Code No.: R25MA101BS	R25	H.T.No.			8	R						

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–I–Semester End Examinations (Regular) - December - 2025
MATRICES AND CALCULUS
(Common for ECE, CSM & CSD)

[Time: 3 Hours] **[Max. Marks: 60]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 10 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A **(10 Marks)**

1. a) Find the value(s) of k such that Rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & k \end{bmatrix}$ is 2. [2M]

b) If λ is an Eigen value of a matrix A corresponding to the Eigen vector X then prove that λ^2 is an Eigen value of a matrix A^2 corresponding to the eigenvector X. [2M]

c) Write the geometrical interpretation of Rolle's theorem. [2M]

d) If $x = r\cos\theta,\ y = r\sin\theta$ then find $\frac{\partial x}{\partial \theta}$ and $\frac{\partial \theta}{\partial x}$. [2M]

e) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\, dx\, dy$. [2M]

PART-B **(50 Marks)**

2. Find the rank of the matrix, by reducing into the normal form $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$. [10M]

OR

3. Solve the following system of equations by Gauss –Seidal iteration method correct to three decimal places $8x - 3y + 2z = 20, 4x + 11y - z = 33, 6x + 3y + 12z = 35$ [10M]

4. Verify the Cayley Hamilton theorem and find A^{-1} for $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$. [10M]

OR

5. Reduce the quadratic form $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ to the canonical form by an orthogonal transformation. [10M]

6.a) Verify Rolle's theorem for the function $(x-a)^m(x-b)^n$ where $m,n>0$ in $[a,b]$. [5M]

b) Obtain the Taylor's series expansion of $\sin 2x$ in powers of $x-\frac{\pi}{4}$. [5M]

OR

7. State Lagrange's mean value theorem. If $a<b$ prove that $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}$ using Lagrange's mean value theorem. Deduce the following [10M]

(i) $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$

(ii) $\frac{\pi}{4}+\frac{1}{5}<\tan^{-1}2<\frac{\pi}{4}+\frac{1}{2}$

8. Verify Euler's theorem for $u=x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$ and prove that [10M]

$\frac{\partial^2 u}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}$

OR

9. A rectangular box open at the top is to have volume of 32 cubic *ft.* find the dimensions of the box requiring least material for the construction. [10M]

10. By changing the order of integration, evaluate $\int_0^1\int_{x^2}^{2-x}xy\,dxdy$. [10M]

OR

11. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y}e^{x+y+z}\,dz\,dy\,dx$. [10M]

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