SSP Assignment

Set-A

Part-A

1) What are Dirichelet's conditions?

A) Dirichelet's condition:-

For any periodic function f(t), the fourier series over the interval $-\infty < t < \infty$ is valid if the series converges at every point. If the function has infinite frequency component at some point, it will not converge at that point. Therefore the conditions imposed on the periodic function f(t) is such that the fourier series should converge at every point over the interval $(-\infty < t < \infty)$. The conditions are called dirichelet's condition, given as:

1. The periodic function f(t) should be absolutely integrable over the interval T sec i.e., $\boxed{\int_0^T |f(t)|\, dT < \infty}$

2. The function f(t) must have finite number of maxima and minima in one period.

3. The function f(t) has a finite number of finite discontinuities.

2) Distinguish between LTI & LTV systems?

A) LTI - Linear time invarient (Linearity & time invarient)

LTV - Linear time varient. (Linearity & time varient).

Linearity:- A system is said to be linear, if it satisfies the scaling property (homogenity) and principle of superposition (additivity).

Additivity:- Given any two signals, $y_1(t) = T\{x_1(t)\}$ and $y_2(t) = T\{x_2(t)\}$, the principle of superposition gives $y_1(t) + y_2(t) = T\{x_1(t) + x_2(t)\}$ — (1)

where $T\{\cdot\}$ = transformation by the system to which the signals are applied.

<u>Homogenity</u>: For any signal $x(t)$, the homogenity property is given by $ay(t) = T[ax(t)]$ —②

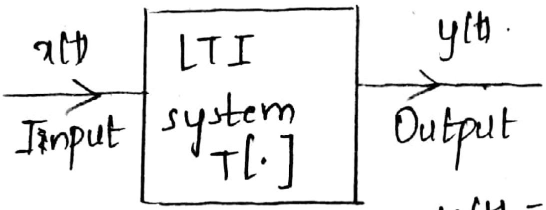
Combining both ① & ②, we get

$$a_1y_1(t) + a_2y_2(t) = T\{a_1x_1(t) + b_2x_2(t)\}.$$

$$= T[a_1x_1(t)] + T[bx_2(t)]$$

$$= aT[x_1(t)] + bT[x_2(t)]$$

$$= ay_1(t) + by_2(t)$$



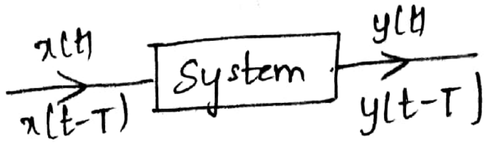
$$y(t) = T[x(t)]$$

$$ax(t) \longrightarrow y(t) = T[ax(t)] = aT[x(t)]$$

<u>Time-invarient</u>:-

If the system properties i.e., input or output characteristics do not change with time, then it is called time invarient system.

<u>Note</u>:- For discrete time system, we are calling shift invarience.



Time invarience.

$$x(t) \rightarrow y(t).$$

$$x(t-T) \rightarrow y(t-T) \quad (TI)$$

$$x(t-T) \not\rightarrow y(t-T) \quad (TV)$$

A continuous-time system described by $y(t) = T\{x(t)\}$ is time invarient if it holds true. the following.

$$y(t-t_0) = T\{x(t-t_0)\}$$

A discrete system described by $y[n] = T\{x[n]\}$ is time invarient if and only if the following holds true

$$y[n-n_0] = T\{x[n-n_0]\} \ \forall n_0.$$

Time varient:- A system which does not satisfy the above conditions for a continuous time system or a discrete-time system is called a time-varying system.

Eg:- A system $y(t) = x(t) + 2x(t-1)$ is T.I. system since if the i/p is shifted by time t_0, the o/p also shifts by the same time as $y(t-t_0) = x(t-t_0) + 2x(t-t_0-1)$,

whereas $y[n] = nx[n]$ is T.V system since,

the i/p $x[n]$ is shifted by n_0, then

$$y_1[n] = n x[x-n_0]$$

o/p $y[n]$ is shifted by n_0, then

$$y_2[n-n_0] = (n-n_0)\, x_1[n-n_0] \neq y_1[n].$$

3) Show that $x(t) * \delta(t) = x(t)$?

A) $x(t) * \delta(t) = x(t)$

Convolution:- $x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$

$$h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

if $h(t) = \delta(t)$

$$\delta(t) * x(t) = \int_{-\infty}^{\infty} \delta(T)\, x(t-T)\, dT$$

$$\delta(t) * x(t) = \int_{-\infty}^{\infty} \delta(T)\, x(t-T)\, dT \quad \text{---} (1)$$

$$\delta(t) y(t) = \delta(t)\, y(0)$$

$$\int_{-\infty}^{\infty} \delta(t) \underbrace{y(t)}_{x(t-T)}\, dt = \int_{-\infty}^{\infty} \delta(t)\, y(0)\, dt = y(0) \int_{-\infty}^{\infty} \delta(t)\, dt$$

$\downarrow$

Constant

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1}$$

$$\int_{-\infty}^{\infty} \delta(t)\, y(t)\, dt = y(0)$$

from (1) $\delta(t) * x(t) = \int_{-\infty}^{\infty} \delta(T)\, x(t-0)\, dT = x(t) \int_{-\infty}^{\infty} \delta(T)\, dT$

$= x(t)$

4) Find the z-transform of the sequence $x[n] = \{5, 3, -2, 0, 4, -3\}$?

