Code No: R22CS305PC

CMR ENGINEERING COLLEGE: HYDERABAD
UGC AUTONOMOUS
II. B.Tech - I - Semester End Examinations (Regular) - December - 2025
DISCRETE MATHEMATICS
(Common for CSE, CSM)

PART A (10 MARKS)

1 a. Write the negation of the statement " if the processor is fast then the printer is slow". [1M]

Ans: $\sim(p \to q) = p \wedge \sim q$

The negation of the statement is:

" The processor is fast and the printer is not slow."

1 b. Show that for two propositions p and q where $(p \text{ XOR } q) \vee (p \leftrightarrow q)$ is a Tautology. [1M]

Ans:

p	q	p XOR q	$p \leftrightarrow q$	$(p \text{ XOR } q) \vee (p \leftrightarrow q)$
T	T	F	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

Since the final column (representing the entire expression) consists of 'T' (true) values ∴ The given expression is Tautology.

1 c. Describe transitive relation. (1M)

Ans: A Relation if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for all elements a, b and c are in the set.

1.(d) How to find The transitive Closure of a Relation R? (1M)

Ans: The transitive Closure "R^+" of a relation R is the Smallest transitive relation that Contains R.

Example: Transitive closure (R^+) = {(A,B), (B,C), (A,C)}

if R = {(A,B), (B,C)}

1.(e) Define permutations. (1M)

Ans: permutations are arrangement of objects in a Specific sequence or order (or)

∴ The number of permutations of n distinct items taken r at a time is given by The formula

$$p(n,r) = \frac{n!}{(n-r)!}$$

1.f Define algebraic System with Example (1M)

Ans: An Algebraic System is a non-empty set, often called a carrier set, Equipped with one or more binary operation.

ex: $(Z, +)$ or $(N, +, *)$ ... etc.

1.g: Define Combinations (1M)

Ans: Combinations are Selections of objects from a Collection where the order of Selection does not matter.

(or)

The number of Combinations of n distinct items taken r at a time is given by the formula $C(n,r) = {}^nC_r = \frac{n!}{(n-r)!r!}$

1.h Define the well-ordering principle (1M)

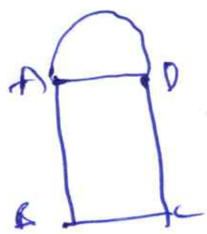
Ans: The well-ordering principle states that every non empty set of positive integers contains a least or minimum element.

1.(i) Difference between a directed and undirected graph (1M)

Ans: The difference is that directed graphs have edges with a specific direction (representing by arrows), indicating a one-way relationship. while undirected graphs have edges without a specific direction, indicating a two-way or symmetric relationship.

1.(J) Define planar Graphs. (1M)

Ans: A planar Graph is a Graph that can be drawn on a plane in such a way that no cross edges in it.



PART - B (50 MARKS)

(2) prove that $((p \to r) \land (q \to r)) \to ((p \lor q) \to r)$ is a Tautology. [10 M]

p	q	r	$p \to r$	$q \to r$	$(p \to r) \land (q \to r)$	$p \lor q$	$(p \lor q) \to r$	$((p \to r) \land (q \to r)) \to ((p \lor q) \to r)$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F	T
T	F	T	T	T	T	T	T	T
T	F	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F	T
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	T

For the given Expression $[(p \to r) \land (q \to r)] \to [(p \lor q) \to r]$ all the final column of Truth values are 'T'

$\therefore$ The given expression is a Tautology.

Note: Here student may use logical Equivalence Formulas Also to prove Tautology.

