CMR Engineering College

II MTech I Semester End Examinations (Reg) Jan-26

Operation Research (R22CS58150E)

Key Paper

PART - A

1 (a) Degeneracy occurs in the simplex method when one or more basic variables take the value zero in a basic feasible solution. (1M)

(b) A slack variable is added to a $\leq$ type constraints to convert it into an equality and represents unused resources. (1M)

(c) The graphical method is limited to linear programming problems with only two decision variables. (1M)

(d) Artificial variables are introduced to obtain an initial basic feasible solution when none exists naturally. (1M)

(e) Kuhn - Tucker conditions are necessary optimality conditions for solving nonlinear programming problems with inequality constraints. (1M)

f) optimistic time is the minimum possible time required to Complete an activity assuming ideal Conditions. (1M)

g) Johnson's algorithm is used to Solve Sequencing problems (especially two machine problems) (1M)

h) Safety stock in the extra inventory maintained to protect against demand (or) Supply uncertainties. (1M)

i) The principle of optimality states that an optimal Solution Contains optimal Solutions to its Sub problems. (1M)

j) Game theory helps in optimal decision making under Competitive and Conflicting Situations (1M)

PART-B

2) Let $x_1\ x_2\ x_3\ x_4$ be the number of parts A, B, C and D manufacured respectively

The objective function is

$$\text{Maximize } Z = 12x_1 + 8x_2 + 14x_3 + 10x_4$$

Subject to resource constraints (2M)

$$5x_1 + 4x_2 + 2x_3 + x_4 \le 100$$

$$2x_1 + 3x_2 + 8x_3 + x_4 \le 75$$

$$x_1\ x_2\ x_3\ x_4 \ge 0.$$

Slack variables S_1 & S_2

$$5x_1 + 4x_2 + 2x_3 + x_4 + S_1 = 100$$

$$2x_1 + 3x_2 + 8x_3 + x_4 + S_2 = 75$$

	C_B	Basis	RHS	x_1	x_2	x_3	x_4	S_1	S_2	
$R_1 \leftarrow$	0	S_1	100	5	4	2	1	1	0	$\frac{100}{2}$
$R_2 \leftarrow$	0	S_2	75	2	3	8	1	0	1	$\frac{75}{8}$ Min
		z_j		0	0	0	0	0	0	
		$c_j - z_j$		12	8	14	10	0	0	
						↑ Max				

(3M)

$R_2 \longrightarrow \frac{R_2}{8}$ $\qquad S_2$ leaves the basis

		C_j	12	8	14	10	0	0
C_B	Basis	RHS	x_1	x_2	x_3	x_4	S_1	S_2
0	S_1	$\frac{325}{4}$	$\frac{9}{2}$	$13/4$	0	$3/4$	1	$-1/4$
14	x_3	$\frac{75}{8}$	$2/8$	$3/8$	1	$1/8$	0	$1/8$

x_3 uses large zinc

zinc constraints becomes tight quickly

producing x_3 limit profit compared to x_4

$\frac{\text{Profit}}{\text{Resource usage}} \Rightarrow x_4$ is most efficient

The simplex iterations finally gives (5M)

$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0 \quad x_4 = 75$

Maximum profit $Z_{max} = 10(75)$

$= 750$ Rs.

3) Maximize $z = 3x_1 + 4x_2$

$$x_1 - x_2 \le 1$$

$$-x_1 + x_2 \le 2$$

Slack variables S_1, S_2

objective function $z = 3x_1 + 4x_2 + 0S_1 + 0S_2$

$$x_1 - x_2 + S_1 = 1$$

$$-x_1 + x_2 + S_2 = 2$$

SM:- (5M)

		c_j	3	4	0	0	
C_B	Basis	RHS	x_1	x_2	S_1	S_2	
0	S_1	1	1	-1	1	0	-1
0	S_2	2	-1	1	0	1	2 ← positive
	z_j		0	0	0	0	
	$c_j - z_j$		3	4 ↑ max	0	0	