Sol:- z-transform of

$$x(n) = \{\overset{x(0),\, x(1),\, \dots\dots\, x(n)}{5, 3, -2, 0, 4, -3}\}$$

$n = 5$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$X(z) = \sum_{n=0}^{5} x(n)\, z^{-n}$$

$$= x(0)\, z^{-0} + x(1)\, z^{-1} + x(2)\, z^{-2} + \dots\dots + x(5) z^{-5}$$

$$X(z) = 5 + 3z^{-1} + (-2) z^{-2} + (0) z^{-3} + 4z^{-4} + (-3)\, z^{-5}$$

$$X(z) = 5 + 3z^{-1} - 2z^{-2} + 4z^{-4} - 3z^{-5}.$$

SET 1

we shown

PART - B

1 A) Explain how a function can be approximated by orthogonal functions.

Ans:- Any signal $x(t)$ can be approximated as a sum of components alongs a set of 'n' mutually orthogonal functions if these functions form a complete set.

Approximation of a signal by a set of orthogonal functions:- with

→ Consider a set of 'n' orthogonal functions $g_1(t), g_2(t)$ ---- $g_n(t)$ ~~over~~ over an interval (t_1, t_2) that is

$$\int_{t_1}^{t_2} g_j(t)\, g_k(t)\, dt = 0 \quad j \neq k$$

$$\int_{t_1}^{t_2} g_j^2(t)\, dt = k_j$$

wave is

Let an arbitrary function $x(t)$ be approximated over an interval (t_1, t_2) along these 'n' mutually orthogonal functions | signals then

$$x(t) = c_1 g_1(t) + c_2 g_2(t) + \cdots + c_n g_n(t).$$

For best approximation we have to find the values of $c_1, c_2, c_3 \cdots c_n$. such that ε the mean square error of $x_e(t)$ is minimized.

(1-2nt) dt

WKT $\quad x(t) = \sum_{r=1}^{n} c_r g_r(t) + x_e(t)$

(or) $\quad x_e(t) = x(t) - \sum_{r=1}^{n} c_r g_r(t).$

$$\varepsilon = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left[x(t) - \sum_{r=1}^{n} c_r g_r(t)\right]^2 dt. \quad \left[\because \varepsilon = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x_e^2(t)\, dt\right]$$

1B) Ans.

(6)

$$\varepsilon = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}\left[(x(t))^2 + \sum_{r=1}^{n} c_r^2 g_r^2(t) - 2x(t)\sum_{r=1}^{n} c_r g_r(t)\right] dt$$

$$\Rightarrow \varepsilon = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2}\left[x^2(t) - 2\sum_{r=1}^{n} x(t)\, c_r g_r(t) + \sum_{r=1}^{n} c_r^2 g_r^2(t)\right] dt.$$

ε is the function of $c_1, c_2 \ldots c_n$ and to minimise ε we have

$$\frac{\partial \varepsilon}{\partial c_1} = \frac{\partial \varepsilon}{\partial c_2} = \frac{\partial \varepsilon}{\partial c_3} \cdots = \frac{\partial \varepsilon}{\partial c_n} = 0.$$

Consider $\frac{\partial \varepsilon}{\partial c_j} = 0.$

$$\Rightarrow \frac{1}{t_2 - t_1}\left[\int_{t_1}^{t_2} \frac{\partial}{\partial c_j} x^2(t)\, dt - 2\int_{t_1}^{t_2} \sum_{r=1}^{n} \frac{\partial}{\partial c_j}[x(t)\, c_r g_r(t)]\, dt + \frac{\partial}{\partial c_j}\int_{t_1}^{t_2} c_r^2 g_r^2(t)\, dt\right] = 0.$$

The ~~fn~~ derivative w.r.t c_j which do not contain terms of c_j are zero.

$$\Rightarrow -2\int_{t_1}^{t_2} x(t)\, g_j(t)\, dt + \int_{t_1}^{t_2} 2c_j g_j^2(t)\, dt.$$

$$\Rightarrow \int_{t_1}^{t_2} x(t)\, g_j(t)\, dt = \int_{t_1}^{t_2} c_j g_j^2(t)\, dt.$$

$$c_j = \frac{\int_{t_1}^{t_2} x(t)\, g_j(t)\, dt}{\int_{t_1}^{t_2} g_j^2(t)\, dt}$$

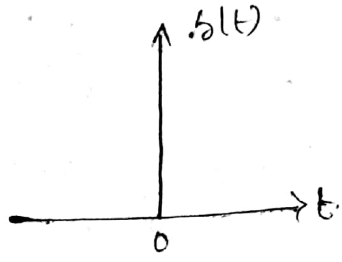
But $\int_{t_1}^{t_2} g_j^2(t)\, dt = K_j$

$$c_j = \frac{\int_{t}^{t_2} x(t)\, g_j(t)\, dt}{K_j}$$

1B) Discuss the concept of impulse function. Explain how signum function is expressed in terms of unit step function.

