Code No.: MA101BS	R20	H.T.No.			8	R					

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–I–Semester End Examinations (Supply) - December- 2025
LINEAR ALGEBRA AND CALCULUS
(Common for all)

[Time: 3 Hours] **[Max. Marks: 70]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 20 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A (20 Marks)

1. a) Define Rank of a matrix. [2M]
b) Find the rank of the Matrix [2M]

$$\begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}.$$

c) State Cayley-Hamilton theorem. [2M]
d) Define Symmetric Matrix and Skew Symmetric matrix. [2M]
e) Explain P –test and give example. [2M]
f) Explain about Convergent and Divergent of the series. [2M]
g) State Lagrange's mean value theorem. [2M]
h) Define Beta function. [2M]
i) If $u = \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. [2M]
j) Define Functional dependence. [2M]

PART-B (50 Marks)

2. Discuss for what values of λ and μ the equations $x+y+z=6, x+2y+3z=10, x+2y-\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. [10M]

OR

3. For the Matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ find the inverse of the matrix by elementary row operations (Gauss Jordan method). [10M]

4. Using Cayley-Hamilton theorem and find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. [10M]

OR

5. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the Canonical form by Orthogonal reduction and find the nature of quadratic form. [10M]

6. Explain Comparison test with example and test the convergence of $\sum \sqrt{n^4+1} - \sqrt{n^4-1}$ [10M]

OR

7. Write about Ratio test and test the convergence $1 + \frac{2x}{2!} + \frac{3^2}{3!}x^2 + \frac{4^2}{4!}x^3 + - - - - - -$ [10M]

-36
-11 36 36
2 | 1 0 2 -18 0
1 -9 18
2(14)+1(-3+1)+1(4-5) 14
42-2-4 15-1 14
+9-1 +15-1 28
9

8. Write the geometrical representation of Rolle's theorem and Verify Rolle's theorem for $f(x) = \frac{\sin x}{e^x}$ in $[0,\pi]$. [10M]

OR

9. Derive the relationship between Beta and Gamma functions. [10M]

10. If $u = log(x^3 + y^3 + z^3 - 3xyz)$. Show that $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 u = \frac{-9}{(x+y+z)^2}$. [10M]

OR

11. Examine the function $x^3 + y^3 - 3axy$ for maxima and minima. [10M]

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$$\begin{pmatrix} +1 & -1 & +1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & -2 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & -1 & 1 \\ 0 & -1 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (0$$

$x + 2 = 0$

$x = -2$

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

CMREC (R-20)

I B Tech I Sem End Exam (Supply) Dec- 2025

Linear Algebra and Calculus

~~Key~~ Scheme of Evaluation

PART-A

1 (a) Definition (2M)

(b) Echelon form (1M)

for Rank (1M)

(c) statement (2M)

(d) Definition -1 (1M)

-2 (1M)

(e) statement (1M)

Example (1M)

(f) Definition - 1 (1M)

-2 (1M)

(g) statement (2M)

(h) Definition (2M)

(i) Differentiation (1M)

Solution (1M)

(j) Definition (2M)

PART-B

(2) Matrix form & Echelon form - (5M)

Explanation - (5M)

(3) Solution — (10M)

(4) find A^{-1} by Cayley Hamilton theorem — (10M)

(5) Eigen values
Eigen Vectors } (5M)
the matrix P (orthogonal)
Canonical form
Nature of the QF } (5M)

(6) Definition (2M)
Solution of the Series (8M)

7) Definition (2M)
Solution of the Series (8M)

8) Definition (3M)
Solution of the Verification (7M)

9) proving the relation (10M)

10) differentiation (3M)
proving the statement (7M)

11) Differentiation
Stationary points } (5M)

Max & Min Values of the function } (5M)

CMREC R-20

I B.Tech I Sem End Exam (Supply) Dec-2025

Linear Algebra and Calculus

Key paper

PART-A

1 (a) The number of non zero rows in a echelon (or) Normal form of the matrix is called the Rank of the matrix.

