# Amplitude Modulation

❖ **Define modulation? Explain different types of modulation.**

Modulation is the process of changing some characteristics (amplitude, frequency & phase) of a carrier wave in accordance with the instantaneous value of the modulating signal.

There are 3 types of modulations:

i) Amplitude modulation
ii) Frequency modulation and
iii) phase Modulation.

i) Amplitude modulation :-

Amplitude modulation is defined as the modulation in which the amplitude of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its (carrier) frequency & phase constant.

ii) Frequency modulation :-

Frequency modulation is defined as the modulation in which the frequency of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its (carrier) amplitude & phase constant.

iii) phase modulation :-

Phase modulation is defined as the modulation in which the phase of the carrier wave is varied in accordance with

❖ **Explain the NEED for modulation?**

❖ **Explain the advantages of modulation?**

The advantages of modulation are

i) Reduces the height of antenna:-

Height of antenna is a function of Wavelength 'λ'. The minimum height of antenna is given by $\lambda/4$.

i.e. height of antenna $= \frac{\lambda}{4} = \frac{c}{4f}$ $\boxed{\because \lambda = \frac{c}{f}}$

Where, $\lambda = \frac{c}{f}$,

$c = 3\times10^8$, velocity of light

$f$ = Transmitting frequency.

ex:- i) $f = 15\,KHz$,

height of antenna $= \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times15\times10^3} = 5000$ meters

ii) $f = 1\,MHz$,

height of antenna $= \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times1\times10^6} = 7$ meters.

From above two examples it is clear that as the transmitting frequency is increased, height of the antenna is decreased.

2) Avoids mixing of Signals:-

All audio (message) Signals ranges from 20 Hz to 20 KHz.

The transmission of message signals from various sources causes the mixing of signals and then it is difficult to separate these signals at the receiver end.

3) Increases the range of communication :-

* Low frequency signals have poor radiation and they get highly attenuated. Therefore baseband signals cannot be transmitted directly over long distances.
* Modulation increases the frequency of the signal and thus they can be transmitted over long distances.

4) Allows multiplexing of signals :-

* Modulation allows the multiplexing to be used. Multiplexing means transmission of two or more signals simultaneously over the same communication channel.

eg :-

1) Number of TV channels operating simultaneously.
2) Number of radio stations broadcasting the signals in MW & SW band simultaneously.

5) Allows adjustments in the bandwidth :-

Bandwidth of a modulated signal may be made smaller or larger.

6) Improves quality of reception :-

Modulation techniques like frequency modulation, pulse

❖ **Define standard form of amplitude modulation and explain the time and frequency domain expression of AM wave**

**July 09- 6M Jan 05 - 4M**

❖ **Define amplitude modulation. Derive the expression on AM by both time domain and frequency domain representation with necessary waveforms.**

**July-08,12M**

· Amplitude modulation is defined as the modulation in which the amplitude of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal keeping its (carrier) frequency & phase constant.

Time-Domain Description :-

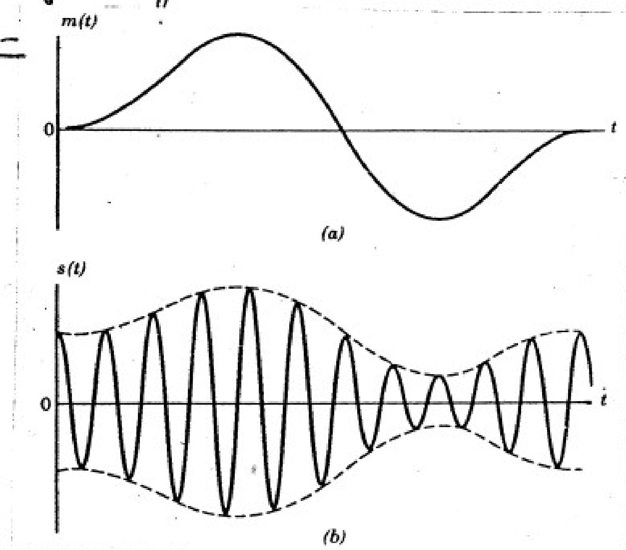



fig (a) Message signal, (b) AM wave $s(t)$.

* The instantaneous value of modulating signal is given by

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (1)$$

Where, $A_m \rightarrow$ maximum amplitude of the modulating Signal

$f_m \rightarrow$ frequency of modulating Signal.

* The Instantaneous value of carrier Signal is given by

$$\boxed{c(t) = A_c \cos(2\pi f_c t)} \longrightarrow (2)$$

Where,

$A_c \rightarrow$ Maximum amplitude of the carrier Signal.