$R_1 \rightarrow R_1 + R_2$

		c_j	3	4	0	0	
C_B	Basis	RHS	x_1	x_2	S_1	S_2	
0	S_1	3	0	0	1	1	$\frac{3}{0} = \infty$ ←
4	x_2	2	-1	1	0	1	$\frac{2}{-1} = -2$
	z_j		-4	4	0	4	
	$c_j - z_j$		7 ↑	0	0	-4	

Since all S_1 entry undefined (5M)

the objective function can increase

indefinitely without violating the Contraints

The problem is unbounded solution

4) $\text{Max } Z = -2x_1 - 3x_2$

S.T

$-x_1 - x_2 \cancel{\geq} -2$

$2x_1 + x_2 \leq 10$

$x_1 + x_2 \leq 8$

Slack variables $S_1\ S_2\ S_3$ $\quad$ $\text{Max } Z = -2x_1 - 3x_2 + 0S_1 + 0S_2 + 0S_3$

$-x_1 - x_2 + S_1 = -2$

$2x_1 + x_2 + S_2 = 10$ (5M)

$x_1 + x_2 + S_3 = 8$

		C_j	-2	-3	0	0	0	
C_B	B	X_B	x_1	x_2	S_1	S_2	S_3	
$R_1 \leftarrow 0$	S_1	-2	-1	-1	1	0	0	2
$R_2 \leftarrow 0$	S_2	10	2	1	0	1	0	5
$R_3 \leftarrow 0$	S_3	8	1	1	0	0	1	8
		Z_j	0	0	0	0	0	
		$C_j - Z_j$	-2	-3	0	0	0	

$R_1 \longrightarrow (-1)R_1$

$R_2 \longrightarrow R_2 - 2R_1$

$R_3 \longrightarrow R_3 - R_1$

		c_j	-2	-3	0	0	0	
C_B	B	X_B	x_1	x_2	s_1	s_2	s_3	
-2	x_1	2	1	$\boxed{1}$	-1	0	0	2 ✗
0	s_2	6	0	$\boxed{-1}$	2	1	0	-6 ✗
0	s_3	$+6$	0	$\boxed{0}$	1	0	1	0
		z_j	-2	-2	2	0	0	
		$c_j - z_j$	0	-1	-2	0	0	

↑

No replacement of x_1 (as x_1 doesn't leave the position

$x_1 = 2 \quad x_2 = 0.$

(5M)

$\text{Max } Z = -2(2) - 0$

$\text{Max } Z = -4$

5) Equality Constraint in Simplex method

(i) Use the Big M method (or) Two phase method

To handle an equality Contraints in the simplex method you must introduce an artificial variable to serve as the initial basic variable for the row since the artificial variable is not part of the original problem. it must be driven to zero in the final solution. The Big M method assigns a very large penality (M) to the artificial variable in the objective function, While the two phase method solves a seperate auxiliary problem first to eliminate all artificial variables. (3M)

(ii) Unrestricted Variable in Simplex method

Replace with the difference of two non negative variables

An unrestricted (or free) variable x_j, Can take any real value (positive, negative (or zero)). To use the simplex method, all variables must be non-negative. The standard procedure is to replace the unstricted variables with two non negative variables x_j' and x_j'' such that $x_j = x_j' - x_j''$

are then included in the liner programming problem

Advantages and applications of duality :- (2M)

Provides economic insight, computational efficiency, and sensitivity analysis.

Duality in liner programming offers serveral advantges and applications.

(i) Economic Interpretation :- The dual variables (shadow prices) provide valuable economic insights into the ~~valuable economic~~ of the value of resources (or) contraints in the original (primal) problem.

