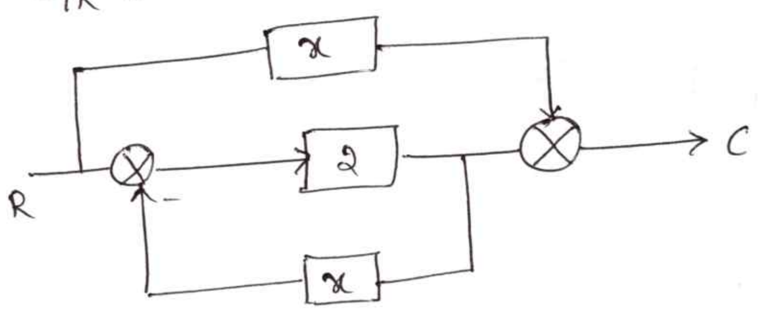
1) a)

$$C/R = \frac{s+9}{(s+5)(s+20)(s+1)}$$

poles → $-5, -20, -1$

zeros → -9

b) $C/R = 10$



$$\frac{C}{R} = \frac{2}{1+2x} + x$$

$$10 = \frac{2 + x + 2x^2}{1+2x}$$

$$10 + 20x = 2x^2 + x + 2$$

$$2x^2 - 19x - 8 = 0$$

$\cancel{x^2 - 8}$ $\quad x = 0.45,\ 9.05.$

c) Rise time (t_r):-

It is the time taken for response to raise from 0 to -100%. for the very first time.

$$t_r = \frac{\pi - \theta}{\omega_d}$$

d) i) All the coefficients of its characteristic polynomial are to be positive

ii) Any coefficients should not miss.

e)

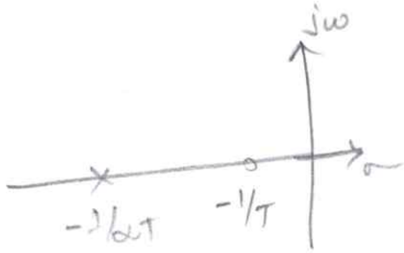
Time Response	Frequency Response
Based on Time domain behavior	Based on sinusoidal steady state.

f) phase Margin

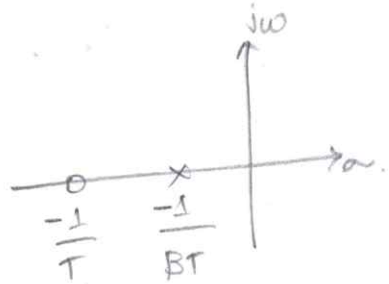
The Phase Margin γ, is defined as the additional phase lag to be added at the gain cross over frequency in order to bring the system to the verge of instability.

$$\gamma = 180^\circ + \phi_{gc}.$$

g) s-plane representation.



Lead Compensator



Lag Compensator

h) P- Controller

→ Improve Speed

→ Increase overshoot

PI Controller

→ Reduce Bandwidth

→ Reduce stability Margin

i) A set of variables that completely describe the system at any time.

j) If the determinant of Q_c is non zero system is Controllable

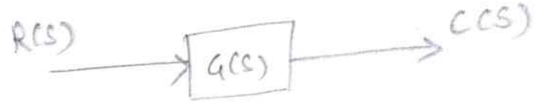
$$Q_c = [B \;\; AB \;\; A^2B \; .. \; A^{n-1}B]$$

PART-B.

2) a) Control System :-

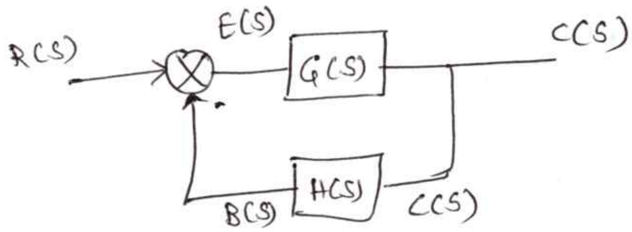
In a system when the output quantity is controlled by varying the input quantity, the system is called control system.

open loop Control system



$$TF = G(S)$$

closed Loop Control system



$$E(S) = R(S) - B(S)$$

$$B(S) = C(S)\,H(S)$$

$$C(S) = E(S)\,G(S)$$

$$= [R(S) - C(S)\,H(S)]\,G(S)$$

$$C(S) = R(S)\,G(S) - C(S)\,H(S)\,G(S)$$

$$C(S) + C(S)\,H(S)\,G(S) = R(S)\,G(S)$$

$$C(S)\,[1 + G(S)\,H(S)] = R(S)\,G(S)$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{[1 + G(S)\,H(S)]}$$

b) characteristics of feedback.