$f_c \rightarrow$ frequency of carrier Signal.

The Standard equation for AM wave is given by

$$\boxed{S(t) = A_c\left[1 + K_a m(t)\right]\cos(2\pi f_c t)} \longrightarrow (3)$$

Where,

$K_a$ is a constant called the amplitude Sensitivity of the modulator.

Substituting eq (1) in eq (3), we get

$$S(t) = A_c\left[1 + K_a A_m \cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

$$S(t) = A_c\left[1 + \mu \cos(2\pi f_m t)\right]\cos(2\pi f_c t)$$

Where, $\boxed{\mu = K_a A_m}$ is called the modulation Index or modulation factor.

$$S(t) = A_c \cos(2\pi f_c t) + \mu A_c \cos(2\pi f_c t) \cdot \cos(2\pi f_m t) \longrightarrow (4)$$

equation (4) can be further expanded, by means of the trigonometric -al relation:

$$\boxed{\cos a \cdot \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)}$$

$$S(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2}\cos\left[2\pi f_c - 2\pi f_m\right]t + \frac{\mu A_c}{2}\cos\left[2\pi f_c + 2\pi f_m\right]t \rightarrow$$

equation (5) is the amplitude modulated Signal, consist of three frequency Component

1) The first term is the carrier itself. It has a frequency '$f_c$' and amplitude '$A_c$'.

2) The 2nd Component is $\frac{\mu A_c}{2}\cos 2\pi(f_c-f_m)t$. It has frequency $(f_c-f_m)$ Called Lower Sideband $(F_{LSB})$ and having amplitude $\frac{\mu A_c}{2}$

3) Similarly 3rd Component is $\frac{\mu A_c}{2}\cos 2\pi(f_c+f_m)t$. It has frequency $(f_c+f_m)$ called upper Sideband $(F_{USB})$ and having - amplitude $\frac{\mu A_c}{2}$.

Frequency - Domain Description :-

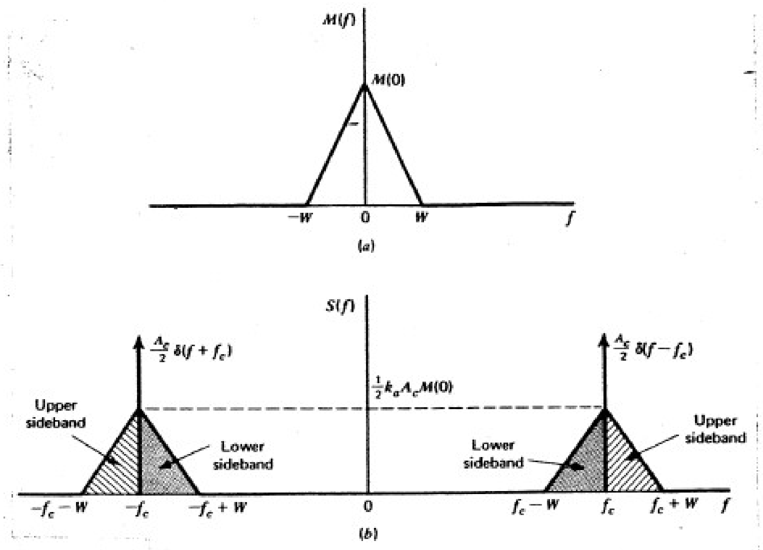
The time domain description of a conventional AM wave is given below:

$$s(t) = A_c[1+K_a m(t)]\cos 2\pi f_c t.$$

$$s(t) = A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t \longrightarrow ①$$

Taking fourier transforms on both the Sides of eq ①, we get

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] + \frac{A_c K_a}{2}\left[M(f-f_c)+M(f+f_c)\right]$$




(a) Spectrum of baseband signal. (b) Spectrum of AM wave.

* The amplitude spectrum of the AM wave has 2 Sidebands on either Sides of $\pm f_c$.
* For +ve frequencies, the highest frequency component of the AM wave equals $f_c+W$, called UPPER Sideband $f_{USB}$ and the lowest frequency component equals $f_c-W$, called LOWER Sideband $f_{LSB}$.

## Transmission Bandwidth ($B_T$):-

The difference between upper Sideband and lower Sideband frequencies defines the transmission bandwidth '$B_T$'.

$$B_T = f_{USB} - f_{LSB}$$
$$= (f_c + f_m) - (f_c - f_m)$$
$$= \cancel{f_c} + f_m - \cancel{f_c} + f_m$$
$$\boxed{B_T = 2f_m}$$

∴ Bandwidth required for transmission of an AM wave is twice the modulating Signal frequency i.e. $2f_m$.