(ii) Computational Efficiency :- Sometimes the dual problem is easier (or) faster to solve then prime problem. especially if the primal has many constraints and few variables

(iii) Sensitive analyses :- Duality theory is fundamental to sensitivity analysis, which examines how changes in the input parameters affect the optimal solution

(iv) Applications :- Duality is used in various fields including operations management (resource allocation) finance (portfolio optimization) and network flow problems. (5M)

6) Kuhn Tucker $\quad \text{Min } z = (x_1-1)^2 + (x_2-2)^2$

$$S.T. \quad -x_1 + x_2 = 1$$
$$x_1 + x_2 \leq 2$$

(2M)

$$F(x_1, x_2) = (x_1-1)^2 + (x_2-2)^2 + \lambda_1(-x_1+x_2-1) + \lambda_2(x_1+x_2-2)$$

Stationary Conditions

$$\frac{\partial F}{\partial x_1} = 2(x_1-1) - \lambda_1 + \lambda_2 = 0 \quad \text{--- } (1)$$

$$\frac{\partial F}{\partial x_2} = 2(x_2-2) + \lambda_1 + \lambda_2 = 0 \quad \text{--- } (2)$$

Complementary slackness

(3M)

$$\frac{\partial F}{\partial \lambda_1} = -x_1 + x_2 - 1 = 0$$
$$-x_1 + x_2 = 1 \quad \text{--- } (3)$$

$$\frac{\partial F}{\partial \lambda_2} = x_1 + x_2 - 2 = 0$$
$$x_1 + x_2 = 2 \quad \text{--- } (4)$$

Solving (3) & (4)

$$x_1 = \frac{1}{2} \qquad x_2 = \frac{3}{2}$$

from (1) & (2)

$$\left.\begin{aligned} -\lambda_1 + \lambda_2 &= 1 \\ \lambda_1 + \lambda_2 &= 1 \end{aligned}\right\} \quad \begin{aligned} \lambda_1 &= 0 \geq 0 \\ \lambda_2 &= 1 \geq 0 \end{aligned}$$

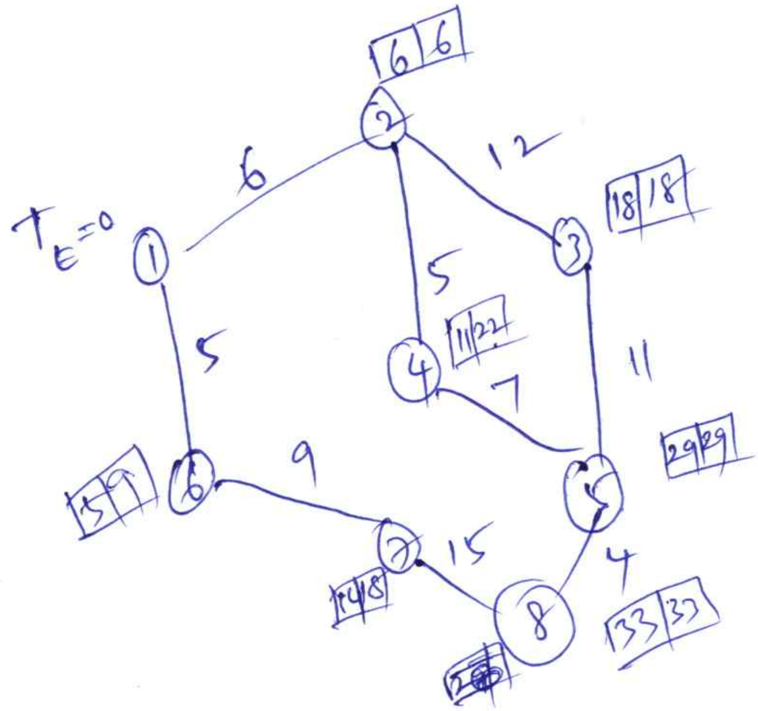
$$\text{Min } z = \left(\frac{1}{2}-1\right)^2 + \left(\frac{3}{2}-2\right)^2 \qquad (5M)$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

(7)

