CMR Engineering College
UGC Autonomous
II - B.Tech II-Semester End Examinations (Supply) Dec-25

DISCRETE MATHEMATICS.
(Common for IT, CSD, CSC)

Time: 3 Hours Max Marks: 60.

PART - A

1 a) Represent the proposition "if you have the flee then you miss the final examination." into symbolic form and also it negation. (1 Mark)

Ans: Let P : "You have the flee"
Q : "You miss the final examination"

proposition: " if you have the flee then you miss the final examination".

$$P \rightarrow Q$$

Negation: $\neg P \rightarrow Q$.

b) Write about well-formed formulae. (1 Mark)

Ans: A well formed formulae (wff) is a syntactically correct expression built from propositional or predicate variables, logical connectives, quantifiers (In predicate logic) and parenthesis according to precise formation rules.

It is well formed in the sense that it can be unambiguously interpreted as a logical statement with a definite truth value under an interpretation.

c) Define Binary relation. (1 Mark)

Ans: A Binary relation is a set of ordered pairs that describes how elements of one set are related to elements of the same or another set.

Given sets A and B, a Binary relation R from A to B is any subset of the Cartesian product of $A \times B$.

Each element of R is an ordered pair (a, b) with $a \in A$ and $b \in B$; if $(a, b) \in R$, we say "a is related to b" and often write aRb

d) Differentiate partial ordering and total ordering relations. (1 Mark)

Ans: A partial order is an order relation where some elements may be incompatible, while a total order is an order relation where every pair of elements is comparable.

A partial order on a set S is a relation $\leq$ that is reflexive, antisymmetric, and transitive.

Ex: On the power set of {1, 2, 3} the relation $\subseteq$ is a partial order because sets like {1, 2}, and {2, 3} are incomparable

In a total order, no such incomparable pair exists, every two elements can be compared in the order

Ex: On the integers Z with $\leq$, any two integers a, b, satisfy $a \leq b$, or $b \leq a$. So, $\leq$ is a total order.

e) Provide an example of partially ordered set (1 Mark)

Ans: The example of POSET is the power set of a set with the subset relation.

Ex: Take the set $A = \{1, 2, 3\}$

Consider its power set.

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\},$
$\{1,2,3\}\}$ with the relation $\subseteq$.

This structure is $(P(A), \subseteq)$ is a partially ordered set because, it is reflexive $(X \subseteq X)$, antisymmetric (if $X \subseteq Y$ and $Y \subseteq X$, then $X = Y$) and transitive
(if $X \subseteq Y$, and $Y \subseteq Z$, then $X \subseteq Z$)

f) Explain the Concept of Duality in the Boolean Algebra. (1 Mark)

Ans Two formulas A and A^* are said to be duals of each other if either one can be obtained from the other by replacing $\wedge$ by $\vee$, $\vee$ by $\wedge$ Connectives.
T by F and F by T

Ex: $P \vee (Q \wedge R)$ its dual is $P \wedge (Q \vee R)$
$P \vee (P \wedge \sim Q)$ its dual is $P \wedge (P \vee \sim Q)$
$P \to Q$ is equivalent to $\sim P \vee Q$ its dual is $\sim P \wedge Q$

g) How many can 10-Similar Coins fall heads, when they are tossed Simultaneously? (1 Mark)

Ans:-

Total possible outcomes equal $2^{10} = 1024$ as each coin has 2 faces.

Favorable outcomes number 1 (all heads).

Thus, probability $= \frac{1}{1024}$

$$P(X=10) = \binom{10}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{0} = 1 \times \frac{1}{1024}$$

The probability of all 10 Similar fair Coins landing heads when tossed Simultaneously is $\frac{1}{1024}$

h) Determine the Co-efficient of x^{12} in $x^3(1-2x)^{10}$ (1 Mark)

Ans. The coefficient of x^{12} in this expression is the Same as the Coefficient of x^9 in the expansion of $(1-2x)^{10}$

Using binomial theorem, the general term of for the expansion of $(1-2x)^{10}$ is given by