Sol) Impulse function :- It is the most widely used function in analysis of signals & systems. It is also called as dirac delta function. It is defined as

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\,dt = 1}$$



$\delta(t) = 0,\ t \neq 0.$

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0. \end{cases}$$

The impulse function has zero amplitude everywhere except at origin where it has infinite amplitude & the area under it is given by unity. $\delta(t)$ can be represented as a limiting case of rectangular pulse function.

$$\cancel{\delta(t)}\ x(t) = \frac{1}{\Delta}\left[u(t) - u(t-\Delta)\right]$$

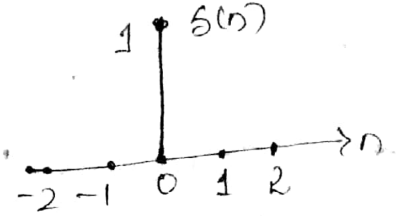
$$\delta(t) = \lim_{\Delta \to 0} \frac{1}{\Delta}\left[u(t) - u(t-\Delta)\right]$$

Properties :-

1. $\delta(t) = \delta(-t)$ ⟹ It is an even function.
2. $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$; $\int_{-\infty}^{\infty} x(t_0)\,\delta(t-t_0)\,dt = x(t_0)$
3. $\delta(at) = \frac{1}{|a|}\,\delta(t)$
4. $x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau.$
5. $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0) = x(t_0).$

Discrete time Impulse function :- $\delta(n)$. It is also called sample sequence

$$\delta(n) = \begin{cases} 1, & n=0 \\ 0, & n=0 \end{cases}$$



Properties of δ(n) :-

① $\delta(n) = u(n) - u(n-1)$

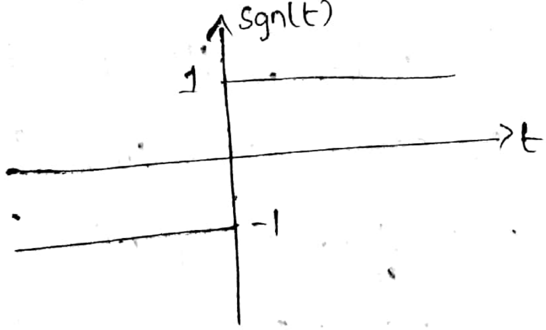
② $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

③ $\delta(n-k) = \begin{cases} 1, & n=k \\ 0, & n \neq k \end{cases}$

④ $x(n_0) = \sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0)$.

(ii) Signum function :- It is denoted as sgn(t).

$$\mathrm{Sgn}(t) = \begin{cases} 1, & t>0 \\ -1, & t<0 \end{cases}$$

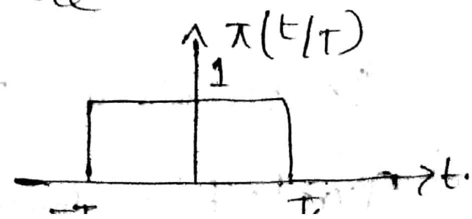


The signum function is expressed in terms of unit step function as $2u(t) - 1$ ~~i.e,~~ or $\mathrm{sgn}(t) = u(t) - u(-t)$.

3 A) Find the fourier transform of gate spectrum & sketch it.

Sol Gate / rect pulse is defined as

$$\pi\left(\frac{t}{T}\right) \text{ or } \mathrm{rect}(t) = \begin{cases} 1, & t \leq \left|\frac{T}{2}\right| \\ 0, & \text{elsewhere} \end{cases}$$



$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \pi\left(\frac{t}{\tau}\right) e^{-j\omega t} dt$$

$$= \int_{-\tau/2}^{\tau/2} (1) e^{-j\omega t} dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\tau/2}^{\tau/2}$$

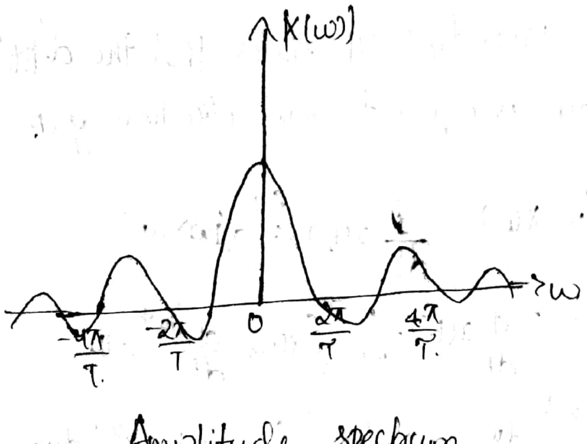
$$= \left[\frac{e^{-j\omega\tau/2} - e^{j\omega\tau/2}}{-j\omega}\right] = \frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j\omega}.$$

$$= \frac{2\tau}{\omega\tau}\left[\frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{2j}\right]$$