③ For Three propositions p, q and r prove that

$$[(p \vee q) \to r] \longleftrightarrow [(p \to r) \wedge (q \to r)] \quad [10M]$$

p	q	r	$p \vee q$	$(p \vee q) \to r$	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

From Colum 5 $[(p \vee q) \to r]$ and Column 8 $[(p \to r) \wedge (q \to r)]$

Both Expression values are having same truth values so the given Expression

$$[(p \vee q) \to r] \longleftrightarrow [(p \to r) \wedge (q \to r)]$$ is

Equivalent to

Note: Here may used Logical Equivalence Formulas Also

(4) Let $A = \{1, 2, 3, 4, 5\}$ and $P = \{A \times A\}$. R is a relation set such that $(x, y) R (x_1, y_1) \Leftrightarrow x y_1 = x_1 y$. Show that R is an Equivalence relation. Compute A/R [10M]

Sol. Given $A = \{1, 2, 3, 4, 5\}$

$P = \{A \times A\}$

$P = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4),$
$(2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2),$
$(4,3), (4,4), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5)\}$

R is a Relation set such that $(x, y) R (x, y_1) \Leftrightarrow \underline{x y_1 = x_1 y}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,5),$
$(2,1), (2,2), (2,3), (2,4), (2,5),$
$(3,1), (3,2), (3,3), (3,4), (3,5),$
$(4,1), (4,2), (4,3), (4,4), (4,5),$
$(5,1), (5,2), (5,3), (5,4), (5,5)\}$

To show that R is Equivalence [8M]

(1) Reflexive property $\forall a$ such that $(a, a) \in R$

$\therefore$ R is reflexive $R = \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$

(2) Check for Symmetry:

if $(x, y) R (x_1, y_1)$ then $xy_1 = x_1 y$ can we show

$(x_1, y_1) R (x, y)$

The condition for $(x_1, y_1) R (x, y)$ is

$$x_1 y = x y_1$$

$\therefore$ R is Symmetric.

Ex:

$(1,2), (2,1) \in R$,

$\vdots$

$(4,5), (5,4) \in R$. also $\underline{xy_1 = yx_1}$

(3) <u>Check for Transitivity:</u>

if $(x, y) R (x_1, y_1)$ and $(x_1, y_1) R (x_2, y_2)$

then $xy_1 = x_1 y$ and $x_1 y_2 = x_2 y_1$ we what to

Show $(x, y) R (x_2, y_2)$ which means

$xy_2 = x_2 y$. R is transitive.

$(1, 2) R (2, 1) \Rightarrow$ ~~(2,1)~~ R (

$\therefore$ $(1,1) \in \underline{R}$.

from (1), (2) and (3) R is an Equivalence

relation

Compute A/R: (Equivalence classes) [2m]:

$A = \{1, 2, 3, 4, 5\}$

The ratios formed distinct Ratio from the Relation R

are $1/1, 1/2, 1/3, 1/4, 1/5, 2/1, 2/3, 2/4, 2/5, 3/1, 3/2, 3/4,$

$4/1, 4/3, 4/5, 5/1, 5/2, 5/3, 5/4$

The Equivalence classes A/R are the sets of Ordered pairs that yield the same ratio

- $[1/1] = \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$

$[1/2] = \{(1,2), (2,4)\}$

$[1/3] = \{(1,3)\}$.

- - - - - and soon for all 19 unique ratios.

(5) With examples differentiate Symmetric and Anti-Symmetric relations [10m].

Ans: Symmetric [5M]:

A relation on set A $\forall a, b \in A$ such that $(a, b) \in R$ and $(b, a) \in R$ then relation R is said to be symmetric relation on the R.

Example:

$A = \{1, 2, 3\}$ Relation on A is given as $R = A \times A$

$R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

if $(a,b) \in R$ then $(b,a) \in R$ ~~then~~ it is Symmetric

relation

$(1,2) \in R$ and $(2,1) \in R$ $\therefore$ R is symmetric

Anti - Symmetric [5 m].

① R is anti - symmetric if $(a,b) \in R$ and $(b,a) \in R$ if and only if $a = b$

(or)

② R is anti - symmetric if $(a,b) \in R$ and $(b,a) \notin R$ if and only if $a \neq b$.

Example: $R = \{(1,1), (1,2)\ (1,3), (2,2)\}$

in Above relation

$(1,1) \in R$ $(1,1) \in R$ if $1=1$

$(1,2) \in R$ $(2,1) \notin R$ if $1 \neq 2$

(6) prove that a group G is abelian if $(a * b)^{-1} = a^{-1} * b^{-1}$ for all a, b belongs to G. [10 M]

Sol: In any Group G, the inverse of a product of elements is given by the General property

$$(a*b)^{-1} = b^{-1} * a^{-1}$$

: To show that Group is Abelian that Group we need to show obey Commutative property.