(or)

Highest Non zero minor of the matrix

(b) Let the matrix $A = \begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$

$R_2 \longrightarrow R_2 + 3R_1$

$R_3 \longrightarrow R_3 + 5R_1$

$\sim \begin{bmatrix} -1 & 0 & 6 \\ 0 & 6 & 19 \\ 0 & 1 & 33 \end{bmatrix}$ $R_3 \longrightarrow 6R_3 - R_2$

$\sim \begin{bmatrix} -1 & 0 & 6 \\ 0 & 6 & 19 \\ 0 & 0 & 179 \end{bmatrix}$ 198 - 19

$\rho(A) = 3$.

(c) Every Square matrix is Satisfies its own Characteristic equation.

(d) Symmetric matrix : $A = A^T$

Skew symmetric matrix $-A = A^T$

(e) p test $\Sigma \frac{1}{n^p} = 1 + \frac{1}{n} + \frac{1}{n^2} + \text{---} \text{---}$

(i) if $p > 1$ the series convergent

(ii) if $p \leq 1$ the series divergent

Example :- $\Sigma \frac{1}{n^2}$ $p = 2 > 1$

the series convergent

(f) Convergent series :-

a series Σa_n is said to be convergent if $\lim_{n \to \infty} a_n = c$

divergent series :-

a series Σa_n which is not convergent ~~is~~ called

divergent series

(g) LMVT :- (i) $f(x)$ is continuous on $[a, b]$

(ii) $f(x)$ is derivable on (a, b)

then there exists at least one c in (a, b)

such ~~that~~ $f'(c) = \frac{f(b) - f(a)}{b - a}$

(h) Beta function :-

if $m > 0, n > 0$ then $B(m, n) = \int_{x=0}^{1} x^{m-1} (1-x)^{n-1} dx$

is called Beta function

(i) $u = \frac{y}{z} + \frac{z}{x}$ is a homogeneny of degree $n=0$

by Euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu$$

$$= (0)u$$

$$= 0.$$

(j) the functions u & v are functionally dependence if the jacobean vanishes.

PART-B

2) Given System is $x+y+z=6$

$x+2y+3z=10$

$x+2y+\lambda z=\mu$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{pmatrix} \begin{matrix} \to R_1 \\ \to R_2 \\ \to R_3 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{pmatrix} \quad \begin{matrix} R_2 \to R_2-R_1 \\ R_3 \to R_3-R_1 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{pmatrix} \quad R_3 \to R_3-R_2$$

(i) if $\lambda-3=0$ $\quad \mu-10 \neq 0$

$\lambda=3$ $\quad \mu \neq 10$

No Solution

(ii) if $\lambda \neq 3$ $\quad \mu \neq 10$ (or $\mu=10$

Unique Solution

(iii) if $\lambda-3=0$ $\quad \mu-10=0$

$\lambda=3$ $\quad \mu=10$

Infinitely many Solution.

3) the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

$$\sim [A : I] \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right)$$

$$R_2 \longrightarrow R_2 - R_1 \quad R_3 \longrightarrow R_3 + 2R_1$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & +2 & 2 & 0 & 1 \end{array}\right)$$

$-6-6$

$$R_1 \longrightarrow R_1 - 2R_2 \quad R_3 \longrightarrow R_3 + R_2$$

$$\sim \left(\begin{array}{ccc|ccc} 2 & 0 & -12 & 3 & -2 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right)$$

$2-3$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -5 & -3 \\ 0 & 2 & 0 & -5 & -1 & -3 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right)$$

$$A^{-1} = \begin{bmatrix} 0 & -5 & -3 \\ -5/2 & -1/2 & -3/2 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

4) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$a_1 = \text{Trace } A = 1 - 2 + 3 = 2$

$a_2 = -6 + 3 - 2 = -5$

$a_3 = 1(-2)(3) = -6$

the characteristic equation of A is $\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$

$$\lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$$

by Cayley Hamilton theorem

$$A^3 - 2A^2 - 5A + 6I = 0.$$

Multiply by A^{-1}

$$A^{-1}(A^3 - 2A^2 - 5A + 6I) = 0$$

$$A^2 - 2A - 5I + 6A^{-1} = 0$$

$$A^{-1} = \frac{1}{6}(-A^2 + 2A + 5I)$$

$$A^2 = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -1 & 8 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6}\left(\begin{bmatrix} -1 & 1 & -8 \\ 0 & -4 & 0 \\ 0 & 0 & -9 \end{bmatrix} + 2\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} + \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}\right)$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 6 & 3 & -4 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(5) $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

the characteristic equation is $\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$.

