Code No: 09A1BS01 **R09**

B. Tech I Year Examinations, May/June -2012
MATHEMATICS-I
(Common to all Branches)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks
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1. a) Find whether the series $\sum (-1)^n \frac{\sin\left(\frac{1}{\sqrt{n}}\right)}{n-1}$ is absolute convergent or conditional convergent.

   b) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$.

   c) Test the convergence of the series $\sum \frac{n^3 - 5n^2 + 7}{n^5 + 4n^4 - n}$. [6+6+3]

2. a) Prove using mean value theorem $|\sin u - \sin v| \le |u - v|$.

   b) If the sum of the three numbers is a constant, then prove that their product is maximum when they are equal.

   c) Prove that the functions $u = xy + yz + zx$, $v = x^2 + y^2 + z^2$, $w = x + y + z$ are functionally dependent and find the relation between them. [6+5+4]

3. a) Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid.

   b) Trace the curve $x = a\cos^3\theta$, $y = b\sin^3\theta$. [8+7]

4. a) Find the volume of the solid generated by the revolution of the Cissoid $y^2 = \frac{x^2}{(2a - x)}$ about its asymptote.

   b) By changing the order of integration evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy\, dx$. [7+8]

5. a) Solve $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$.

   b) Obtain the orthogonal trajectories of the family of curves $r(1 + \cos\theta) = 2a$. [7+8]

6. a) By using variation of parameters solve $\frac{d^2y}{dx^2}+4y=\tan 2x$.

b) Solve $\left((2x-1)^3\frac{d^3y}{dx^3}+(2x-1)\frac{dy}{dx}-2y=x\right)$. [8+7]

7. a) Find the Laplace transform of $te^{2t}\sin 3t$.

b) Use Laplace Transforms, to solve $\left(D^2+1\right)x=t\cos 2t$ given $x(0)=x^1(0)=0$. [7+8]

8. a) Verify divergence theorem for $2x^2yi-y^2j+4xz^2k$ taken over the region of first octant of the cylinder $y^2+z^2=9$ and x=2.

b) Find the directional derivative of $\nabla.\nabla\phi$ at the point (1, -2, 1) in the direction of the normal to the surface $xy^2z=3x+z^2$ where $\phi=2x^3y^2z^4$. [8+7]

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