Code No.: R22MA201BS	R22	H.T.No.			8	R						

CMR ENGINEERING COLLEGE: : HYDERABAD
UGC AUTONOMOUS
I–B.TECH–II–Semester End Examinations (Supply) – December - 2025
VECTOR CALCULUS AND TRANSFORMS
(Common for all)

[Time: 3 Hours] **[Max. Marks: 60]**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 10 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A **(10 Marks)**

1. a) Find $L(5\ sint + 2\ sin3t)$. [1M]
   b) Find the Laplace transform of $e^{-3t}sin2t$. [1M]
   c) Find the inverse Laplace transform of $\frac{1}{(s+a)(s+b)}$. [1M]
   d) Find $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$. [1M]
   e) Find $\beta\left(\frac{3}{2},\frac{1}{2}\right)$. [1M]
   f) Compute $\Gamma\left(-\frac{1}{2}\right)$. [1M]
   g) Define divergence of vector point function. [1M]
   h) Prove that $\bar{F} = yz\bar{\imath} + zx\bar{\jmath} + xy\bar{k}$ is irrotational vector. [1M]
   i) Define line integral. [1M]
   j) State Gauss divergence theorem of transformation between surface integral and volume integral. [1M]

PART-B **(50 Marks)**

2.a) Find the Laplace transform of $e^{4t}(sin^3 3t + cosh^3 3t)$. [5M]
   b) Find $L(tcos^2 3t)$. [5M]

OR

3.a) Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$. [5M]
   b) Evaluate $L\left(e^{-2t}\int_0^t tsint\ dt\right)$. [5M]

4. Find inverse Laplace transform of the function $\frac{s^2}{(s^2+a^2)^2}$ using convolution theorem. [10M]

OR

5. Solve $\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0$, where $y = 1, \frac{dy}{dt} = 2, \frac{d^2y}{dt^2} = 2$ at t=0, using Laplace transform method. [10M]

6. To prove $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. [10M]

OR

7. Express the integral $\int_0^1 x^m(1-x^n)^p dx$ in terms of gamma function and hence $\int_0^1 x^{3/2}\left(1-x^{1/2}\right)^{1/2} dx$. [10M]

8. Find constants *a,b,c* so that the vector $\bar{A} = (x+2y+az)\bar{\imath} + (bx-3y-z)\bar{\jmath} + (4x+cy+2z)\bar{k}$ is irrotational. Also find φ such that $\bar{A} = \nabla\emptyset$. [10M]

OR

9. Prove that $\nabla \times (\nabla \times \bar{a}) = \nabla(\nabla.\bar{a}) - \nabla^2\bar{a}$. [10M]

10. Verify Green's theorem in the plane for $\int_C (x^2 - xy^3)dx + (y^2 - 2xy)\,dy$ where C is a square with vertices $(0,0), (2,0), (2,2), (0,2)$. [10M]

OR

11. Verify Stokes theorem for $\bar{F} = (y-z+2)\bar{\imath} + (yz+4)\bar{\jmath} - xz\bar{k}$ where S is the surface of the cube $x=0, y=0, z=0$ and $x=2, y=2, z=2$ above the xy-plane. [10M]

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