To show Abelian it must be prove that

$a*b = b*a$ for each $a, b \in G$.

given $(a*b)^{-1} = b^{-1} * a^{-1} \rightarrow (1)$

$(a*b)^{-1} = a^{-1} * b^{-1} \rightarrow (2)$

$\therefore b^{-1} * a^{-1} = a^{-1} * b^{-1} \rightarrow$ from (1) and (2).

Multiply both sides of the equation on the left by b and on the right by b.

$$b*(b^{-1} * a^{-1})*b = b*(a^{-1} * b^{-1}) * b$$

$$\underline{b*b^{-1}} * a^{-1} * b = b * a^{-1} * \underline{b^{-1} * b}$$

$$e * a^{-1} * b = b * a^{-1} * e$$

$$a^{-1} * b = b \cdot a^{-1}$$

This shows that a^{-1} and b commute for any $a, b \in G$.

$\therefore$ given group G $(a*b)^{-1} = b^{-1} * a^{-1}$ is an Abelian group.

7(a) Explain Groups and types of Groups with Examples [5M]

Ans: A Group $(G, *)$ is a set G and a binary operation $*$ that combine any two elements to form a third and satisfying closure, Associativity, identity and inverse properties called Group.

Types of Groups:

① Abelian Group: A group where the operation is commutative $(a*b = b*a)$

Example: $(\mathbb{Z}, +)$ is Abelian $3+5 = 5+3$.

② Cyclic Group: A Group that can be generated by a single element (a generator)

Example: $(\mathbb{Z}_5, +)$ (integers mod 5) is cyclic.

7 (b) Explain Semigroups and monoids [5m].

Ans: Semigroup:

A non empty set S, (S, *) is called a Semigroup if it follows the following axioms.

- ~~Closure~~ ① Closure : $(a*b) \in S$ for $a, b \in S$

② Associativity : $a*(b*c) = (a*b)*c$ when a, b, c belongs to S.

Semigroup Follows closure and Associativity.

monoid.

A non-empty set S, (S, *) is called a monoid if it follows following axioms.

① Closure : $\forall a, b \in S$ then $a*b \in S$

② Associativity : $\forall a, b, c \in S$ when $a*(b*c) = (a*b)*c$.

③ Identity element : $\forall e \in S$ such that $a*e = e*a$.

⑧ solve in a class of 52 Students, 30 are Studying C++, 28 are Studying Pascal and 13 are Studying both Languages. How many in this class are studying at least one of these languages. [10M].

Sol:- Total Number of Student in a Class $n(S) = 52$.

no of ~~Students~~ Studying C++ $\Rightarrow n(C^{++}) = 30$

no. of Students Studying in pascal $\Rightarrow n(P) = 28$

no. of Students Studying both C++ and Pascal

$$n(C^{++} \cap P) = 13.$$

At least one of these language $n(C^{++} \cup P)$

$$\therefore \; n(C^{++} \cup P) = n(C^{++}) + n(P) - n(C^{++} \cap P)$$

$$= 30 + 28 - 13$$

$$= 30 + 15$$

$$= 45$$

$\therefore$ The no. of Students Studying at least one of these langues are $\underline{\underline{45}}$.

9) Describe the Binomial theorem with an Examples
Theorem [8m] + Example [2m].

Ans:

An Expression Consisting of two terms, connected by + or − signs is called a binomial Expression

Binomial Theorem.

If a and b are real numbers and n is a positive integer then.

$$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1} b + {}^nC_2\, a^{n-2} b^2 + \cdots$$

$$+ \cdots + {}^nC_r\, a^{n-r} b^r + \cdots + {}^nC_n\, b^n.$$

where $${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

The General term or $(r+1)^{th}$ term in the Expansion is given by

$$T_{r+1} = {}^nC_r\, a^{n-r} b^r.$$

Example:

$$(x+y)^5 = {}^5C_0 x^5 + {}^5C_1 x^4 y + {}^5C_2 x^3 y^2 + {}^5C_3 x^2 y^3 + {}^5C_4 x y^4 + {}^5C_5 y^5.$$