$T_{r+1} = \binom{n}{r} a^{n-r} b^r$, where $n = 10$, $a = 1$, $b = -2x$

$$T_{r+1} = \binom{10}{r} (1)^{10-r} (-2x)^r$$

$$T_{r+1} = \binom{10}{r} (-2)^r x^r$$

Step 3: Find the Coefficient of x^9

To find the Coefficient of x^9, set $r = 9$

$$T_{10} = \binom{10}{9} (-2)^9 x^9$$

The Coefficient is $\binom{10}{9} (-2)^9$

Step 4: Calculate the Value

$$\binom{10}{9} = \frac{10!}{9!(10-9)!} = \frac{10!}{9!\,1!} = 10$$

$$(-2)^9 = -512$$

$$\text{Coefficient} = 10 \times (-512) = -5120.$$

The Coefficient of x^{12} in $x^3(1-2x)^{10}$ is -5120

Define Bipartite graph and Isomorphic graph (1 mark)

i) Define Bipartite graph

Ans:- A Bipartite graph is a graph whose vertices can be divided into two disjoint and independent sets U and V, such that every edge connects a vertex in U to one in V. there are no edges connecting vertices within the same set.

Isomorphic graphs are the graphs that are structurally the same. A Graph G_1 is isomorphic to Graph G_2, if there is a bijection (one to one correspondence) between their vertex sets that preserves adjacency..

j) Define Planar graph. Is $K_{2,3}$ planar graph

(1 Mark)

Ans :-

A planar graph is a graph that can be drawn on a plane without any edges crossing.

$K_{2,3}$ is a Planar graph. It can be drawn without any edge crossings.

PART - B

2) obtain PCNF and PDNF for the formula

$(\sim P \rightarrow \cancel{R}) \wedge (q \leftrightarrow P)$ (10 marks)

Ans: PCNF

Given that $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$

$\rightarrow \neg(\neg P) \vee R) \wedge [(Q \rightarrow P) \wedge (P \rightarrow Q)]$ $(\because \neg(\neg P) = P)$

$\rightarrow (P \vee R) \wedge [(\neg Q \vee P) \wedge (\neg P \vee Q)]$ $(\because P \rightarrow Q = \neg P \vee Q)$

$\rightarrow (P \vee R \vee F) \wedge [((\neg Q \vee P) \vee F)) \wedge ((\neg P \vee Q) \vee F)]$ $(\because P \vee F = P)$

$\rightarrow ((P \vee R) \vee (Q \wedge \neg Q)) \wedge ((\neg Q \vee P) \vee (R \wedge \neg R)) \wedge (\neg P \vee Q) \vee (R \wedge \neg R)$

$\rightarrow ((P \vee Q \vee R) \wedge (P \vee \neg Q \vee R)) \wedge ((P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)) \wedge$

$((\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R))$

$\rightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge$

$(\neg P \vee Q \vee \neg R)$

This is required PCNF

PDNF:

Conversion of the given DNF into PDNF

First replace the implications and biconditional.

→ Given that the formula $F = (\neg p \to r) \wedge (q \leftrightarrow \neg p)$

$$F = (\neg(\neg p) \vee r) \wedge ((q \wedge p) \vee (\neg q \wedge \neg p))$$

$$F = (p \vee r) \wedge ((q \wedge p) \vee (\neg q \wedge \neg p))$$

Apply the distributive law

$$\to (p \wedge ((q \wedge p) \vee (\neg q \wedge \neg p))) \vee (r \wedge ((q \wedge p) \vee (\neg q \wedge \neg p)))$$

$$\to (p \wedge q \wedge p) \vee (p \wedge \neg q \wedge \neg p) \vee (r \wedge q \wedge p) \vee (r \wedge \neg q \wedge \neg p)$$

After simplification.

$$\to (p \wedge q) \vee F \vee (p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$$

$$\to (p \wedge q) \vee (p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$$

→ After expansion

$$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$$

→ After Removing duplication.

$$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$$

The PDNF is

$$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$$

3a) Show that $R \to S$ can be drawn from the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q (5 marks)

Ans :-

Instead of deriving $R \to S$, we shall include R as an additional premise and show S first.

{1}	(1)	$\neg R \vee P$	Rule Premise
{2}	(2)	R	Rule P (Assumed Premise)
{1,2}	(3)	$R \to P$	Rule T, (1) and E_{11}, ($\neg R \vee P \Leftrightarrow R \to P$)
{1,2}	(4)	P	Rule T, 2, 3, and I_{11} ($P, P \to Q \Leftrightarrow Q$)
{5}	(5)	$\underset{(P)}{P} \to \underset{(Q)}{(Q \to S)}$	Rule P
{1,2,5}	(6)	$Q \to S$	Rule T, 4, 5 and I_{11}
{7}	(7)	Q	Rule P
{1,2,5,7}	(8)	S	Rule T, 6, 7, and I_{11}
{1,2,5,7}	(9)	$R \to S$	Rule - CP

$\therefore R \to S$ is a valid conclusion.