i) Feedback Control System

→ Positive feedback

→ Negative feedback

ii) Effects of feedback

Reduction of parameters Variations by using feedback

$$\text{sensitivity} = \frac{\text{\% change in } A}{\text{\% change in } K}$$

$$S_K^A = \frac{\partial A}{\partial K} \times \frac{K}{A}$$

a) forward path

OLTF $S_G^T = 1$

CLTF $S_G^T = \frac{1}{1+G(S)H(S)}$

b) feedback Path

CLTF $S_H^T = \frac{-G(S)H(S)}{1+G(S)H(S)}$

(iii) Effect of feedback on time Constant of a Control system

(iv) Effect of feed back on Gain

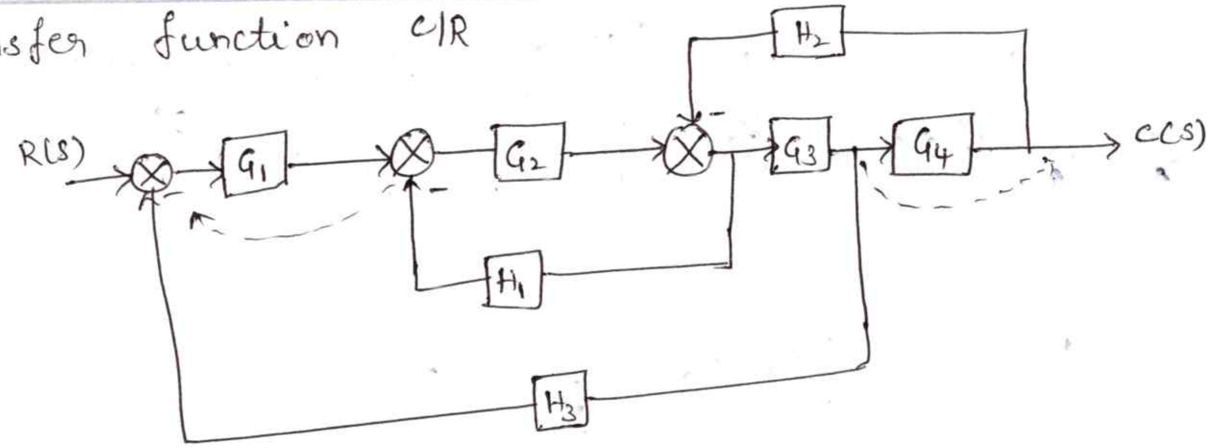
Due to feedback Gain reduced.

v) Effect of feedback on stability

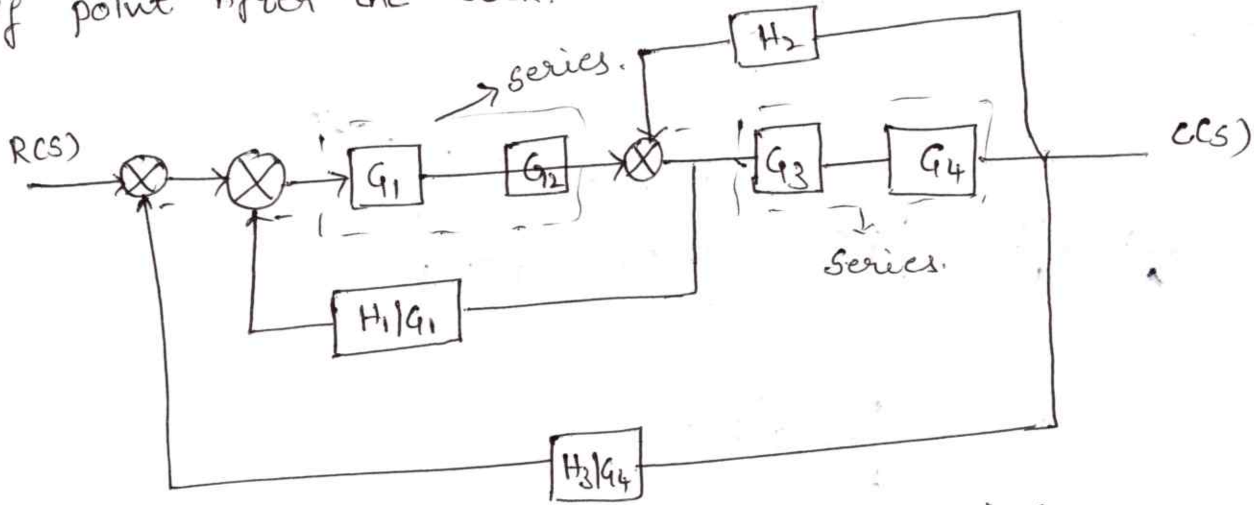
$$c(t) = K \cdot e^{-(U+K)t}$$

AS K↑, System dynamics ↑ and transient response decays.

