

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART-A

(25 Marks)

- 1.a) Prove that $x=2$ is a regular singular point for the differential equation $x(2-x)y'' - 2(x-1)y' + 2y = 0$ [2]
- b) Find the particular integral of $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 8y = x$ [3]
- c) Evaluate $\int_{-1}^1 P_0(x) dx$ [2]
- d) Prove that $J_0(0) = 1$ [3]
- e) Find the value of a if $\cos ax \sin by$ is harmonic. [2]
- f) Find the points at which $f(z) = \frac{z}{(z^2 - z)}$ is not analytic. [3]
- g) Find the residue of $\frac{2z+3}{z^2 - z - 2}$ at $z = -1$ [2]
- h) Expand $z \cos \frac{1}{z}$ [3]
- i) The fixed points of $f(z)$ are the points where $f(z) = z$ [2]
- j) Find the critical points of $w = \sin z$ [3]

PART-B

(50 Marks)

2. Solve the differential equation in series. $2x \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - y = 0$ around $x = 0$ [10]

OR

3. Solve the differential equation in series. $\frac{d^2y}{dx^2} + xy = 0$ [10]

4.a) Prove that $J_{n-1} = \frac{2}{x} (nJ_n - (n+2)J_{n+2} + (n+4)J_{n+4} - \dots)$

b) Express $x^2 - 3x + 4$ in terms of Legendre Polynomials [5+5]

OR

5.a) Prove that $\frac{d}{dx} [x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$

b) Express $x^2 - 4x + 7$ in terms of Legendre Polynomials. [5+5]

6.a) Find an analytic function whose real part is $e^{-x}(x \sin y - y \cos y)$

b) Evaluate the integral $\int_C \frac{\sin^2 z}{(z - \frac{\pi}{6})^3} dz$ where $C: |z|=1$ [5+5]

OR

7.a) Find the analytic function whose real part is $e^{2x}(x \cos 2y - y \sin 2y)$

b) Evaluate $\int_C \frac{(z^3 - z) dz}{(z-2)^3}$ where C is $|z|=3$ [5+5]

8. Expand $\frac{1}{z^2 - 3z + 2}$

a) $|z| > 2$ b) $1 < |z| < 2$ [5+5]

OR

9. Find the residue at the singular points of the function $\frac{z^2}{(z-1)^2(z+2)}$. [10]

10.a) Find the image of $1 < x < 2$ under the transformation $w = \frac{1}{z}$

b) Find the bilinear mapping which maps the points $z=1, i, -1$ into $0, 1, \infty$. [5+5]

OR

11.a) Find the image of the infinite stripe $\frac{1}{4} \leq y \leq \frac{1}{2}$ under the mapping $w = \frac{1}{z}$

b) Find the image of $|z| < 1$ and $|z| > 1$ under the transformation $W = \frac{iz+1}{z+i}$ [5+5]