Activity i	j	Estimate t_o	t_m	t_p	$t_e = \frac{t_o+4t_m+t_p}{6}$	$\sigma = \frac{t_p-t_o}{6}$	σ^2
1	2	3	6	9	6	1	1
1	6	2	5	8	5	1	1
2	3	6	12	18	12	2	4
2	4	4	5	6	5	2/3	4/9
3	5	8	11	14	11	1	1
4	5	3	7	11	7	8/6 = 4/3	16/9
6	7	3	9	15	9	2	4
5	8	2	4	6	4	4/6	4/9
7	8	8	16	18	15	10/6	25/9

(5M)



Critical path

1 — 2 — 3 — 5 — 8

T_E = 33 weeks

(1M)

Project Variance σ_p^2

$1+4+1+\frac{4}{9} = \frac{58}{9}$

(2M)

standard deviation

$\sigma_p = \sqrt{\frac{58}{9}} = 2.54$ weeks

(2M)

8) The smallest element is 2 it is Machine - I (first Job A)

The smallest element is 3 it is Machine - II

E & G (Do this Job Last)

The smallest element is 4 (M - I) place early

C & I (After first Job)

Next smallest element is 5 B, H (M - I) place next

Next smallest element is 4 (M - II) place late

(5M)

Remaining Job is F

Optimal Job sequence A - C - I - B - H - F - D - G - E

Optimal seq (Tasks)	Machine - I In	Machine - I Out	Machine - II In	Machine - II Out
A	0	2	2	8
C	2	6	8	15
I	6	10	15	26
B	10	15	26	34
H	15	20	34	42
F	20	28	42	51
D	28	37	51	55
G	37	44	55	58
E	44	50	58	61

Minimum total elapsed time 61 hours (5M)

(9) $\text{Min } f = x_1 x_2^2 x_3^{-1} + 2x_1^{-1} x_2^3 x_4 + 10 x_3$

ST $3x_1^{-1} x_3^{-2} x_4 + 4x_3 x_4 \le 1$

$5x_1 x_2 \le 1$

GP form

objective function $f = T_1 + T_2 + T_3$

where $T_1 = x_1 x_2^2 x_3^{-1}$

$T_2 = 2x_1^{-1} x_2^3 x_4$

$T_3 = 10 x_3$

$g_1 = 3x_1^{-1} x_3^{-2} x_4 + 4x_3 x_4 \le 1$

$g_2 = 5x_1 x_2 \le 1$ (3M)

Degree of difficulty :-

DOD = (No of terms) − (No. of Variables) − 1

= 6 − 4 − 1

= 1

the problem can be solved using dual formulation

Let dual variables be $w_1 w_2 w_3$ objective terms

$w_4 w_5$ first constraints

w_6 second constraints (2M)

Normality Condition

$w_1 + w_2 + w_3 = 1$

orthogonality Condition $w_1 - w_2 - w_4 + w_6 = 0$

x_2: $2w_1 + 3w_2 + w_6 = 0$

x_3 $-w_1 + w_5 + w_3 - 2w_4 = 0$

x_4 $w_2 + w_4 + w_5 = 0$

Solving the above equations along with normality condition gives

$$w_1 = \frac{1}{6} \quad w_2 = \frac{1}{6} \quad w_3 = \frac{2}{3}$$

minimum value

$$f^* = \pi \left(\frac{c_i}{w_i}\right)^{w_i}$$

$$= \left(\frac{1}{\frac{1}{6}}\right)^{1/6} \left(\frac{2}{1/6}\right)^{1/6} \left(\frac{10}{2/3}\right)^{2/3}$$

$$f^* = 15$$ (5M)

the optimal solution is

$$x_1 = 0.2 \quad x_2 = 1 \quad x_3 = 0.1 \quad x_4 = 0.5$$

(10) three stages

stage 1 : Decision on x_1

2 : on x_2

3 : on x_3 Let total resoure = 12

at stage 3:— $x_3 = 12 - x_1 - x_2$

Cost function $f_3 = x_3^2 = (12 - x_1 - x_2)^2$

stage 2:— $R_2 = 12 - x_1$

$$f_2 = x_2^2 + (R_2 - x_2)^2$$

Diff wrt x_2 $\frac{\partial f_2}{\partial x_2} = 2x_2 - 2(R_2 - x_2) = 0$

$$x_2 = \frac{R_2}{2}$$

$$x_2 = \frac{12 - x_1}{2}$$ (5M)