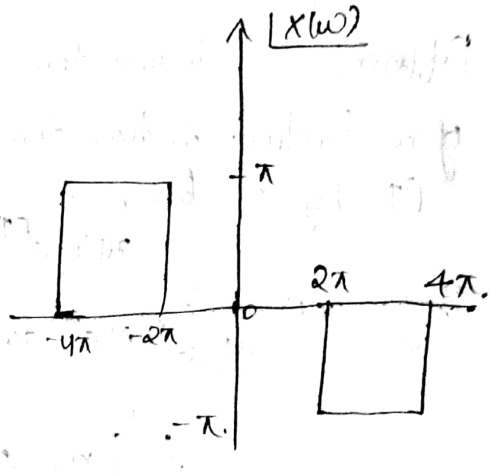
$$= \tau\left[\frac{\sin\omega(\tau/2)}{\omega(\tau/2)}\right] = \tau \operatorname{sinc}\frac{\omega\tau}{2}$$

$$F\left[\pi\left(\frac{t}{\tau}\right)\right] = \tau \operatorname{sa}\left(\frac{\omega\tau}{2}\right).$$

$$\operatorname{rect}\left(\frac{t}{\tau}\right) = \pi\left(\frac{t}{\tau}\right) \xleftrightarrow{F.T} \tau \sin\frac{\omega\tau}{2}.$$



Amplitude spectrum



phase spectrum

3B) State and prove time convolution & time diff^n property of FT.

So. Convolution Property:- It states that the convolution of two signals in time domain is equivalent to the multiplication of their spectra in frequency domain.

7. If $x_1(t) \xleftrightarrow{F.T} X_1(\omega)$ $x_2(t) \xleftrightarrow{FT} X_2(\omega)$

Then $x_1(t) * x_2(t) \xleftrightarrow{F.T} X_1(\omega) X_2(\omega)$

Proof :- WKT $x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau$.

$$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau \right] e^{-j\omega t} dt.$$

Interchanging the order of integration

$$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau) \left[\int_{-\infty}^{\infty} x_2(t-\tau) e^{-j\omega t} dt \right] d\tau.$$

put $t - \tau = p$

$\Rightarrow dt = dp \Rightarrow t = \tau + p$.

$$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau) \left[\int_{-\infty}^{\infty} x_2(p) e^{-j\omega \tau} \cdot e^{-j\omega p} dp \right] d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau) e^{-j\omega \tau} d\tau \, (X_2(\omega))$$

$$\therefore \boxed{F[x_1(t) * x_2(t)] = X_1(\omega) X_2(\omega)}$$

Differentiation in time domain property :- It states that the diffⁿ of a function in time domain is equal to multiplication of its FT by a factor $j\omega$.

$x(t) \xleftrightarrow{FT} X(\omega)$. $\frac{d}{dt} x(t) \xleftrightarrow{FT} j\omega X(\omega)$

Proof :- $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$. $\Rightarrow \frac{d}{dt} x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \frac{d}{dt} e^{j\omega t} d\omega$

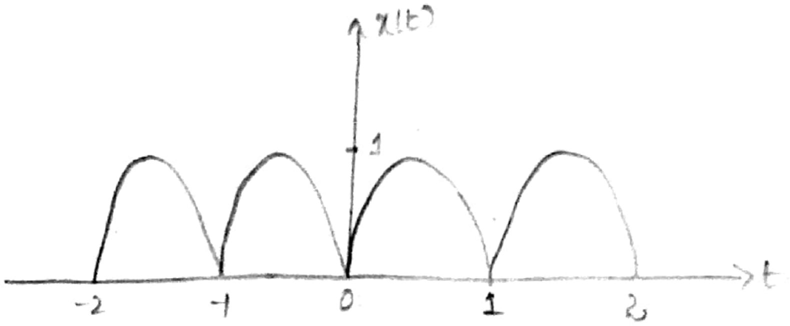
$$= \cancel{\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \frac{e^{j\omega t}}{j\omega} d\omega} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) j\omega e^{j\omega t} d\omega$$

$$= j\omega \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \right] = j\omega \cdot F^{-1}[X(\omega)].$$

$$\Rightarrow F[x(t)] = j\omega X(\omega)$$

$$\boxed{\frac{d}{dt}(x(t)) \longleftrightarrow j\omega X(\omega)}$$

2a) Find the exponential fourier series for the rectified sine wave shown in fig.



Sol) The waveform shown in fig is a part of sine wave with period = 2

$x(t) = A\sin \omega t \quad 0 \le t \le 1$ i.e, $\frac{2\pi}{2} = \pi$.

$x(t) = A\sin \pi t \quad 0 \le t \le 1$ where $\boxed{A = 1}$

The period of the rectified sine wave is $T = 1$.

