

**R16**

Code No: 132AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year II Semester Examinations, August - 2018

**MATHEMATICS – II**  
(Common to EEE, ECE, CSE, EIE, IT, ETM)

Time: 3 hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.  
Part A is compulsory which carries 25 marks. Answer all questions in Part A.  
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

**PART-A****(25 Marks)**

- 1.a) Find the Laplace transform of the function  $f(t) = t^2$ . [2]
- b) Find Laplace transform of  $4 \sin(t - 3)$ . [3]
- c) Show that  $\Gamma(n) = 2 \int_0^\infty e^{-x^2} x^{2n-1} dx$ . [2]
- d) Show that  $\beta(p, q) = \beta(p + 1, q) + \beta(p, q + 1)$ . [3]
- e) Find the area bounded by the curves  $y = x, y = x^2$ . [2]
- f) Evaluate  $\int_0^1 \int_0^1 x^2 y^2 dx dy$  [3]
- g) If  $\phi = x^2 y^2 z^2$  then find Grad  $\phi$ . [2]
- h) Find a unit normal vector to the surface  $x^2 + y^2 + 2z^2 = 26$  at the point  $(2, 2, 3)$ . [3]
- i) Find curl  $\vec{F}$  when  $\vec{F} = 3x^2 i + (2xz - y)j + zk$ . [2]
- j) Is the work done by a force in moving a particle from one point to another point in an irrotational field is independent of the path of integration? Justify the answer. [3]

**PART-B****(50 Marks)**

2. Use Laplace transforms, solve  $y''(t) + 5y'(t) + 6y(t) = t, y(0) = 1, y'(0) = 1$ . [10]
- OR**
3. Solve by using Laplace transforms  $y'' + 4y' + 3y = e^{-t}$  with  $y(0) = y'(0) = 1$ . [10]
4. Prove that  $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$  using  $\beta - \Gamma$  functions. [10]
- OR**
- 5.a) Prove that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
  - b) Prove that  $\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$ . [5+5]

6. The plane  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  meets the axes in  $A, B$  and  $C$ . Find the volume of the tetrahedron  $OABC$ . [10]

OR

7. Evaluate  $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^2 yz \, dz \, dy \, dx$ . [10]

8. Prove that if  $\vec{r}$  is the position vector of any point in space then  $r^n \vec{r}$  is irrotational and is solenoidal if  $n = -3$ . [10]

OR

- 9.a) Evaluate  $\nabla \cdot \left( r \nabla \left( \frac{1}{r^3} \right) \right)$  where  $r = \sqrt{x^2 + y^2 + z^2}$ . [5+5]  
b) If  $\vec{R} = x\vec{i} + y\vec{j} + z\vec{k}$ , then find  $\nabla \cdot \vec{R}$  and  $\nabla \times \vec{R}$ .

10. Verify Stoke's theorem for the vector field  $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$  integrated round the rectangle in the plane  $z = 0$  and bounded by the lines  $x = 0, y = 0, x = a, y = b$ . [10]

OR

11. Verify divergence theorem for  $2x^2y\vec{i} - y^2\vec{j} + 4xz^2\vec{k}$  taken over the region of first octant of the cylinder  $y^2 + z^2 = 9$  and  $x = 2$ . [10]

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