3b) Prove or Disprove the Validity of the following arguments using the rules of inference. All men are fallible. All kings are men. Therefore, all kings are fallible (5 Marks)

Ans:

Let us consider:

$M(x)$: x is a men

$F(x)$: x is fallible

$K(x)$: x is a king.

1. $\forall x \ (M(x) \rightarrow F(x))$
2. $\forall x \ (K(x) \rightarrow M(x))$
3. $\forall x \ (K(x) \rightarrow F(x))$

(1) $\forall(x) \ K(x) \rightarrow M(x))$ Rule P

$K(y) \rightarrow m(y)$.

4Q) Explain about relations with examples and give suitable examples for equivalence relation. (10 Marks)

Ans :- Relation.

- Relation can be described as a collection of ordered pairs
- It is used to relate an object from one set to other set, and the sets must be non empty.
- The relation can contain two or more than two sets
- Suppose there are two sets A & B for instance.

set A contain an object a and set B contain an object b. The object will be related to each other if there is a relation of ordered pairs (a,b) with the help of a Cartesian product.

Types of relations

1> Empty relation
2> Universal relation
3> Identity relation
4> Inverse relation
5> reflexive relation
6> Symmetric relation
7> transitive relation
8> Equivalence relation.

1> Empty relation: void relation is one in which there is no relation between any element of a set i e $R = \phi$ which is a subset of $A \times A$

2> Universal relation: A relation R is the set A is universal relation if each element of A is related to every element of A ..., is $R = A \times A$ (full relation)

3> Identity relation: Relation R is said to be an identity relation if all the elements are related to itself(I)

If relation I is identity, all elements in set P are related with itself.

Such that for every $x \in P$ it is $(x, x) \in I$

ex. in the set $P = \{1, 2, 3, 4\}$

Identity relation would be

$$I = \{(1,1), (2,2), (3,3), (4,4)\}$$

4) Inverse relation:

If a set with elements the inverse pair of other set then relation is called Inverse relation.

Ex: If $R = \{(a, b) : a \in P$ and $b \in Q\}$ then from set Q to P is inverse relation

$$R^{-1} = Q \to P$$
$$= \{(b, a), a, b \in R\}$$

The range of relation R and domain are same

Ex: $R = \{(a, x), (b, y)\}$

$R^{-1} = \{(x, a), (y, b)\}$

5) Reflexive relation:

It is same as identity. A relation R is said to be reflexive if all the elements of any set are mapped to itself

Ex: if $P = \{1, 2\}$ then $R = \{(1,1), (2,2)\}$ is reflexive relation

$\therefore$ Every element of P is mapped itself such that $1 \in P$, $(1,1) \in R$

5a) Let $f(x) = x+4$, $g(x) = 4x$ for $x \in R$, where R is the set of real numbers. Find $f \circ g$ and $g \circ f$. (4 Marks)

Ans :- Given $f(x) = x+4$

$g(x) = 4x$

Calculate $f \circ g$.

To find the composite function $f \circ g(x)$. substitute $g(x)$ into the function $f(x)$

$$f(g(x)) = f(4x)$$

$$f(4x) = 4x+4$$

Calculate $g \circ f$

To find the composite function of $g \circ f(x)$, substitute $f(x)$ into the function $g(x)$

$$g(f(x)) = g(x+4)$$

$$g(x+4) = 4(x+4)$$

$$g(x+4) = 4x+16$$

$f \circ g(x) = 4x+4$ and

$g \circ f(x) = 4x+16$

5b) Let $R = \{(b,c),(b,e)(c,e),(d,a)(c,b)(c,c)\}$ be a relation on the set $A = \{a,b,c,d,e\}$ find the transitive ~~relation~~ closure of the relation R.

[5 Marks]

<u>Ans :-</u>

Given Relation R

$R = \{(b,c)(b,e)(c,e)(d,a)(c,b)(c,c)\}$ on the set $A = \{a,b,c,d,e\}$

Identify the pairs needed for transitivity

Condition: check for missing pairs (x,z) when (x,y) and (y,z) are in R.