stage 1:— $f_1 = x_1^2 + \left(\frac{12 - x_1}{2}\right)^2 + \left(\frac{12 - x_1}{2}\right)^2$

$$f_1 = x_1^2 + \left(\frac{12 - x_1}{2}\right)^2$$

Diff wrt x_1 $\frac{\partial f_1}{\partial x_1} = 2x_1 - (12 - x_1) = 0$

$$\boxed{x_1 = 4}$$

$$\boxed{x_2 = 4} \quad \boxed{x_3 = 4}$$

Minimal Value $f_{min} = 4^2 + 4^2 + 4^2$

$= 48$ (5M)

(11)

$$\begin{array}{cc|cccc} & & & P\text{-}B & & \\ & & I & II & III & IV \\ \hline & I & 3 & 2 & 4 & 0 \\ & II & 3 & 4 & 2 & 4 \\ P\text{-}A & III & 4 & 2 & 4 & 0 \\ & IV & 0 & 4 & 0 & 8 \end{array}$$

Sol:- This matrix has no saddle point

given payoff matrix by using the Concept of dominance.

row I is dominated by III

$$\begin{array}{c|cccc} & I & II & III & IV \\ \hline II & 3 & 4 & 2 & 4 \\ III & 4 & 2 & 4 & 0 \\ IV & 0 & 4 & 0 & 8 \end{array}$$

Column I is dominated by III.

$$\begin{array}{c|ccc} & II & III & IV \\ \hline II & 4 & 2 & 4 \\ III & 2 & 4 & 0 \\ IV & 4 & 0 & 8 \end{array}$$

(5M)

Column II is dominated by the average of Colums III & IV

$$\begin{array}{c|cc} & III & IV \\ \hline II & 2 & 4 \\ III & 4 & 0 \\ IV & 0 & 8 \end{array}$$

row II is dominated by the average of III & IV

$$\begin{array}{ccc} & \underline{\text{III}} & \underline{\text{IV}} \\ \underline{\text{III}} & 4 & 0 \\ \underline{\text{IV}} & 0 & 8 \end{array}$$

Solved by arithmetic method

$$\begin{array}{ccccc} & \underline{\text{III}} & \underline{\text{IV}} & & \\ \underline{\text{III}} & 4 & 0 & 8 & 2/3 \\ \underline{\text{IV}} & 0 & 8 & 4 & 1/3 \\ & 8 & 4 & & \\ & 2/3 & 1/3 & & \end{array}$$

optimal strategy for P-A $(0, 0, 2/3, 1/3)$

P-B $(0, 0, 2/3, 1/3)$ (5M)

Value of the game for A $\dfrac{8\times4+0\times4}{8+4} = 8/3$

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Operation Research

Scheme of evaluation

PART- A

1) (a) Explanation 1M

(b) definition 1M)

(c) Explanation 1M

(d) Explanation 1M

(e) Statement 1M

(f) Explanation 1M

(g) Algorithm 1M

(h) Definition 1M

(i) Explanation 1M

(j) Explanation 1M

PART- B

2) Formation of the problem 5M

and Solution of the problem 5M

3) the Solution of the problem (10M)

4) Solution of the problem 10M

5) Explanation of the Constraint 5M
& unrestricted variable 5M

6) Solution of the problem 10M

7) for problem solving 5M
time
variance
standard deviation } 5M

8) Explanation 5M
& Solution of the minimizes 5M

9) Solution by Geometric programing method 10M

10) Solution by dynamic programming 10M

11) Solution of the pay off matrix 10M)