❖ **Define modulation index and percentage modulation index.**

The ratio of change in amplitude of modulating Signal to the amplitude of carrier wave is Known as modulation Index or modulation factor or modulation Co-efficient or depth of - modulation or degree of modulation '$\mu$'.

$$\boxed{\mu = \frac{A_m}{A_c}} \quad \text{or} \quad \boxed{\mu = K_a A_m}$$

percentage modulation index

$$\boxed{\% \ \mu = \left(\frac{A_m}{A_c}\right) \times 100}$$

NOTE :-

* If $A_m$ is greater than $A_c$ then distortion is introduced into the System.
* The modulating Signal voltage '$A_m$' must be less than carrier signal voltage '$A_c$' for proper amplitude modulation.

❖ **Obtain the expression for total transmitted power of AM wave.**

W.K.T

$$S(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi [f_c - f_m] t + \frac{\mu A_c}{2} \cos 2\pi [f_c + f_m] t$$

The AM wave has three components : unmodulated Carrier, Lower Sideband and upper Sideband.

∴ The total power of AM wave is the Sum of the carrier Power '$P_c$' and powers in the two Sidebands i.e. $P_{USB}$ & $P_{LSB}$

$$P_T = P_c + P_{USB} + P_{LSB}$$

* The average carrier power

$$P_c = \frac{(A_c/\sqrt{2})^2}{R}$$

$$\boxed{P_c = \frac{A_c^2}{2R}}$$

W.K.T.
RMS value $V_{rms} = \frac{V_m}{\sqrt{2}}$

$^{||ly}$ $A_{c_{rms}} = \frac{A_c}{\sqrt{2}}$

W.K.T
Power 'p' $= \frac{V_{rms}^2}{R}$

$$P_c = \frac{A_{c_{rms}}^2}{R}$$

$$P_c = \frac{(A_c/\sqrt{2})^2}{R}$$

* The average Sideband power:

$$P_{USB} = P_{LSB} = \frac{(\mu A_c/2\sqrt{2})^2}{R}$$

$$= \frac{\frac{\mu^2 A_c^2}{4\times 2}}{R}$$

$$\boxed{P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}}$$

$$P_{USB} = P_{LSB} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R}$$

$$= \frac{\frac{\mu^2 A_c^2}{4\times 2}}{R}$$

$$P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}$$

∴ The average total power,

$$P_T = P_c + P_{USB} + P_{LSB}$$

$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right]$$

$$\boxed{\frac{\mu^2}{4} + \frac{\mu^2}{4} = \frac{\mu^2}{2}}$$

$$\boxed{P_T = P_c\left[1 + \frac{\mu^2}{2}\right]}$$

For 100% modulation $\mu = 1$, we have

$$P_T = P_c\left[1 + \frac{1^2}{2}\right]$$

❖ **Derive modulation index using AM wave.**

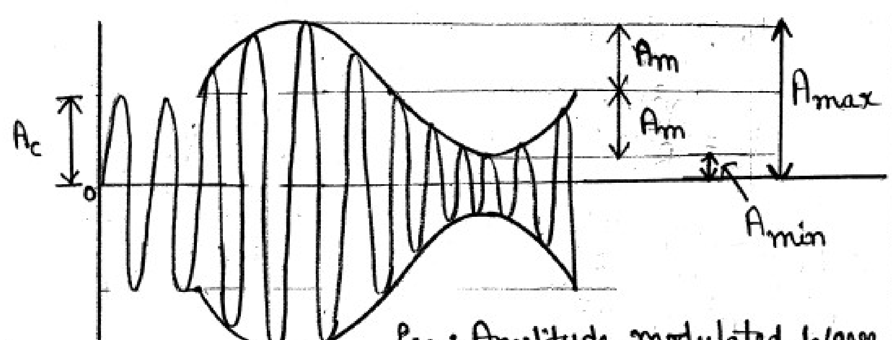



Fig : Amplitude modulated Wave

We can calculate the modulation Index from the amplitude modulate wave.

W.K.T $\boxed{\mu = \frac{A_m}{A_c}}$

From figure,

$$\boxed{A_m = \frac{A_{max} - A_{min}}{2}} \rightarrow (1)$$

$$A_c = A_{max} - \underline{A_m} \rightarrow (2)$$

Substituting equation (1) in equation (2)

$$A_c = A_{max} - \left[\frac{A_{max} - A_{min}}{2}\right]$$

$$A_c = \frac{2A_{max} - A_{max} + A_{min}}{2}$$

$$\boxed{A_c = \frac{A_{max} + A_{min}}{2}}$$

$$\therefore \mu = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}/\cancel{2}}{A_{max} + A_{min}/\cancel{2}}$$

$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

❖ **Explain amplitude modulation for single tone information.**

A Single-tone modulating Signal $m(t)$ has a Single (tone) frequency Component '$f_m$' and is defined as follows:

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \rightarrow (1)$$

Where $A_m$ is the amplitude of the modulating wave and $f_m$ is the frequency of the modulating wave.