$(b,c) \in R$ and $(c,e) \in R \Rightarrow (b,e) \in R$

$(b,c) \in R$ and $(c,b) \in R \Rightarrow (b,b) \in R$

$(c,b) \in R$ and $(b,c) \in R \Rightarrow (c,c) \in R$

$(c,b) \in R$ and $(b,e) \in R \Rightarrow (c,e) \in R$

$(e,c) \in R$ and $(c,b) \in R \Rightarrow (e,b) \in R$

$(e,c) \in R$ and $(c,e) \in R \Rightarrow (e,e) \in R$

check the Pairs for transitivity

$(e,b) \in R$ & $(b,c) \in R \Rightarrow (e,c) \in R$

$(e,b) \in R$ & $(b,e) \in R \Rightarrow (e,e) \in R$

$(e,b) \in R$ & $(b,b) \in R \Rightarrow (e,b) \in R$

(b,b) (c,c) (e,e) do not create new Paths

The transitive closure is

$R^+ = \{(b,c),(b,e)(c,e)(d,a)(c,b)(e,c)(b,b)(c,c)(e,b)(e,e)\}$

6> Equivalence relation.

(If it is reflexive, Symmetric and transitive) then it is equivalence relation

Ex: {1, 2, 3, 4, 5, 6}

R = {(1,1), (1,3), (1,5) (3,5), (3,3) (5,5), (2,6)
(2,2), (6,6) (6,2), (3,1), (5,1) (5,3) (4,4)}

Lets check reflexive

(1,1) ∈ R
(2,2) ∈ R
(3,3) ∈ R
(4,4) ∈ R
(5,5) ∈ R
(6,6) ∈ R

Hence proved. R is reflexive relation of x

Lets check symmetric

if (x,y) ∈ R then (y,x) ∈ R

(1,3) ∈ R & (3,1) ∈ R
(1,5) ∈ R & (5,1) ∈ R
(3,5) ∈ R & (5,3) ∈ R
(2,6) ∈ R & (6,2) ∈ R

Hence proved R is symmetric to X.

lets check transitive.

if (x,y) ∈ R & (y,z) ∈ R then (x,z) ∈ R

(1,1) & (1,3) → (1,3) ∈ R
(1,1) & (1,5) → (1,5) ∈ R
(2,6) & (6,6) → (2,6) ∈ R
(2,6) & (6,2) → (2,2) ∈ R
(2,2) & (2,6) → (2,6) ∈ R

(2,2) → (2,2) ∈ R

7) Symmetric Relation.

$X = \{a, b, c, d\}$

$R = \{(a,a), (b,c), (c,b)\ (d,d)\}$

$(a,a) \in R$ & $(a,a) \in R$

$(b,c) \in R$ & $(c,b) \in R$

$(d,d) \in R$ & $(d,d) \in R$

So R is symmetric to set X

If $R = \{(a,a)\ (b,c), (c,b) (d,d) (a,b) (a,d)\}$

here $(a,b) \in R$ but $(b,a) \notin R$

Hence R is not symmetric to set X.

8) Antisymmetric relation

$R = \{(2,1), (2,3), (1,1)\}$

Condition $(x,y) \in R$ & $x \neq y$ then $(y,x) \notin R$

if $(x,y) \in R$ & $x = y$ then $y, x \in R$.

Hence R is Anti symmetric

9) Transitive relation

$R = \{(a,a), (b,c), (c,b) (d,d).\}$

So Here Set be $X = \{a, b, c, d\}$

6) Define the terms POSET and Hasse Diagram. Determine if the set $S = \{2, 4, 8, 16\}$ with the Divisibility relation / is a partially ordered set and Draw Hasse Diagram [10 Marks]

Ans :- POSET

If "$\leq$" is a partial ordering on a set P then order pair $(P, \leq)$ is called POSET.

Hasse Diagram.

Hasse Diagram is a pictorial representation of POSET (Partial ordered set) POSET is denoted by $(A, \leq)$

Procedure
1. Draw diagraph of POSET
2. Remove self loops
3. Remove transitive edges
4. Arrange all edges upwards
5. Remove all arrows heads
6. Each Vertex is represented by dot.

