

**R15**

Code No: 123AH

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2016

MATHEMATICS-III

(Common to EEE, ECE, EIE, ETM)

Time: 3 Hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

**PART-A****(25 Marks)**

- 1.a) Solve  $(x^2 D^2 + xD - 4)y = 0$ . [2]
- b) Find the particular solution of  $4x^2 \left( \frac{d^2 y}{dx^2} \right) + 8x \left( \frac{dy}{dx} \right) + y = \frac{4}{\sqrt{x}}$ . [3]
- c) Express  $x^2 - 1$  in terms of  $P_n(x)$ . [2]
- d) Express  $J_2$  in terms of  $J_0$  and  $J_1$ . [3]
- e) Show that  $f(z) = z |z|$  is not analytic anywhere. [2]
- f) Find the harmonic conjugate of  $u = 2xy + 3y$ . [3]
- g) Expand  $\frac{1}{(z+1)}$ , when  $z > 1$ . [2]
- h) Find the co-efficient of  $z^3$  in the expansion of  $\frac{1}{z^2(1-z)}$ . [3]
- i) Evaluate the residue of  $\frac{e^z}{z^2(z^2+9)}$  at  $z = 0$ . [2]
- j) Find the image of  $c < y < d$  under the transformation  $w = e^z$ . [3]

**PART-B****(50 Marks)**

2. Solve in series  $3x^2 \left( \frac{d^2 y}{dx^2} \right) + x \left( \frac{dy}{dx} \right) + y = x$ . [10]

**OR**

3. Solve  $(1+x)^2 \left( \frac{d^2 y}{dx^2} \right) + (1+x) \left( \frac{dy}{dx} \right) + y = 4 \cos \log(1+x)$ . [10]

4.a) Prove that  $\cos(x \cos \theta) = J_0 - 2J_2 \cos 2\theta + 2J_4 \cos 4\theta - \dots$

b) Prove that  $\sin(x \cos \theta) = 2J_1 \cos 3\theta + 2J_3 \cos 5\theta - \dots$ . [5+5]

**OR**

5. Show that  $\int_{-1}^1 p_m(x) p_n(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2}{2n+1} & \text{if } m = n \end{cases}$ . [10]

6.a) Find the analytic function whose real part is  $\left(r - \frac{1}{r}\right) \sin \theta$ .

b) Evaluate  $\int x^2 y dx + (x^2 - y^2) dy$  from (0,0) to (1,3) along  $y=x^2$ . [5+5]

OR

7) If  $F(z) = \int \frac{(3z^2 + 7z + 1)}{(z-a)} dz$  using Cauchy's integral formula where C is  $|z|=2$ .

Find  $F(1)$ ,  $F(3)$ ,  $F''(1-i)$ . [10]

8. Expand  $\frac{z}{(z+1)(z-3)}$  where (a)  $|z| > 3$  (b)  $|z| < 1$ . [10]

OR

9. Expand  $f(z) = \frac{z+3}{z(z^2-z-2)}$  in power of  $z$

(a)  $0 < |z| < 1$  (b)  $1 < |z| < 2$  (c)  $|z| > 2$ . [10]

10.a) Prove that under the transformation  $w=1/z$ , the image of the lines  $y=x-1$  and  $y=0$  are the circle  $u^2 + v^2 - u - v = 0$  and the line  $v=0$ , respectively.

b) Find the bilinear transformation which maps the points  $(-1, \infty, 1)$  to  $(-1, -2, i)$ . [5+5]

OR

11.a) Find the image of the triangle with vertices  $i, 1+i$  and  $1$  in  $Z$ -plane under the transformation  $w=3z+4-2i$ . [5+5]

b) Show that the transformation  $w = \frac{5-4z}{4z-2}$  transforms the circle  $|z|=1$  into a circle of radius unity in  $w$ -plane and find the centre of the circle. [5+5]

---ooOoo---