Let $$\boxed{c(t) = A_c \cos(2\pi f_c t)} \rightarrow (2)$$

Where $A_c$ is the amplitude of the carrier wave and $f_c$ is the frequency of the carrier wave.

* The time-domain expression for the Standard AM wave is

$$\boxed{S(t) = A_c\left[1 + K_a \underline{m(t)}\right] \cos 2\pi f_c t} \rightarrow (3)$$

Substituting eq (1) in eq (3), we get

$$S(t) = A_c\left[1 + \underline{K_a A_m} \cos 2\pi f_m t\right] \cos 2\pi f_c t$$

Since, the modulation Index $\boxed{\mu = K_a A_m}$

We get
$$S(t) = A_c\left[1 + \mu \cos 2\pi f_m t\right] \cos 2\pi f_c t. \rightarrow (4)$$

equation (4) can be further expanded, by means of the trigonometric -al relation

$$\boxed{\cos a . \cos b = \frac{1}{2} \cos(a-b) + \frac{1}{2} \cos(a+b)}$$

$$S(t) = A_c \cos(2\pi f_c t) + \mu A_c \underbrace{\cos(2\pi f_c t)}_{\cos a} \cdot \underbrace{\cos(2\pi f_m t)}_{\cos b}$$

$$S(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos[2\pi f_c - 2\pi f_m]t + \frac{\mu A_c}{2} \cos[2\pi f_c + 2\pi f_m]t \rightarrow (5)$$

Taking Fourier transform on both sides of eq (5), we get

$$\boxed{S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{\mu A_c}{4}\left\{\delta[f-(f_c-f_m)] + \delta[f+(f_c-f_m)]\right\} + \frac{\mu A_c}{4}\left\{\delta[f-(f_c+f_m)] + \delta[f+(f_c+f_m)]\right\}}$$

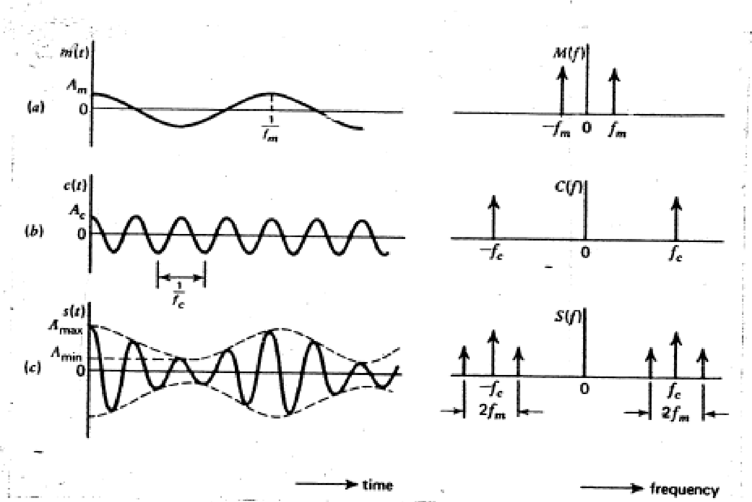



Fig (1) Illustrating the time-domain (on the left) and frequency domain (on the right) characteristics of a standard amplitude modulation produced by a single tone.
(a) Modulating wave (b) Carrier wave (c) AM wave..

* In practice, the AM wave $s(t)$ is a voltage or current wave. The average power delivered by an AM wave to a 1-ohm resistor is calculated as follows:

Average carrier power '$P_c$' $= \frac{A_c^2}{2}$

'$P_{USB}$', Upper Side-frequency power $= \frac{\mu^2 A_c^2}{8}$

'$P_{LSB}$', Lower Side-frequency power $= \frac{\mu^2 A_c^2}{8}$

The transmission efficiency '$\eta$' is the ratio of the total Sideband power to the total power in the modulated wave

$$\eta = \frac{\text{Power in Sidebands}}{\text{Total power } (P_T)} = \frac{P_{USB} + P_{LSB}}{P_c\left[1+\frac{\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2 A_c^2}{8} + \frac{\mu A_c^2}{8}}{P_c\left[1+\mu^2/2\right]} = \frac{\frac{\cancel{2}\,\mu^2 A_c^2}{{}_4\cancel{8}}}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2 A_c^2}{4}}{P_c\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{2}\left[\frac{A_c^2}{2}\right]}{P_c\left[\frac{2+\mu^2}{2}\right]}$$

$$= \frac{\frac{\mu^2}{2}\cancel{P_c}}{\cancel{P_c}\left[\frac{2+\mu^2}{2}\right]} = \frac{\frac{\mu^2}{\cancel{2}}}{\frac{2+\mu^2}{\cancel{2}}}$$

$$\boxed{\eta = \frac{\mu^2}{\mu^2+2}}$$

If $\mu=1$, that is, 100 percent modulation is used, the total power in the two side frequencies of the resulting AM wave is only $1/3^{rd}$ of the total power in the modulated wave as shown in fig ②.