Draw the Hasse Diagram for $S = \{2, 4, 8, 16)$ for Divisibility

$S = \{(2,4), (2,8) (2,16) (4,8), (4,16) (8,16)\}$

7 a) Define a Semigroup and Monoid. Give an example of a Monoid which is not group. Justify your answer (6 Marks)

Ans:- A Semi group is a set with an associative binary operation. A monoid is a semigroup with an identity element. The set of positive integers under multiplication is a monoid but not a group.

A Semigroup is an algebraic structure consisting of a set S and a binary operation that satisfies the associative property.

For all $a, b, c \in S$ $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

A monoid is a semigroup with an identity element in addition to associativity. It satisfies the identity property.

There exists an element $e \in M$ such that for every $a \in M$, $e \cdot a = a \cdot e = a$

The set of positive integers Z^+ under multiplication

→ It is a monoid because multiplication is associative, and the Identity element $e = 1$ is in Z^+

→ It is not a group because not every element has an inverse within the set. For Instance, the element 2 has an inverse of $1/2$ under multiplication but $1/2$ is not a positive integer so, it is not in the set Z^+.

7b) Let $G = \{-1, 0, 1\}$ Verify whether G forms a group under addition. (4 Marks)

Ans:

Given that

$$G = \{-1, 0, 1\}$$

Verify whether G forms a group under addition

→ check for closure

The set $G = \{-1, 0, 1\}$ must be closed under addition

- Consider the sum of 2 elements $1 + 1 = 2$.
- The result 2 is not in the G
- Therefore (∴) ~~G is~~

the set G is not closed under addition

8Q) State and Prove Binomial theorem (10 Marks)

Ans

The Binomial theorem states that for any positive integer n, the expansion of $(a+b)^n$ is given by the formula

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$ where

$\binom{n}{k}$ is the binomial coefficient defined as

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Proof:

Step 1: $(a+b)^n$

if $n = 1$
no of terms = 2

if $n = 2$
no of terms = 3

if $n = 3$
no of terms = 4

if $n = n$
no of terms in expansion is $(n+1)$

Step 2

If you add the powers of every term, should be equal to n ie
Adding powers of every term should be equal to n

for ex:
In $(a+b)^3 = a^3b^0 + 3a^2b^1 + 3a^1b^2 + a^0b^3$

Step 3

for every term a power will decrease and b power will increase

ie
Starting "a" with power n & b with power 0
From the next term power of "a" decreased by 1 and power of "b" increased by 1.

Step 4

Coefficients Calculation

$^nC_0$ = first term = $^3C_0 = \frac{3!}{(3-0)!\,0!} = \frac{3!}{3!} = 1$

$^nC_1$ = Second term = $^3C_1 = \frac{3!}{(3-1)!\,1!} = \frac{3\times 2!}{2!} = 3$

$^nC_2$ = Third term = $^3C_2 = \frac{3!}{(3-2)!\,2!} = \frac{3\times 2!}{2!} = 3$

⋮

^nC_n = n^{th} term

$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \dots + {}^nC_n b^n$

$$\boxed{(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r}$$

9a) Express how many ways are there to seat 10 boys and 10 girls around a circular table if boys and girls ~~are~~ seat alternatively (5 Marks)

Ans.

Total ways to seat 20 people around a circullar table is

The no of ways to arrange n district objects in a circle is $(n-1)!$

Total ways $= (20-1)! = 19!$

Total number of ways to seat 10 boys and 10 girls alternatively

First seat the 10 boys in a circle $(10-1)!$ ways i.e $9!$ ways.

This creates 10 spaces between them

Next, seat 10 girls in the 10 spaces

The girls can be arranged in those specific spaces in $10!$ ways.

ways to seat alternatively is

$9! \times 10!$ ways

9b) The letters of the word VICTORY are rearranged in all possible ways and the words thus obtained are arranged in a dictionary, what is the rank of the given word. (5 Marks).

Ans: Rank of the word VICTORY

The letters of the word VICTORY are

C I O R T V Y the total number of letters (n) = 7

→ Counting the words starting with V.

For each of these letters, the remaining letters can be arranged in 6! ways

no of words starting with C : 6! = 720

" " " " " I : 6! = 720

O : 6! = 720

R : 6! = 720

T : 6! = 720

Total words before V = 5 × 6! = 3600

→ Counting the words starting with VI

no of words starting with VC = 5! = 120.