$t_0 = 0$.

$t_0 + T = 1$.

The fundamental frequency of the rectified sine wave is

$$\omega_0 = \frac{2\pi}{T} = 2\pi.$$

The exponential fourier series is

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n t}$$

$$C_n = \frac{1}{T}\int_{t_0}^{t_0+t} x(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{1}\int_0^1 \sin\pi t\, e^{-j2\pi n t}\, dt$$

$$= \frac{1}{2j}\left[\int_0^1 e^{j\pi t} \cdot e^{-j2n\pi t} - e^{-j\pi t} \cdot e^{-j2n\pi t}\, dt\right] = \cancel{A\, \frac{1}{2j}\left[\int_0^1 e^{j\pi[(1-2n)t]}\, dt\right.}$$

$$= \frac{1}{2j}\left[\int_0^1 e^{j\pi(1-2n)t}\, dt - \int_0^1 e^{-j\pi(1+2n)t}\, dt\right]$$

$$= \frac{1}{2j}\left[\left(\frac{e^{j\pi(1-2n)t}}{j\pi(1-2n)}\right)_0^1 - \left(\frac{e^{-j\pi(1+2n)t}}{-j\pi(1+2n)}\right)_0^1\right]$$

$$= \frac{2}{\pi(1-4n^2)}$$

$$C_0 = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\,dt = \frac{1}{1}\int_0^1 \sin\pi t\,dt = \left[\frac{-\cos\pi t}{\pi}\right]_0^1$$

$$C_0 = \frac{2}{\pi}.$$

The EFS (exp. fourier series) is :-

$$x(t) = \frac{2}{\pi} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2}{\pi(1-4n^2)} e^{j2n\pi t}.$$

2b) Trignometric fourier series is given by :-

$$\boxed{x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt.$$

Exponential fourier series is given by

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}.}$$

$$C_n = \frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t}\,dt = \frac{1}{T}\int_0^T x(t)\left[\cos n\omega_0 t - j\sin n\omega_0 t\right] dt.$$

$$= \frac{1}{2}\left[\frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\,dt - j\frac{2}{T}\int_0^T x(t)\sin\omega_0 n t\,dt\right] = \frac{1}{2}\left[a_n - jb_n\right].$$

$$C_{-n} = \frac{1}{T}\int_0^T x(t) e^{jn\omega_0 t}\,dt = \frac{1}{T}\int_0^T x(t)\left(\cos n\omega_0 t + j\sin n\omega_0 t\right] dt.$$

$$= \frac{1}{2}\left[\frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\,dt + j\frac{2}{T}\int_0^T x(t)\sin n\omega_0 t\,dt\right]$$

$$= \frac{1}{2}\left[a_n + jb_n\right].$$

$$C_0 = \frac{1}{T}\int_0^T x(t)\,dt = a_0.$$

So the formulae for conversion of trigonometric series to exponential series are

$$\boxed{\begin{aligned} C_0 &= a_0 \\ C_n &= \frac{1}{2}\left[a_n - jb_n\right] \\ C_{-n} &= \frac{1}{2}\left[a_n + jb_n\right]. \end{aligned}}$$

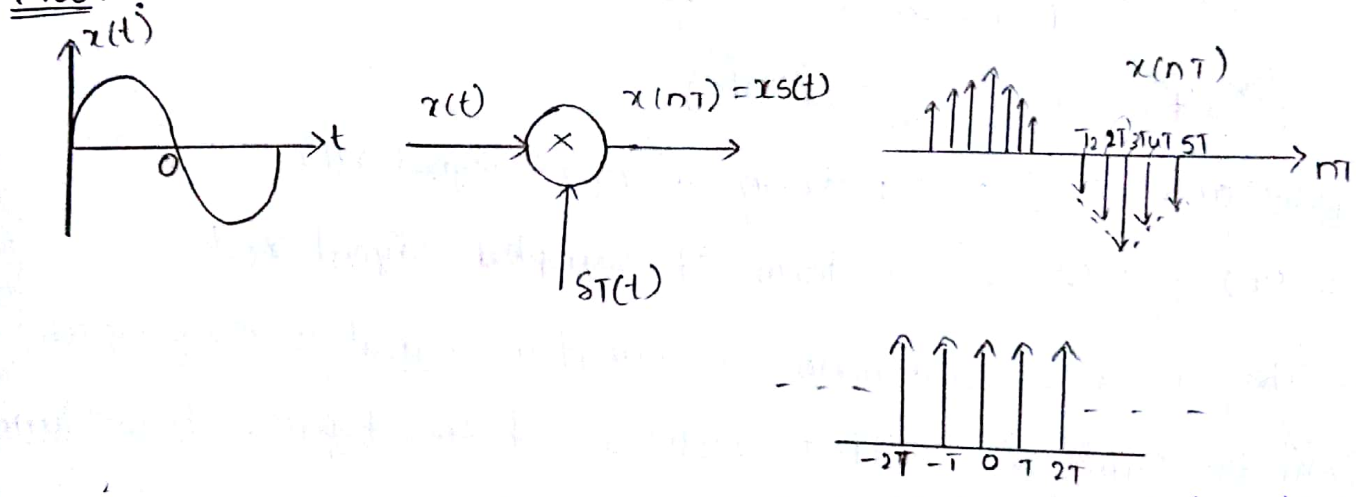
SET-1

1) A) State and Prove Sampling Theorem for band limited signals using analytical approach.