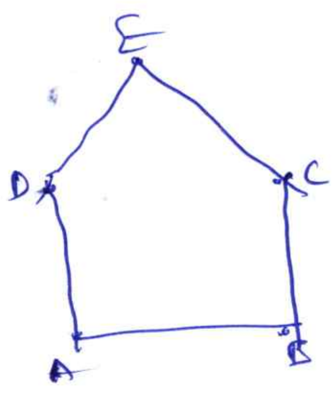
Whe ${}^5C_0$, ${}^5C_1$, ${}^5C_2$, ${}^5C_3$, ${}^5C_4$, ${}^5C_5$ are Coefficient

(10) What is a planar Graph? Draw a planar graph of order 5, 8, 6, 12 if possible [10M].

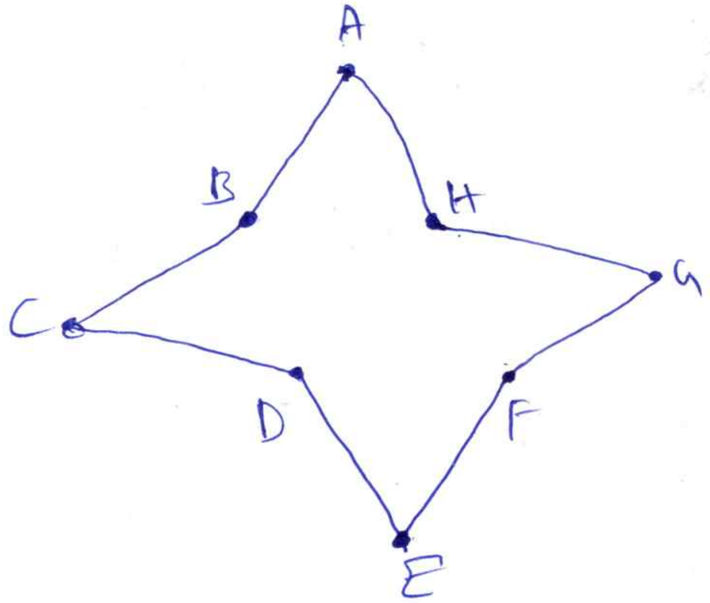
Ans:- Planar Graph (2 M)

A planar graph is a graph that can be drawn on a flat surface without any edges crossing over one another.

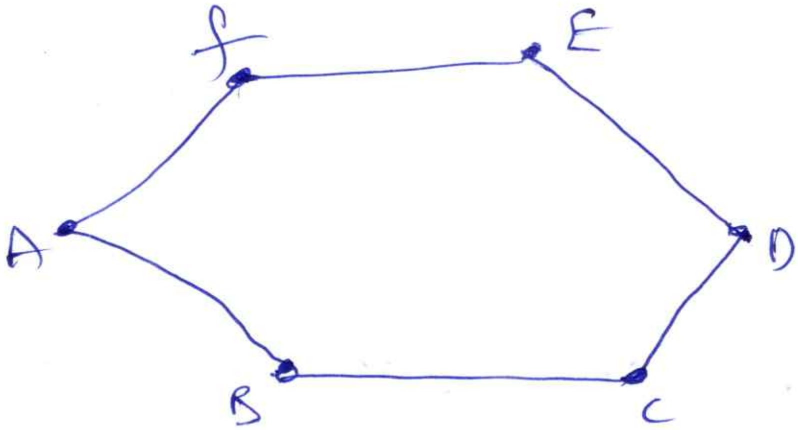
planar Graph order 5 [2m]



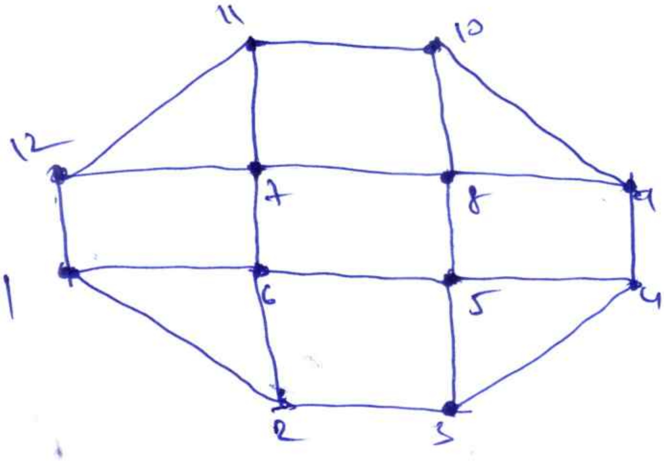
planar Graph of order (8) [2M]



planar Graph of order (6) [2M]



planar Graph of order (12) [2M].