$$(\lambda - 2)(\lambda^2 - 9\lambda + 18) = 0$$

$$(\lambda - 2)(\lambda - 3)(\lambda - 6) = 0.$$

$\lambda = 2, 3, 6$ are eigen values

Case (i) $\lambda = 2$ $(A - \lambda I)X = 0$

$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ is an eigen Vector for $\lambda = 2$

$$e_1 = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{pmatrix}$$

(ii) $\lambda = 3$

$\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$ is an eigenvector $\begin{pmatrix} -1/\sqrt{3} \\ -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{3}} \end{pmatrix} = e_2$

(iii) $\lambda = 6$

$\begin{pmatrix} \pm \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector $\begin{pmatrix} +1/\sqrt{6} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix} = e_3$

The matrix $P = [e_1 \; e_2 \; e_3]$ is an orthogonal matrix

$$P^T A P = D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= 2y_1^2 + 3y_2^2 + 6y_3^2$$

be the canonical form.

positive definite of the matrix.

(6) $\Sigma u_n \leq \Sigma v_n$ If Σu_n is cgt then Σv_n is cgt

$\Sigma u_n \geq \Sigma v_n$ if Σu_n is dgt then Σv_n is dgt

$$\Sigma \sqrt{n^4+1} - \sqrt{n^4-1}$$

$$\text{let } a_n = \sqrt{n^4+1} - \sqrt{n^4-1} \times \frac{\sqrt{n^4+1} + \sqrt{n^4-1}}{\sqrt{n^4+1} + \sqrt{n^4-1}}$$

$$= \frac{\cancel{n^4}+1 - \cancel{n^4}+1}{\sqrt{n^4+1} + \sqrt{n^4-1}}$$

$n^4+1 \neq n^4$

$n^4+1 \approx n^4$ $\frac{1}{\sqrt{n^4}} = 1/n^2$

$$= \frac{2}{\sqrt{n^4+1} + \sqrt{n^4-1}}$$

$$= \frac{2}{n^2\left(\sqrt{1+1/n^4} + \sqrt{1 - \frac{1}{n^4}}\right)}$$

$v_n = \frac{1}{n^2}$ v_n is cgt by p series

$p = 2 > 1$

by Comparison test

u_n is cgt

(7) $\cancel{\sum \frac{1}{r^n} = 1 + \frac{1}{r} + \frac{1}{r}}$

<u>G.S</u>: $\sum r^n = 1 + r + r^2 + -$

if $r < 1$ cgt

if $r > 1$ dgt

ratio test: —

$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = l$

(i) $l > 1$ $\sum a_n$ is cgt

(ii) $l < 1$ $\sum a_n$ is dgt

(iii) $l = 1$ test fails

the series $1 + \frac{2x}{2!} + \frac{3^2}{3!} x^2 + \frac{4^3}{4!} x^3 + - - -$

$$= \sum \frac{n^{n-1}}{n!} x^{n-1}$$

$$a_n = \frac{n^{n-1} x^{n-1}}{n!} \qquad a_{n+1} = \frac{(n+1)^n x^n}{(n+1)!}$$

$$\lim \frac{a_{n+1}}{a_n} = \frac{(n+1)^n x^n}{n^{n-1} x^{n-1}} \times \frac{n!}{(n+1)!}$$

$$= \frac{(n+1)^n}{n^n n^{-1}} \times \frac{x}{(n+1)}$$

$$= \frac{(n+1)^{n-1}}{n^{n-1}} \times x$$

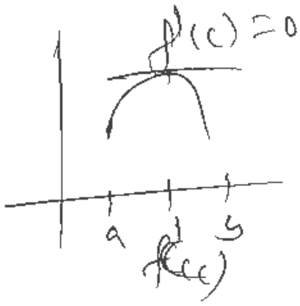
$$= \left(1 + \frac{1}{n}\right)^{n-1} x \longrightarrow \cancel{ex}\ ex$$

