R16

Code No: 132AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year II Semester Examinations, August - 2018

MATHEMATICS – II

(Common to EEE, ECE, CSE, EIE, IT, ETM)

Time: 3 hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) Find the Laplace transform of the function $f(t) = t^2$. [2]

b) Find Laplace transform of $4\sin(t-3)$. [3]

c) Show that $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1} dx$. [2]

d) Show that $\beta(p,q) = \beta(p+1,q) + \beta(p,q+1)$. [3]

e) Find the area bounded by the curves $y = x, y = x^2$. [2]

f) Evaluate $\int_0^1 \int_0^1 x^2 y^2 dxdy$ [3]

g) If $\phi = x^2 y^2 z^2$ then find Grad ϕ. [2]

h) Find a unit normal vector to the surface $x^2 + y^2 + 2z^2 = 26$ at the point (2,2,3). [3]

i) Find curl $\vec{F}$ when $\vec{F} = 3x^2 i + (2xz - y)j + zk$. [2]

j) Is the work done by a force in moving a particle from one point to another point in an irrotational field is independent of the path of integration? Justify the answer. [3]

PART-B

(50 Marks)

2. Use Laplace transforms, solve $y''(t) + 5y'(t) + 6y(t) = t,\ y(0) = 1,\ y'(0) = 1$. [10]

OR

3. Solve by using Laplace transforms $y'' + 4y' + 3y = e^{-t}$ with $y(0) = y'(0) = 1$. [10]

4. Prove that $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$ using $\beta - \Gamma$ functions. [10]

OR

5.a) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

b) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. [5+5]

6. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B and C. Find the volume of the tetrahedron $OABC$. [10]

OR

7. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^2yz\, dz\, dy\, dx$. [10]

8. Prove that if $\vec{r}$ is the position vector of any point in space then $r^n\vec{r}$ is irrotational and is solenodial if n = -3. [10]

OR

9.a) Evaluate $\nabla \cdot \left(r\nabla\left(\frac{1}{r^3}\right)\right)$ where $r=\sqrt{x^2+y^2+z^2}$.

b) If $\overline{R}=x\bar{i}+y\bar{j}+z\bar{k}$, then find $\nabla.\overline{R}$ and $\nabla\times\overline{R}$. [5+5]

10. Verify Stoke's theorem for the vector field $\vec{F}=(x^2-y^2)i+2xyj$ integrated round the rectangle in the plane $z=0$ and bounded by the lines $x=0, y=0, x=a, y=b$. [10]

OR

11. Verify divergence theorem for $2x^2yi-y^2j+4xz^2k$ taken over the region of first octant of the cylinder $y^2+z^2=9$ and $x=2$. [10]

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