Statement : It states that "A band limited signals $x(t)$ with $X(\omega)=0$ for $|\omega|>\omega_m$ can be represented into samples and uniquely determined from its sample $x(nT)$ If the sampling frequency $f_s \geq 2f_m$. where f_m is the heighest frequency component present in it.

i.e for signal recovery the sampling frequency must be atleast twice the highest frequency present in the signal.

Proof :



$x(t)$ is a continuous time band limited to be sampled which has no spectral. components above f_m Hz that means $x(\omega)=0$ for $|\omega|>\omega_m$.

* $S_T(t)$ is an impulse strain which samples at a rate of f_s Hz.
* Here $f_s(t)$ is the sampled signal. T is the sampled Period $f_s = 1/T$.
* $x_s(t)$ is The Product of signal $x(t)$ and impulse train $S_T(t)$.

$x_s(t) = x(t)\,\delta_T(t)$ where $\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$.

$\therefore\; x_s(t) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT)$

{From fourier integral}.

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(t)\,\frac{e^{jn\omega_s T}}{T}$$

$$x_s(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty} x(t)\,e^{jn\omega_s T}$$

Taking Fourier Transform on b/s.

$$F[x_s(t)] = \frac{1}{T}\sum_{n=-\infty}^{\infty} F\left[x(t)\,e^{jn\omega s t}\right]$$

by using frequency shifting. Property of F.T.

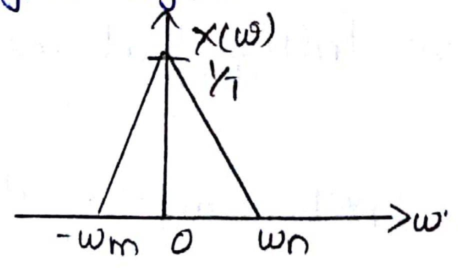
$$X_s(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s).$$

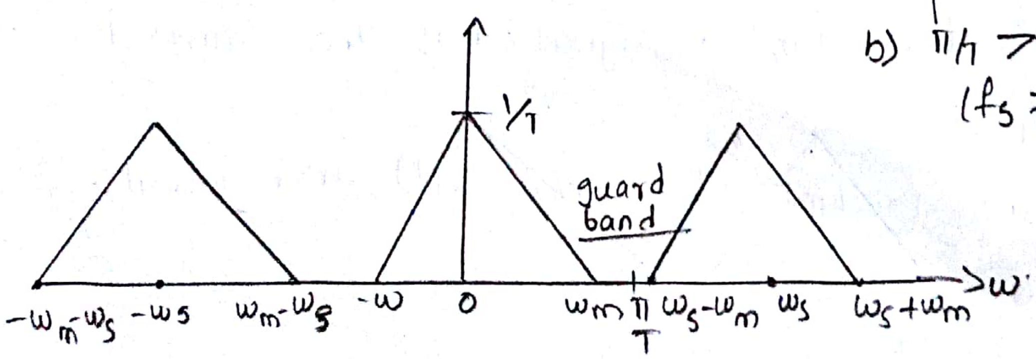
$X(\omega)$ and $X(f)$ are spectrum of input signal $x(t)$.

$X_s(\omega)$ & $X_s(f)$ are spectrum of sampled signal $x_s(t)$.

* The Fourier transform of sampled signal is given by an infinite sum of shifted replicas of the Fourier transform of the original signal.



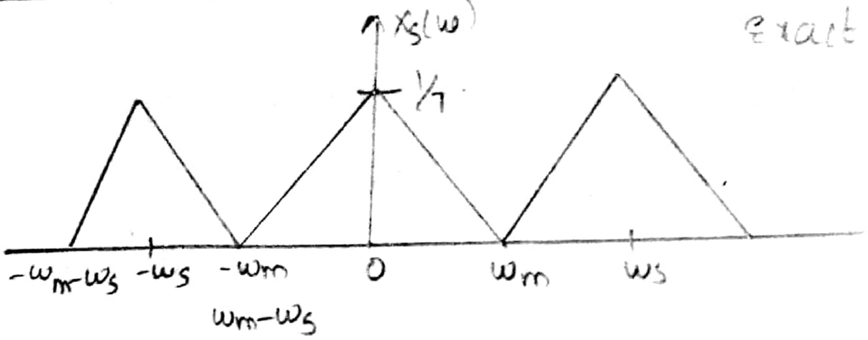
a) frequency spectrum of $x(t)$.