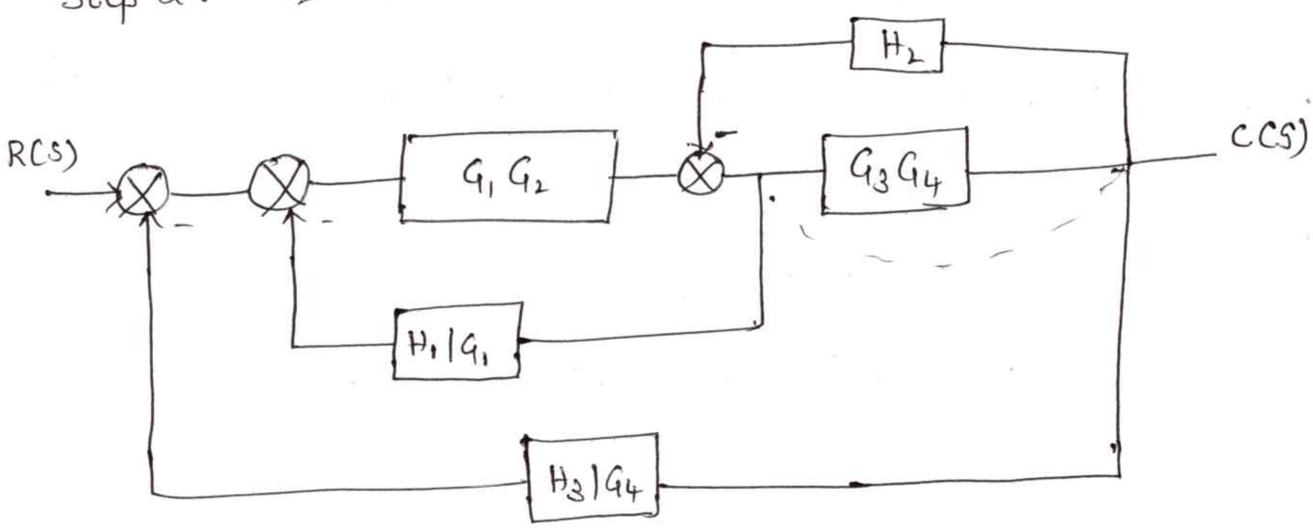
3) Transfer function C/R



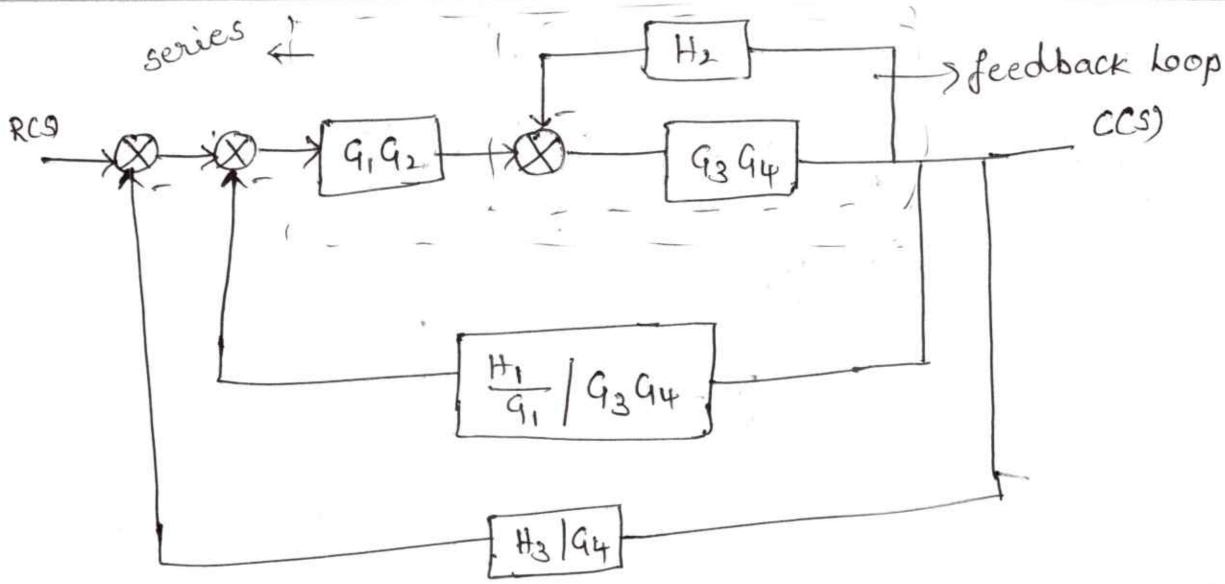
step 1 :- Moving summing point before block & moving take-off point After the block.



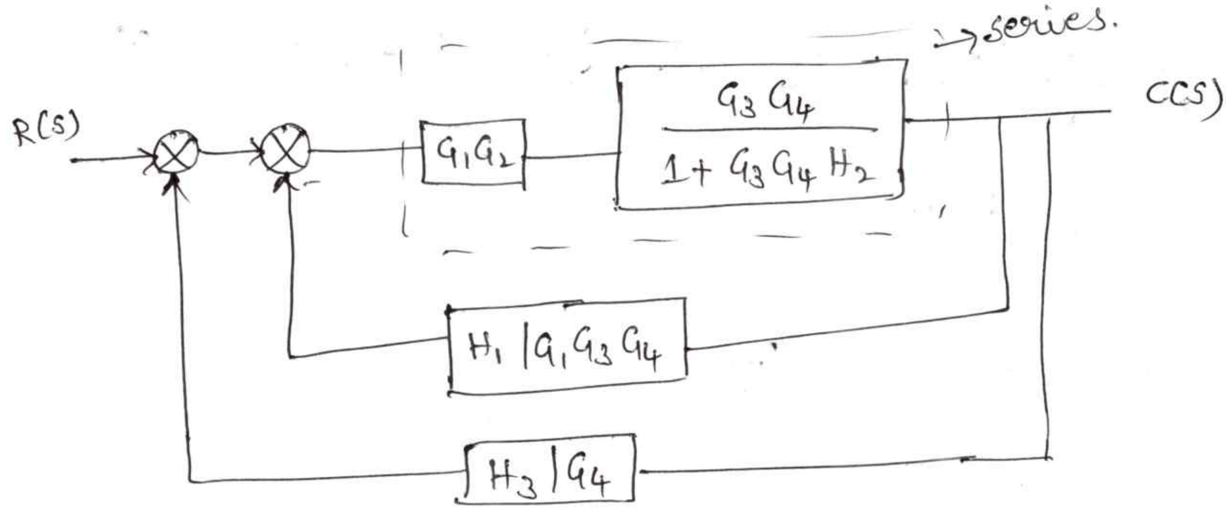
step 2 :- G_1 & G_2 in series & G_3 & G_4 in series.



step 3 :- Moving take off After the block



step 4 :- Remove feedback Loop & series Combination.



step 5 :- Combine series blocks and remove the feedback Loops.

$$TF = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 G_3 H_3 + G_2 H_1 + G_3 G_4 H_3.}$$

4) a) RH criteria

$$4s^4 + 8s^3 + 2s^2 + 10s + 3 = 0$$

s^4	4	2	3
s^3	8	10	0
s^2	-3	3	0
s^1	+18	0	0
s^0	3		

comment:-

→ 2 sign changes in first column.

→ 2 Roots lies on RHS and 2 Roots lies on LHS of s-plane.

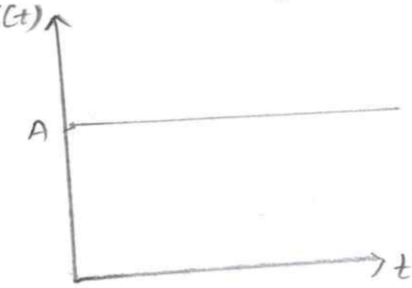
→ system is unstable.

b) step signal:-

The step signal is a signal whose value changes from zero to A at $t=0$ and remains at A for $t>0$.