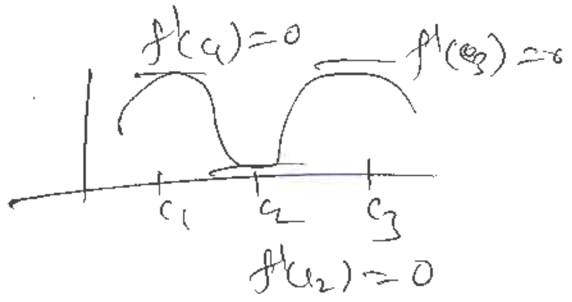
ratio test $ex < 1$ cgt $\quad x > 1/e$ dgt

$x < \frac{1}{e}$ cgt

$x = 1/e$ test fails

8)





$$f(x) = \frac{\sin x}{e^x} \quad \text{on } [0,\pi]$$

(i) $f(x)$ is Continuous on $[0,\pi]$

(ii) $f(x)$ is derivable on $(0,\pi)$

$$f(0) = \frac{\sin 0}{e^0} = \frac{0}{1} = 0$$

$$f(\pi) = \frac{\sin\pi}{e^\pi} = \frac{0}{e^\pi} = 0$$

$$f(0) = f(\pi)$$

$$f(x) = \frac{\sin x}{e^x}$$

$$f'(x) = \frac{e^x \cos x - \sin x \, e^x}{(e^x)^2}$$

$\exists\, c \in (0,\pi) \ni f'(c) = 0$

$$\frac{e^c \cos c - e^c \sin c}{(e^c)^2} = 0$$

$$e^c(\cos c - \sin c) = 0$$

$e^c \neq 0$ for any c $\quad \cos c - \sin c = 0$

$$\tan c = 1$$

$$c = \pi/4 \in (0,\pi)$$

Verified the Rolles theorem

(9) by def $\Gamma(m) = \int_0^\infty e^{-x} x^{m-1}\, dx$

put $x = yt$

$dx = y\,dt$

$$\Gamma(m) = \int_0^\infty e^{-yt} y^{m-1} t^{m-1} y\, dt$$

$$\frac{\Gamma(m)}{y^m} = \int_0^\infty e^{-yx} x^{m-1}\, dx$$

multiply $e^{-y} y^{m+n-1}$ & Integrating w.r.t y from 0 to ∞

$$\Gamma(m) \int_0^\infty e^{-y} y^{n-1}\, dy = \int_0^\infty \left\{\int_0^\infty e^{-y(1+x)} y^{m+n-1} x^{m-1}\, dx\right\} dy$$

$$\Gamma(m)\,\Gamma(n) = \int_0^\infty \left(\int_0^\infty e^{-y(1+x)} y^{m+n-1}\, dy\right) x^{m-1}\, dx$$

$$= \int \frac{\Gamma(m+n)}{(1+x)^{m+n}} x^{m-1}\, dx$$

$$\Gamma(m)\Gamma(n) = \Gamma(m+n) \quad B(m,n)$$

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

(10) $u = \log(x^3+y^3+z^3-3xyz)$

$$\frac{\partial u}{\partial x} = \frac{3x^2-3yz}{x^3+y^3+z^3-3xyz}$$

$$\frac{\partial u}{\partial y} = \frac{3y^2-3xz}{x^3+y^3+z^3-3xyz}$$

$$\frac{\partial u}{\partial z} = \frac{3z^2-3xy}{x^3+y^3+z^3-3xyz}$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

(11) $f(x,y) = x^3+y^3-3axy$

$f_x = 3x^2-3ay = 0 \qquad f_y = 3y^2-3ax = 0$

Solving $\quad 3x(x^3-a^3) = 0$

$x=0 \quad x=a$

$\Rightarrow y=0 \quad y=a$

The stationary points are $(0,0)$ (a,a)

$r = f_{xx} = 6x \quad s = f_{xy} = -3a \quad t = f_{yy} = 6y$

at $(0,0)$ $\quad rt-s^2 = (6x)(6y)-(3a)^2$

$= 36xy - 9a^2 < 0$

at (a,a) $\quad rt-s^2 = 36a^2-9a^2 = 27a^2 > 0$

$r = 6a > 0$. If $a > 0$

$r = 6a < 0$ If $a < 0$

If $a<0$ then $z = -a^3$ the maximum. If $a>0$ then $z = a^3$ the minimum