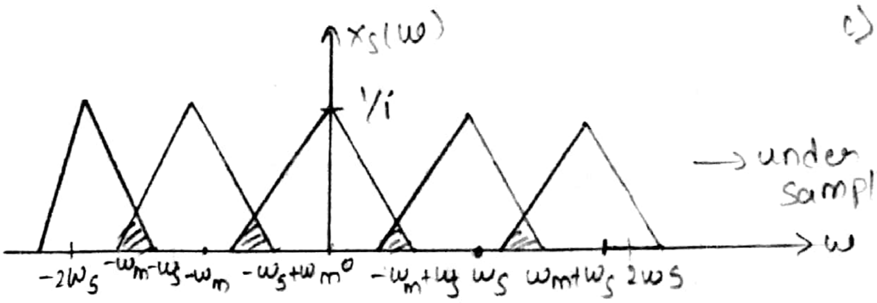
b) $\pi/T > \omega_m$ → over sampling.

$(f_s > f_m)$.

Exact sampling
b) $\pi/T = \omega_m$
$(f_s = f_m)$.



c) $\pi/T < \omega_m$
$(f_s < f_m)$.



→ under sampling

↳ aliasing

(1) $\omega_s - \omega_m > \omega_m$

$\omega_s > 2\omega_m$

$f_s > 2f_m$

2) $\omega_s - \omega_m = \omega_m$

$\omega_s = 2\omega_m$

$f_s = 2f_s$

3) $\omega_s - \omega_m < \omega_m$

$\omega_s < 2\omega_m$

$(f_s < 2f_m)$

The distortion occur in the undersampling condition is called aliasing $(f_s < 2f_m)$

⇒ From fig(b) and (c) if $\pi/T \geq \omega_m$ $(\omega_s \geq 2\omega_m)$ the replicas will not overlap and as a result $x(\omega)$ can be recoverd from $X_s(\omega)$ by passing it through a LPF which has a sharp cut off frequency at $\omega = \pi/T$.

⇒ From fig(d) if $\pi/T < \omega_m$ $(\omega_s < 2\omega_m)$ The successive frequency spectra will over lap and the original signal cannot be recorect from sample signal.

⇒ For signal recovery $\omega_s - \omega_m \geq \omega_m$.

$$\omega_s \geq 2\omega_m$$

$$(\text{or})\ f_s \geq 2f_m$$

$$(\text{or})\ T \leq 1/2f_m.$$

Note :-

From fig (b). i.e when $\omega_s > 2\omega_m$ The spectral components (replicas) have a larger separation between them known as guard band, which makes the process of filtering much easier and effective.

⟹ when $\omega_s = 2\omega_m$ (fig (c)) Their is no separation b/w the replicas i.e noguard band exist and $x(\omega)$ can be obtained from $x_s(\omega)$ by using an Ideal LPF with sharp cut off frequency.

⟹ when $\omega_s < 2\omega_m$ fig (d) The low frequency component in $x_s(\omega)$ overlap on the high frequency component of $x(\omega)$ so Their is a distortion and hence $x(\omega)$ cannot be recovered from $x_s(\omega)$ by using any filter This type of distortion is called aliasing.

⟹ Aliasing can be avoided if $f_s \geq 2f_m$. (or) $T \leq \frac{1}{2f_m}$.

B) Explain The conditions for distortion less transmission from through a system.

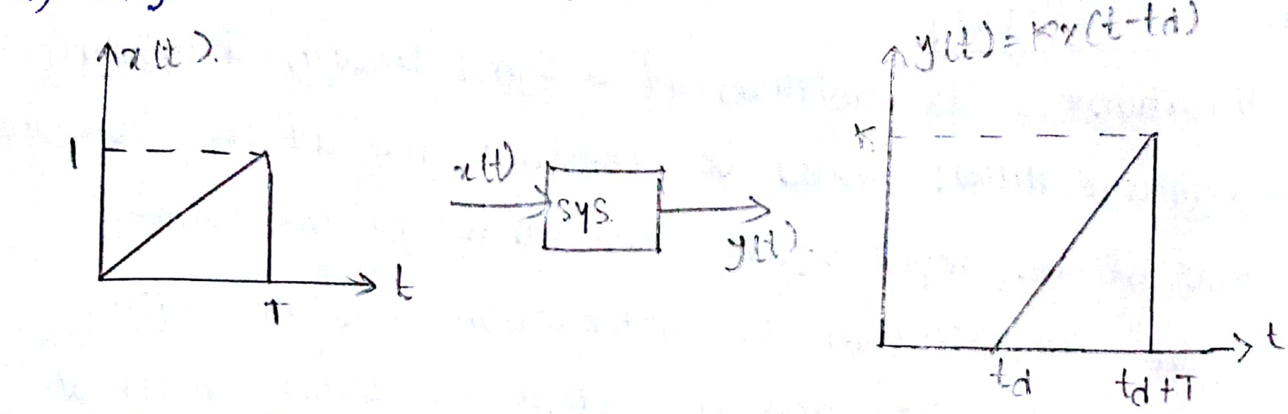
1) ⟹ The change of shape of the signal when it is transmitted through a system is called distortion.