The mathematical representation of the step signal is

$$r(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

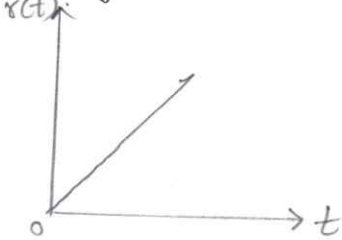


Ramp signal :-

The ramp signal is a signal whose value increases linearly with time from an initial value of zero at $t=0$.

Mathematical representation

$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$$



5) open loop transfer function

$$G(s) = \frac{25}{s(s+8)}$$

The closed loop TF

$$G(s) = \frac{25}{s^2+8s+25}$$

compare with characteristic Eq^n

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = 25$$

$$\boxed{\omega_n = 5}$$

$$\cancel{2}\xi\omega_n = \cancel{8}\,4$$

$$\xi = \frac{4}{5} = 0.8$$

i) Peak overshoot (Mp):-

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}} \times 100$$

$$= e^{-0.8\times3.14/\sqrt{1-(0.8)^2}} \times 100$$

$$M_p = 65\,\%.$$

ii) Peak time (tp):-

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{3.14}{5\sqrt{1-(0.5)^2}}$$

$$t_p = 1.04 \text{ sec}$$

6) open Loop transfer function

$$G(s) = \frac{1}{s(1+0.1s)(1+0.01s)}$$

sol:-

substitute $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega(1+0.1j\omega)(1+0.01j\omega)}$$

Corner frequencies

$$\omega_{c_1} = \frac{1}{0.1} = 10 \text{ rad/sec}$$

$$\omega_{c_2} = \frac{1}{0.01} = 100 \text{ rad/sec.}$$

Term	corner freq rad/sec	slope db/dec	change in slope db/dec
$\frac{1}{j\omega}$	-	-20	-
$\frac{1}{1+0.1j\omega}$	$\omega_{c_1} = 10$	-20	$-20-20=-40$
$\frac{1}{1+0.01j\omega}$	$\omega_{c_2} = 100$	-20	$-40-20=-60$

choose ω_l & ω_h

$\omega_l = 1$ rad/sec $\quad \omega_h = 500$ rad/sec

At $\omega = \omega_l$

$$A = 20\log\left|\frac{1}{j\omega}\right| = 20\log\left|\frac{1}{1}\right| = 0$$

At $\omega = \omega_{c_1}$

$$A = 20\log\left|\frac{1}{j\omega}\right| = 20\log\left|\frac{1}{10}\right| = -20db$$

At $\omega = \omega_{c_2}$

$$A = \left[\text{slope from } \omega_{c_1} \text{ to } \omega_{c_2} \times \log\frac{\omega_{c_2}}{\omega_{c_1}} \times \log\frac{\omega_{c_2}}{\omega_{c_1}}\right] + A\Big|_{\omega=\omega_{c_1}}$$

$= -40 \log \frac{\cancel{100}^{10}}{\cancel{10}} - 20$

$= -40 - 20 = -60$ db

At $\omega = \omega_h$

$A = \left[\text{slope from } \omega_q \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_q}\right] + A|_{\omega = \omega_q}$

$= -60 \log \frac{500}{100} + [-60]$

$= -41.93 - 60 = -101.93$ db.

phase Angle

$\angle G(j\omega) = -90 - \tan^{-1}(0.1\omega) - \tan^{-1}(0.01\omega)$.

ω rad/sec	$\tan^{-1}(0.1\omega)$	$\tan^{-1}(0.01\omega)$	$\phi = \angle G(j\omega)$
1	5.7	0.6	-96.3°
10	45	5.7	-140.7°
100	84.3	45°	-219.3°
500	88.9	78.7	-257.8°

i) Gain Margin $= \frac{1}{|G(j\omega)|}\Big|_{\omega = \omega_{pc}}$

At -180°

GM = 4dB.

ii) Phase Margin $= 180^\circ + \phi_{gc}$

$\phi_{gc} = -90^\circ$

PM = 90°

7) Nyquist plot

$$G(s)\,H(s) = \frac{K(s-4)}{(s+1)^2}$$

$$= \frac{-4K\,(1-0.25s)}{(1+s)\,(1+s)}$$

substitute $s = j\omega$

$$G(j\omega)\,H(j\omega) = \frac{-4K\,(1-0.25j\omega)}{(1+j\omega)\,(1+j\omega)}$$

$$G(j\omega)\,H(j\omega) = \frac{-4K}{1+2j\omega-\omega^2} + \frac{4K\times0.25j\omega}{1+2j\omega-\omega^2}$$

For ω_{pc}, Imaginary part $=0$

$$\left.\frac{4K * 0.25j\omega}{1+2j\omega-\omega^2}\right|_{\omega_{pc}} = 0$$

$$\omega_{pc} = 0$$

Substitute ω_{pc} in Real part

$$G(j\omega)\,H(j\omega) = \frac{-4K}{1+0} \Rightarrow -4K$$

Magnitude

$$|G(j\omega)\,H(j\omega)| = \frac{\sqrt{1+(0.25\omega)^2}}{\sqrt{1+\omega^2}\,\sqrt{1+\omega^2}}$$

