

CMR ENGINEERING COLLEGE: : HYDERABAD

UGC AUTONOMOUS

I-B.TECH-I-Semester End Examinations (Supply) - December- 2025

LINEAR ALGEBRA AND CALCULUS

(Common for all)

[Time: 3 Hours]

[Max. Marks: 70]

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 20 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A**(20 Marks)**

1. a) Define Rank of a matrix. [2M]
- b) Find the rank of the Matrix [2M]

$$\begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$$
- c) State Cayley-Hamilton theorem. [2M]
- d) Define Symmetric Matrix and Skew Symmetric matrix. [2M]
- e) Explain P-test and give example. [2M]
- f) Explain about Convergent and Divergent of the series. [2M]
- g) State Lagrange's mean value theorem. [2M]
- h) Define Beta function. [2M]
- i) If $u = \frac{y}{x} + \frac{z}{x}$, find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$. [2M]
- j) Define Functional dependence. [2M]

PART-B**(50 Marks)**

2. Discuss for what values of λ and μ the equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. [10M]

OR

3. For the Matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ find the inverse of the matrix by elementary row operations (Gauss Jordan method). [10M]

4. Using Cayley-Hamilton theorem and find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. [10M]

OR

5. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the Canonical form by Orthogonal reduction and find the nature of quadratic form. [10M]

6. Explain Comparison test with example and test the convergence of $\sum \sqrt{n^4 + 1} - \sqrt{n^4 - 1}$. [10M]

OR

7. Write about Ratio test and test the convergence $1 + \frac{2x}{2!} + \frac{3^2}{3!}x^2 + \frac{4^3}{4!}x^3 + \dots$. [10M]

8. Write the geometrical representation of Rolle's theorem and Verify Rolle's theorem [10M]
for $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$.

OR

9. Derive the relationship between Beta and Gamma functions. [10M]

10. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$. Show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$. [10M]

OR

11. Examine the function $x^3 + y^3 - 3axy$ for maxima and minima. [10M]