2) Transmittion of a signal - through a system is said to be distortionless if the out is an exact replica of Input signal.

i) This replica may have different magnitude and also different time decay.

3) ii) A constant change in magnitude and a constant time decay are not considered as distortional.

iii) only shape of the signal is important.



* mathematically a signal $x(t)$ is transmitted without distortionally with the output $y(t) = Kx(t-t_d)$ —(1)

* where K is a constant representing the change in amplitude (amplification or attenuation) and t_d is decay time.

apply Fourier Transform to equation ①.

$$Y(\omega) = K \cdot e^{-j\omega t_d} \cdot X(\omega)$$

(time shifting Property of F.T).

* For distortionless transmittion the transfer function of The system must be

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = Ke^{-j\omega t_d} \quad —(2)$$

w.K.T

=) Frequency domain Represented by

$$H(\omega) = |H(\omega)|\, e^{j\theta(\omega)} \quad —(3)$$

$$\left.\begin{aligned} &\text{magnitude is } |H(\omega)| = K \\ &\text{Phase is } \theta(\omega) = -\omega t_d \end{aligned}\right\} \quad —(4)$$

The impulse response is

$h(t) = F^{-1}[H(\omega)]$

$h(t) = F^{-1}[K.e^{-j\omega td}]$

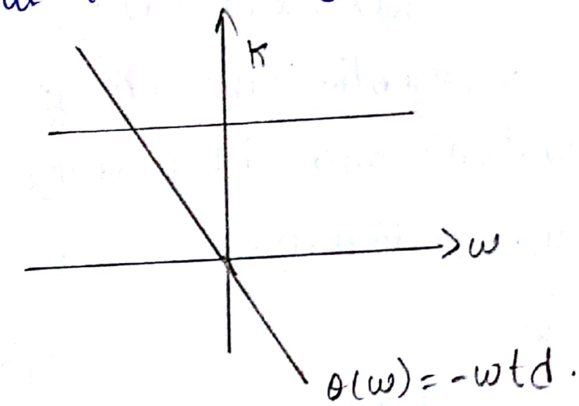
$\boxed{h(t) = K\delta(t-td)}$

$x(t-to) \longrightarrow e^{-j\omega to} X(\omega)$

$\delta(t-to) \longrightarrow e^{-j\omega to}$

$F[\delta(t)] = 1$

For distortionless transmittion of a signal through a system, the magnitude $|H(\omega)|$ should be constant i.e all the frequency components of the input signal must under go the same amount of amplification or attenuation. i.e The system band with is infinite and the phase spectrum should be proportional to frequency.



5 a) Prove the initial value Theorem and Final value Theorem in Laplace transform.

Sol Initial value Theorem:-

The initial value theorem enables us to calculate the initial value of the function $x(t)$. i.e $x(0)$ directly from its transform $X(s)$ without the need of finding The inverse transform of $X(s)$

$x(t) \longleftrightarrow X(s)$.

$$\boxed{\lim_{t \to 0} x(t) = x(0) = \lim_{s \to \infty} sX(s)}$$

Proof:- W.K.T

$$L\left\{\frac{d}{dt}x(t)\right\} = \int_0^{\infty} d/dt\, x(t)\, e^{-st}\, dt$$

$$= sX(s) - x(0^-)$$

taking $s \to \infty$ on both sides

$$\int_0^{\infty} d/dt\, x(t) \left\{\lim_{s\to\infty} e^{-st}\right\} dt = \lim_{s\to\infty} \left\{sX(s) - x(0^-)\right\}$$

$$0 = \lim_{s\to\infty} sX(s) - x(0^-)$$

$$x(0^-) = \lim_{s\to\infty} sX(s)$$

$$\boxed{x(0) = \lim_{s\to\infty} sX(s)}$$

Final value Theorem:-

if the $x(t) \xleftrightarrow{L.T} X(s)$

$$\lim_{t\to\infty} x(t) = x(\infty) = \lim_{s\to 0} sX(s)$$

$$L\left[\frac{d}{dt}x(t)\right] = \int_0^{\infty} d/dt\, x(t)\, e^{-st}\, dt = sX(s) - x(0^-)$$

taking $s \to 0$ on both sides

$$= \int_0^{\infty} d/dt\, x(t) \left\{\lim_{s\to 0} e^{-st}\right\} dt = \lim_{s\to 0} \left[sX(s) - x(0^-)\right]$$

$$= \int_0^{\infty} \left\{\frac{d}{dt} x(t)\right\} dt = \lim_{s\to 0} sX(s) - x(0)$$

$$= x(t)\Big|_0^{\infty} = \lim_{s\to 0} sX(s) - x(0)$$

$$x(\infty) - x(0) = \lim_{s\to 0} sX(s) - x(0)$$

$$\boxed{x(\infty) = \lim_{s\to 0} sX(s)}$$