phase Angle

$$\angle G(j\omega)\,H(j\omega) = -\tan^{-1}(0.25\omega) - 2\tan^{-1}\omega$$

$\omega \to 0$, Mag $= 1$

phase $= -90 - 180 = -270$

$\omega \to \infty$, Mag $= 1$

phase $= 0$

Mapping C_2 :-

$$G(S)H(S) = \frac{-4K(1-0.25j\omega)}{(1+j\omega)(1+j\omega)}$$

$$1+ST = ST$$

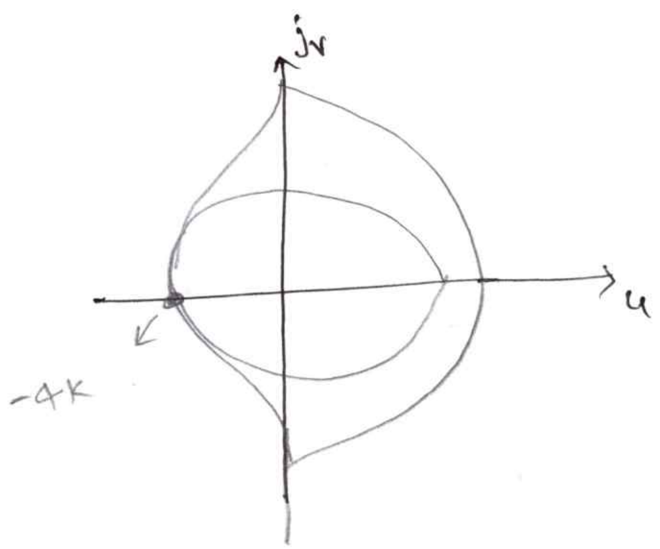
$$S = \underset{R\to\infty}{Lt}\ Re^{j\theta} \qquad \theta = \frac{\pi}{2} \text{ to } -\frac{\pi}{2}$$

Mapping C_4

$$G(S)H(S) = \frac{-4K(1-0.25j\omega)}{(1+j\omega)(1+j\omega)}$$

$$1+ST = 1$$

$$S = \underset{R\to 0}{Lt}\ Re^{j\theta} \qquad \theta = -\frac{\pi}{2} \text{ to } \frac{\pi}{2}$$

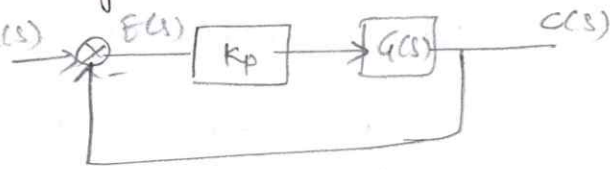


$$-4K = -1$$

$$K = 1/4 = 0.25$$

8) i) Proportional Controller:-

In this Controller activate signal is proportional to Error signal.



$$e_a(t) \propto e(t)$$

$$e_a(t) = K_p\, e(t)$$

Fig:- P - Controller

Loop gain increases, sensitivity decreases, stability increase, steady state errors are not effected is disadvantage.

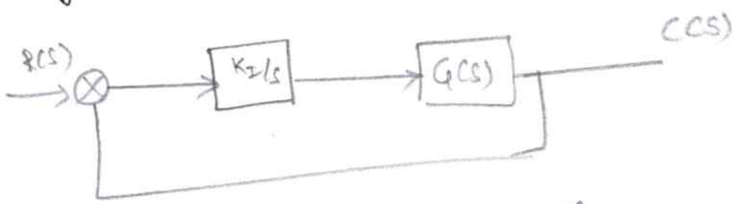
(ii) Integral Controller:-

In this ~~error~~ signal directly proportion to the Integral of error signal

$$e_a(t) \propto \int e(t)\, dt$$

$$e_a(t) = K_I \int e(t)\, dt.$$

Apply L.T



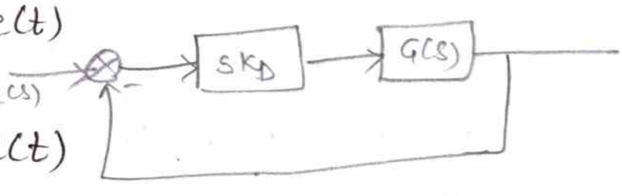
$$\frac{E_a(S)}{E(S)} = \frac{K_I}{S}$$

Fig:- Integral Controller.

(iii) Derivative Controller:-

In this Controller activate signal is proportional to Derivative of Error signal.

$$e_a(t) \propto K_D \frac{d}{dt} e(t)$$



$$e_a(t) = K_D \frac{d}{dt} e(t)$$

Apply Laplace Transform.

Fig:- Derivative Controller.

$$\frac{E_a(S)}{E(S)} = S K_D.$$

(iv) Proportional plus Integral Controller

The Controller signal directly proportional to the proportional plus Integration of the error signal.

$$e_a(t) \propto e(t) + \int e(t)\,dt$$

$$e_a(t) = K_p\, e(t) + \frac{K_p}{S}\, e(t)\,dt$$

Apply Laplace Transform

$$\frac{E_a(S)}{E(S)} = K_p + \frac{K_I}{S}$$

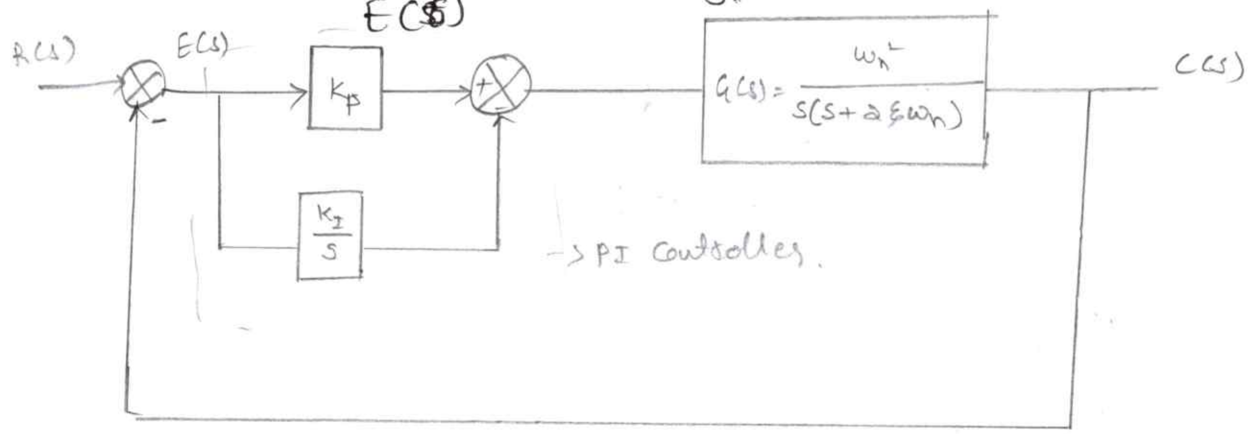


Fig:- PI Controller.

