Code No.:R22MA201BS | R22 | H.T.No. | | | 8 | R | | | | | |

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–II–Semester End Examinations (Supply) - January- 2025
VECTOR CALCULUS AND TRANSFORMS
(Common for all)

[Time: 3 Hours] **[Max. Marks: 60]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 10 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A **(10 Marks)**

1. a) Define the conditions under which the Laplace transform of f(t) exists. [1M]
   b) Define the Laplace transformation of periodic functions. [1M]
   c) Find $L^{-1}\{\frac{1}{s^{n+1}}\}$ [1M]
   d) Define Convolution product. [1M]
   e) Define the Gamma function. [1M]
   f) Write the relation between Beta and Gamma function. [1M]
   g) Define the Gradient of a scalar point function. [1M]
   h) Define Solenoidal Vector. [1M]
   i) State Gauss-divergence Theorem. [1M]
   j) State Green's Theorem. [1M]

PART-B **(50 Marks)**

2.a) $Find\ L\{\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^3\}$ [5 M]

b) $Find\ L\{e^{3t}\sin^2 t\}$ [5 M]

OR

3. Using Laplace transform, Evaluate [10M]
$$\int_0^\infty \frac{\cos at - \cos bt}{t}\,dt.$$

4. Using Convolution Theorem, find [10M]
$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$$

OR

5. Solve the differential equation $\frac{d^2x}{dt^2}+9x=\sin t$ [10M]
Using the Laplace given that x(0), x(π/2)=1.

6.a) Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx$ in terms of Beta function. [5M]

b) Evaluate $\int_0^\infty 3^{-4x^2} dx$ [5M]

OR

7. Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy \, dxdy$ and hence evaluate of double integral. [10M]

8.a) Prove that $\nabla(r^n)=n.r^{n-2}\bar{r}$ [5M]

b) Find the directional derivative of $f=xy^2+yz^3$ at the point (2,-1, 1) in the direction of the i+2j+2k. [5M]

OR

9. Show that the vector $(x^2-yz)i+(y^2-zx)j+(z^2-xy)k$ is Irrotational and Find its Scalar potential. [10M]

10. Verify Green Theorem in the plane for $\oint (x^2 - xy^3)dx + (y^2 - 2xy)dy$, Where C is Square with vertices (0,0), (2,0),(0,2) & (2,2). [10M]

OR

11. Verify Stoke's Theorem for $F=-y^3i+x^3j$, where S is the circular disc $x^2+y^2 \leq 1$, $z=0$. [10M]

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