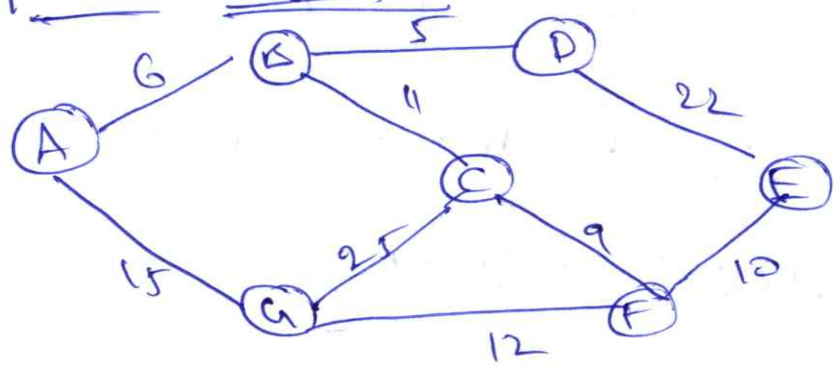


(11) Write kruskal's Algorithm. Find Minimum Spanning tree using kruskal's algorithm for following graph: [10M].

Ans: **Kruskal's Algorithm [2M].**

- kruskal's Algorithm finds a minimum spanning tree for connected, edge-weighted.
- Sort all edges in non-decreasing order of their weights.
- Initialise a spanning forest where each vertex is its own component.
- Iterate through the sorted edges from lowest weight to highest.
- Avoid the edges which we forms a cycle.
- Repeate step until all vertices are in a single component or all edges have been processed.

- problem solving [8M]

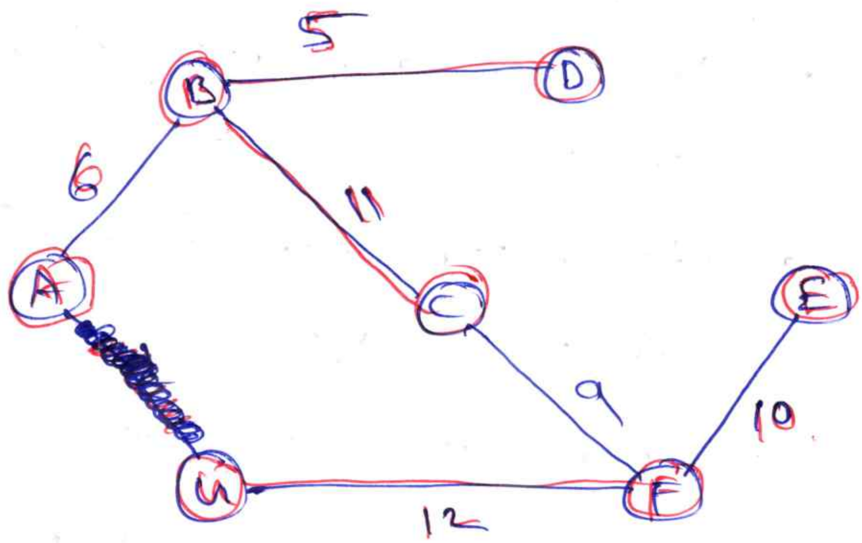


Step①: list and sort the edges with their weights increasing order.

(B - D) = 5
(A - B) = 6
(C - F) = 9
(E - F) = 10
(B - C) = 11
(F - G) = 12

(A - G) = 15 ×
(C - D) = 17 ×
(D - E) = 22 ×
(C - G) = 25 ×

Step②.



It colors all edges without forming a cycles. Step by step. And the above figure is the Minimum Spanning tree using Kruskal's Algorithm.