$$\text{open loop TF} = \frac{(K_I + SK_p)\,\omega_n^2}{S^2(S+2\xi\omega_n)}$$

$$\text{closed loop TF} = \frac{(K_I + SK_p)\,\omega_n^2 / S^2(S+2\xi\omega_n)}{1 + (K_I + SK_p)\,\omega_n^2 / S^2(S+2\xi\omega_n)}$$

$$TF = \frac{(K_I + SK_p)\,\omega_n^2}{S^3 + 2S^2\xi\omega_n + (K_I + SK_p)\,\omega_n^2}$$

The drawback of PI Controller is system is not stable. Since order of the system is increases, so that stability is decreased.

9) Lag Compensator:-

A Compensator having a characteristics of a Lag N/w is called Lag Compensator. Lag Compensator improves the steady state response of the system. Lag Compensator is a Low pass filter.

s-Plane represented Lag Compensator

Lag Compensator has pole at

$s = -1/\beta T$ & zero at

$s = -1/T$

$$\text{Transfer function} = \frac{s + Z_c}{s + P_c}$$

$$= \frac{s + 1/T}{s + 1/\beta T} \qquad T > 0 \;\&\; \beta > 1$$

Design of Lag Compensator using Root Locus:-

step 1:- Draw the root Locus of uncompensated system.

step 2:- Determine the dominant pole s_d. Draw a straight line through the origin with an angle $\cos^{-1}\xi$ with respect to negative real axis. The intersection point of the straight line with root locus gives the dominant pole s_d.

Step 3:- Determine the open Loop gain of the uncompensated system at $s = s_d$. Let this gain be K. The Loop gain K at $s = s_d$ on RL is

$$K = \frac{\text{Product of Vector Lengths from } s_d \text{ to open Loop poles.}}{\text{product of Vector lengths from } s_d \text{ to open Loop zeros.}}$$

step 4 :- Calculate the Parameter β of the Compensator

Let

K_{vu} = Velocity error Constant of uncompensated system

K_{vd} = Desired velocity error Constant

$$K_{vu} = \lim_{s\to 0} G(s)$$

$$A = K_{vd}/K_{vu}$$

$$\beta = (1.1 \text{ to } 1.2) \times A.$$

step 5 :- Determine the TF of lag Compensator. The zero of the lag Compensator is choosen to be 10% of the 2nd pole of uncompensated system.

$$Z_c = (-1/T) = 0.1 \times (\text{Second Pole of } G(s))$$

$$T = \frac{1}{[-0.1 \times (\text{second pole of } G(s))]}$$

$$P_c = -1/\beta T$$

$$\text{Transfer function } G_c(s) = \frac{s + 1/T}{s + 1/\beta T} = \frac{\beta(1 + sT)}{(1 + s\beta T)}$$

step 6 :- Determine the OLTF of Compensated system

$$G_o(s) = G_c(s)\, G(s) = \frac{(s + 1/T)}{(s + 1/\beta T)} G(s)$$

step 7 :- check weather the Compensated system satisfies the steady state error requirement.

If it is satisfied, then the design is accepted otherwise repeat the design by modifying the locations of poles & zeros of the Compensator.

10) a) Given system Matrix

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

sol:- state transition Matrix

$$\phi(t) = L^{-1}[SI - A]^{-1}$$

$$SI - A = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$[SI - A]^{-1} = \frac{\operatorname{adj}(SI - A)}{\det(SI - A)}$$

$$\operatorname{adj}(SI - A) = \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}$$

$$|SI - A| = S(S+3) + 2$$

$$\phi(t) = \frac{L^{-1}\begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}}{S^2 + 3S + 2}$$

$$= L^{-1}\begin{bmatrix} \dfrac{S+3}{S^2+3S+2} & \dfrac{1}{S^2+3S+2} \\ \dfrac{-2}{S^2+3S+2} & \dfrac{S}{S^2+3S+2} \end{bmatrix}$$

$$L^{-1}\left[\frac{S+3}{(S+1)(S+2)}\right] = L^{-1}\left[\frac{2}{S+1} - \frac{1}{S+2}\right] = 2e^{-t} - e^{-2t}$$

$$L^{-1}\left[\frac{1}{(S+1)(S+2)}\right] = L^{-1}\left[\frac{1}{S+1} + \frac{2}{S+2}\right] = e^{-t} + 2e^{-2t}$$

$$L^{-1}\left[\frac{-2}{(S+1)(S+2)}\right] = L^{-1}\left[\frac{-2}{S+1} + \frac{2}{S+2}\right] = -2e^{-t} + 2e^{-2t}$$

$$L^{-1}\left[\frac{S}{(S+1)(S+2)}\right] = L^{-1}\left[\frac{-1}{S+1} + \frac{2}{S+2}\right] = -e^{-t} + 2e^{-2t}$$

$$\phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} + 2e^{-2t} \\ -2e^{t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

b) Controllability :-

The Controllability Matrix Q_c is

$$Q_c = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$$

For a single I/p system Q_c is $n \times n$ square Matrix.

If the determinant of Q_c is non zero then the system is controllable.