→ Counting the words with VIC.

No of words starting with VICO = 3! = 6

VICR : 3! = 6

Total words before VICT = 2 × 3! = 12

→ Counting words starting with VICT

No of words starting with VICTO : 2! = 2

No " " " " VICTR : 2! = 2!

Total words before VICTO = 2 × 2! = 4

→ Counting words starting with VICTO

no of words starting with VICTOY!!! = 1.

→ Count the words starting with VICTOR

The word VICTORY is the first word.

→ Calculating the Rank.

Rank = (words before V) + (words before VICT)
+ (words befor VICTO) + (words before VICTOR) + 1

Rank = 3600 + 12 + 4 + 1 + 1 = $\underline{\underline{3618}}$

The Rank of the word VICTORY is 3618

10Q) In any Planar graph, show that
$|V| - |E| + |R| = 2$. Prove that complete graph of
5 vertices is non Planar (10 Marks).

Ans
Prove that $|V| - |E| + |R| = 2$

step 1: Euler's formula
Euler's formula for any connected Planar
graph is given by
$|V| - |E| + R = 2$ where V is the
no of Vertices, $|E|$ is the no of edges R is the no of
Regions. (faces)

→ K_5 is non-Planarity

To prove that a Complete graph with 5 Vertices (K_5) is non-planar. Following Conditions for Planar graphs are used.

Euler's formula = $V - E + R = 2$

for a Simple planar graph with no loops or multiple edges. every region must be bounded by atleast 3 edges so $2|E| \geq 3R$

Step 1: Calculate edges in K_5

for K_5, $V = 5$ The no of edges in a Complete graph is given by

$$\frac{|V|(|V|-1)}{2}$$

$$E = \frac{5(5-1)}{2} = \frac{20}{2} = 10$$

Step 2 Apply Euler's formula

Substitute $V = 5$ & $E = 10$ into Euler's form to find the required no of regions R

$$5 - 10 + R = 2$$
$$-5 + R = 2$$
$$R = 7$$

Step 3 : check the region condition.

Now check if the condition $2|E| \geq 3R$ holds for K_5

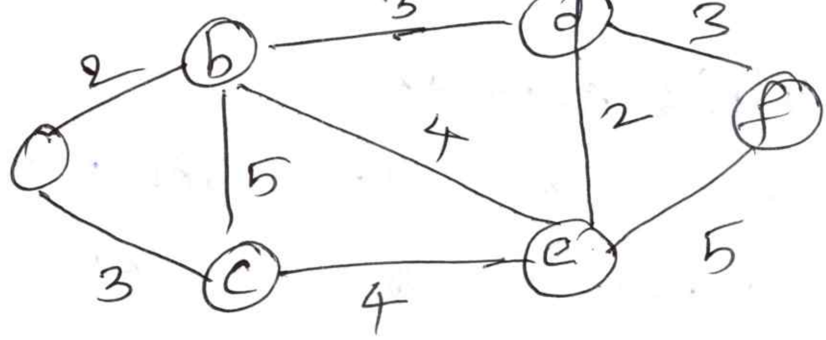
$$2(10) \geq 3(7)$$

$$20 \geq 21$$

This statement is false. The number of edges required to bound 7 regions is 21 but K_5 only has 20 (regions) edges.

Since, K_5 does not satisfy the necessary condition for a planar graph. It is non-planar.

11 Q) Apply Kruskal's algorithm and Prim's Algorithm to obtain minimal spanning tree and also find minimal cost (10 Marks)



Ans: Kruskal's Algorithm

Step 1: Sort edges by weight

Sort all the edges in non-decreasing order of their weights

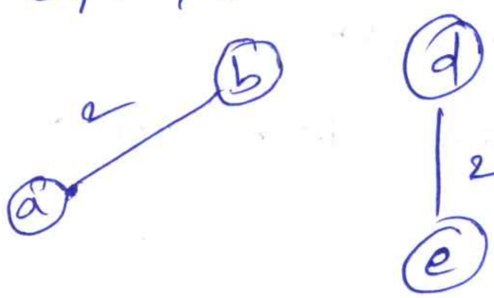
S.No	Edges	Weight
1	a, b	2
2	d, e	2
3	b, d	3
4	d f	3
5	a, c	3
6	b, c	5
7	c, e	4
8	e, f	5
9	b, e	4