Eg:- $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = [1 \quad 2]$ $D = 0$.

$$Q_c = [B \quad AB]$$

$$AB = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix}$$

$\det[Q_c] = 0$

System is not Controllable.

11)a) i) state :-

The state of the system represents the minimum amo-
-unt of information needed to know about a system at $t=t_0$ such that the function behaviour of the system ($t \geq 0$) can be determined with response of the I/p.

ii) stat space :-

If $x_1(t), x_2(t), x_3(t) \ldots x_n(t)$ are the minimum no. of state variables which are necessary to describe the dynamics of a control system, then n-dimensional space whose coordinate axes consist of x_1 axis, x_2 axis ... x_n axis is called state space.

b) state Model of the system for the given transfer fun-
-ction

$$\frac{Y(s)}{U(s)} = \frac{10}{s^3+4s^2+2s+1}$$

$$Y(s)\left[s^3+4s^2+2s+1\right] = 10U(s)$$

$$s^3 Y(s) + 4s^2 Y(s) + 2s\, Y(s) + Y(s) = 10\, U(s)$$

Take inverse L.T on both sides

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 10u$$

Assign the state variables.

$$y = x_1$$

$$\frac{dy}{dt} = \dot{x}_1 = x_2$$

$$\frac{d^2y}{dt^2} = \dot{x}_2 = x_3$$

$$\frac{d^3y}{dt^3} = \dot{x}_3$$

$$\dot{x}_3 + 4x_3 + 2x_2 + x_1 = 10u$$

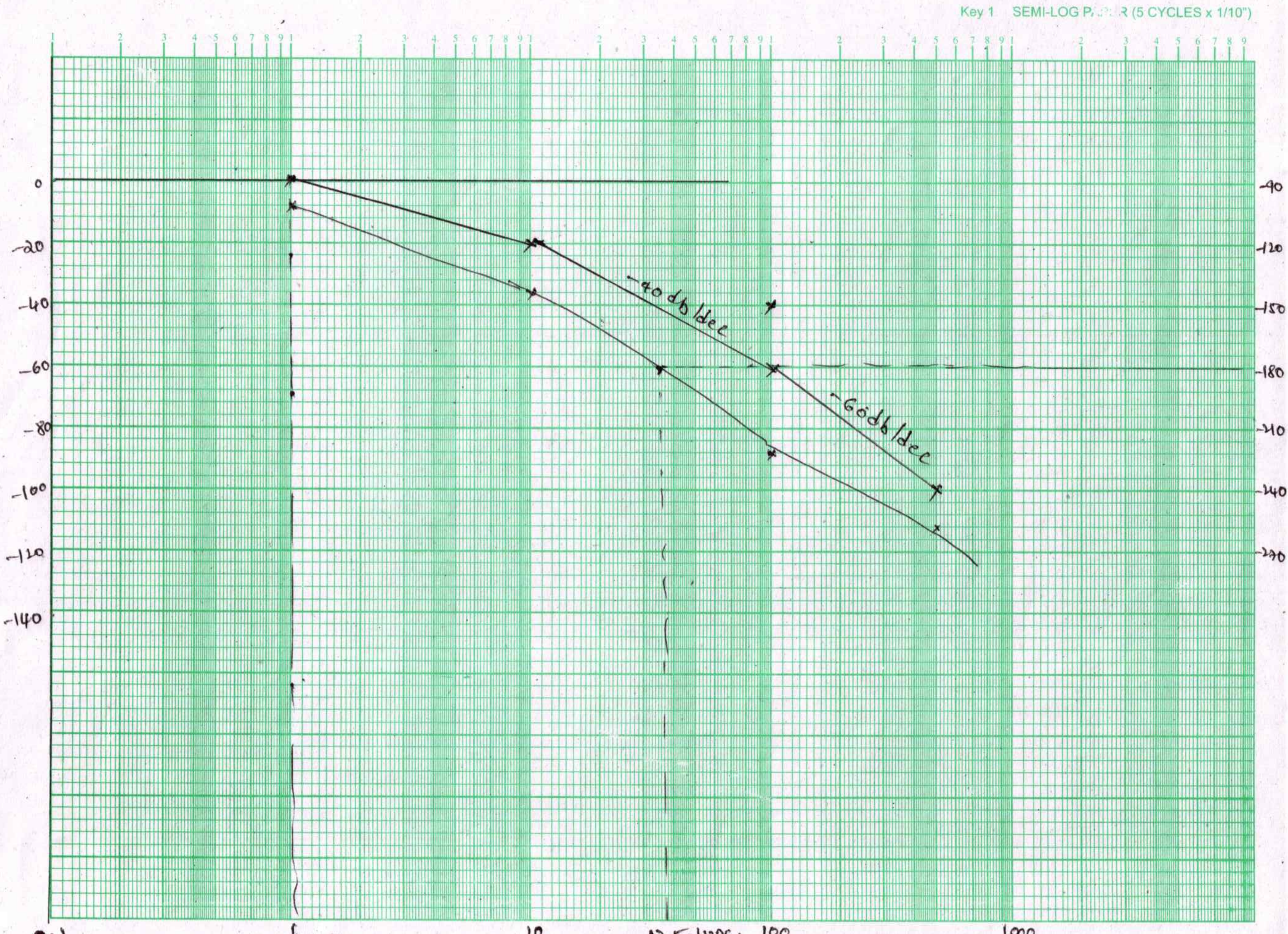
$$\dot{x}_3 = -[x_1 + 2x_2 + 4x_3] + 10u$$

$$\dot{x}_1 = x_2$$

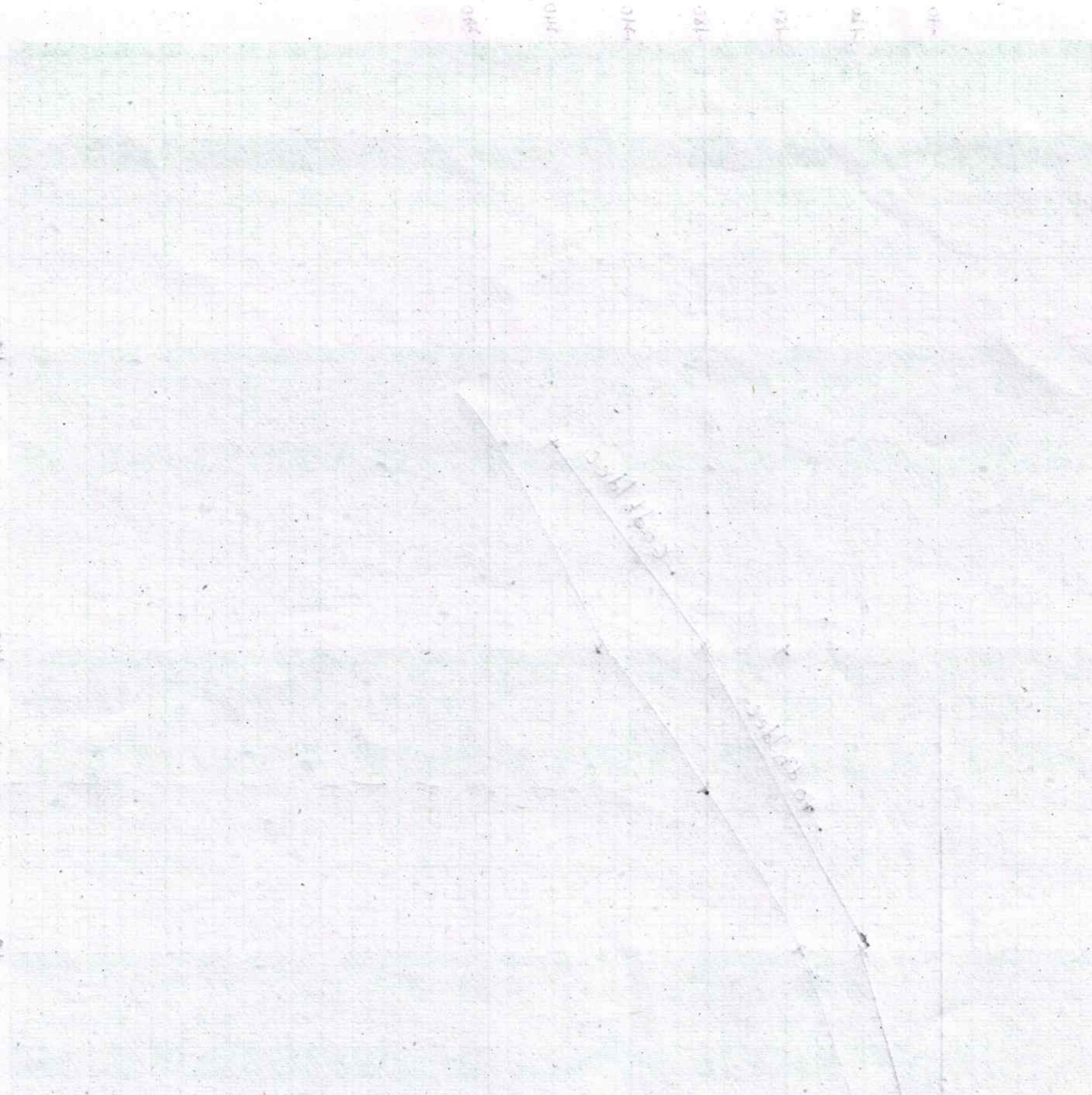
$$\dot{x}_2 = x_3 \quad \& \quad y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



Key 1 SEMI-LOG PAPER (5 CYCLES x 1/10")
0
-20
-40
-60
-80
-100
-120
-140
-90
-120
-150
-180
-210
-240
-270
-40db/dec
-60db/dec
10
100
1000



CMR Engineering College

III - B.Tech - I Sem End Examinations (Regular) - Dec - 2025

Control Systems.

R22.

Scheme of Evaluation

PART - A

1) a) Calculation of poles & zeros → 1M

b) calculation of x value → 1M
[upto characteristic Eqⁿ].

c) Definition of rise time → 1M

d) Condition for Routh's Stability → 1M

e) Compare time & Frequency response → 1M
[1 point].

f) Definition of phase Margin → 1M

g) S-plane Representation - diagram → 1M
[Lead (or) Lag]

h) Effect of P, PI Controllers → 1M
[1 point]

i) Definition of State Variable → 1M

j) Definition of Controllability → 1M

PART - B.

2) a) Control system Definition → 1M

calculation of TF for open loop & Closed loop } → 4M

b) characteristics of feed back → 5M
[Explanation].

3) Block diagram reduction → 10M.

4) a) Routh array formation & calculation } → 5M

comment on stability → 1M.

b) step signal - Explanation, Graph → 2M

Ramp signal, statement, Graph → 2M

5) calculation of

Damping ratio → 2M

peak overshoot → 3M

peak time → 3M

natural frequency → 2M

6) Bode plot

calculation → 8M

Gain Margin → 1M

phase Margin → 1M

7) Nyquist plot

procedure → 4M

Calculation → 4M

stability → 2M

8) Explanation of

i) Proportional → 2M

ii) Integral → 2M

iii) Derivative → 2M

iv) Proportional plus Integral → 4M.

9) Lag Compensator Explanation → 2M

Design procedure for Lag Compensator using root Locus } → 8M

10) a) State transition Matrix

calculation → 8M

b) Controllability

formula → 1M

Example → 1M

11) a) Definitions of

State → 1M

State Space → 1M

b) calculation of State-Model } → 8M.