Step 2 Build MST

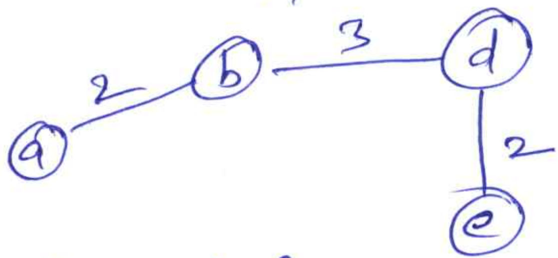
Add edges one by one to the MST if they do not form a cycle

→ Add ab

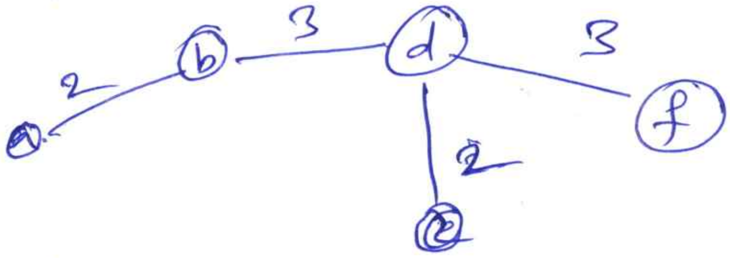
add edges d, e



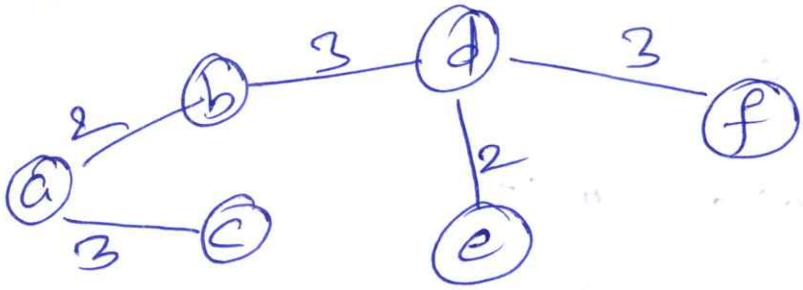
add edges b, d.



add edges d, f



add edges a, c



skip the edges which forms cycle.

step 3: Calculate Minimal Cost

The edges in the MST are (a, b), (d, e)(b, d) (d, f) and (a, c). Kruskal's Algorithm is the minimal cost using

$2+2+3+3+3 = \underline{13}$

Prim's Algorithm

Step 1:
Start with vertex a V_{MST} = a Cost 0

Step 2 Add next minimum edges

→ edges from (a): (a,b) weight 3
(a,c) " 3

choose (a,b) V_{MST} = (a,b) Cost = 3

→ Edges from (a,b), (a,c) weight 3
(b,d) weight 3
(b,e) " 4
(b,c) " 5

choose (a,c) or (b,d)

choose (a, c) V_{MST} = (a,b,c) cost 3+3=6

→ Edges from (a,b,c) (b,d) weight 3
(c,e) weight 4
(b,e) weight 4
(b,c) " 5

Choose b,d V_{MST} = (a,b,c,d) cost 6+3=9

→ Edges from (a, b, c, d) (d, e) weight 2

(d, f) weight 3

(c, e) weight 4

(b, e) " 4

(b, c) " 5

Choose (d e) V_{MST} = (a, b, c, d, e) cost 9 + 2 = 11

→ Edges from (a, b, c, d, e) (d, f) weight 3

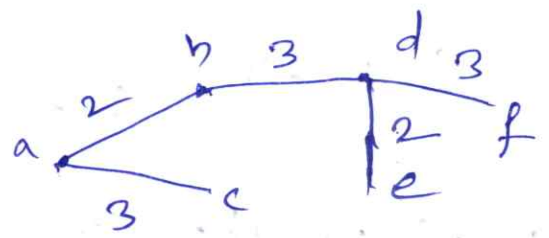
(c, e) weight 4

(b, e) " 4

(e, f) " 5

Choose (d, f) V_{MST} = (a, b, c, d, e, f)

Cost = 11 + 3 = 14.



Min Cost = 2 + 3 + 3 + 3 + 2 = 13

The minimal cost using prim's Algorithm

is 2 + 3 + 3 + 2 + 3 = 13