❖ **Derive an expression for multitone amplitude modulation, total transmitted power and total modulation index.**

W.K.T an amplitude modulated wave is expressed as:

$$S(t) = A_c\left[1 + K_a m(t)\right]\cos 2\pi f_c t.$$

For simplicity consider two modulating signal:

$$m_1(t) = A_{m1}\cos 2\pi f_{m1} t$$

$$m_2(t) = A_{m2}\cos 2\pi f_{m2} t.$$

$$\therefore \quad S(t) = A_c\left[1 + K_a\left(m_1(t) + m_2(t)\right)\right]\cos 2\pi f_c t.$$

$$= A_c\left[1 + K_a\left(A_{m1}\cos 2\pi f_{m1} t + A_{m2}\cos 2\pi f_{m2} t\right)\right]\cos 2\pi f_c t.$$

$$= A_c\left[1 + \underbrace{K_a A_{m1}}_{\mu_1}\cos 2\pi f_{m1} t + \underbrace{K_a A_{m2}}_{\mu_2}\cos 2\pi f_{m2} t\right]\cos 2\pi f_c t.$$

$$S(t) = A_c\left[1 + \mu_1\cos 2\pi f_{m1} t + \mu_2\cos 2\pi f_{m2} t\right]\cos 2\pi f_c t.$$

$$S(t) = A_c\cos 2\pi f_c t + \mu_1 A_c\cos 2\pi f_c t\cdot\cos 2\pi f_{m1} t + \mu_2 A_c\cos 2\pi f_c t\cdot\cos 2\pi f_{m2} t$$

W.K.T

$$\boxed{\cos A\cdot\cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$S(t) = A_c\cos 2\pi f_c t + \frac{\mu_1 A_c}{2}\cos 2\pi\left[f_c - f_{m1}\right]t + \frac{\mu_1 A_c}{2}\cos 2\pi\left[f_c + f_{m1}\right]t$$
$$+ \frac{\mu_2 A_c}{2}\cos 2\pi\left[f_c - f_{m2}\right]t + \frac{\mu_2 A_c}{2}\cos 2\pi\left[f_c + f_{m2}\right]t \rightarrow (5)$$

From equation (5) it is clear that, when we have two modulating frequencies, we get four additional frequencies, two upper-sidebands (USB) $f_c + f_{m1}$, $f_c + f_{m2}$ and two lower sidebands 'LSB' $f_c - f_{m1}$, $f_c - f_{m2}$.

## Total transmitted power:-

The total power in the amplitude modulated wave is calculated as follows:

$$P_T = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

$$= \frac{(A_c/\sqrt{2})^2}{R} + \frac{\mu_1 A_c^2}{8R} + \frac{\mu_1^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R} + \frac{\mu_2^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{2R} + \cancel{2}\cdot\frac{\mu_1 A_c^2}{4\cancel{8}R} + \cancel{2}\cdot\frac{\mu_2^2 A_c}{4\cancel{8}R}$$

$$= \frac{A_c^2}{2R} + \frac{\mu_1^2 A_c^2}{4R} + \frac{\mu_2^2 A_c}{4R}$$

$$= \frac{A_c^2}{2R}\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$

$$\boxed{P_T = P_c\left[1 + \frac{\mu_t^2}{2}\right]}$$

$$\boxed{P_c = \frac{A_c^2}{2R}}$$

Where, $\frac{\mu_t^2}{\cancel{2}} = \frac{\mu_1^2}{\cancel{2}} + \frac{\mu_2^2}{\cancel{2}}$

$$\mu_t^2 = \mu_1^2 + \mu_2^2$$

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2}}$$

In general, Total modulation index is given by

$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \cdots + \mu_n^2}}$$

❖ **Explain generation of AM wave using SQUARE- LAW modulator helps to produce AM wave. Derive the related equations and draw the waveforms** **July-05,8M**

❖ **Explain the generation of AM wave using SQUARE- LAW modulator along with relevant diagram & analysis.** **July-08,10M**

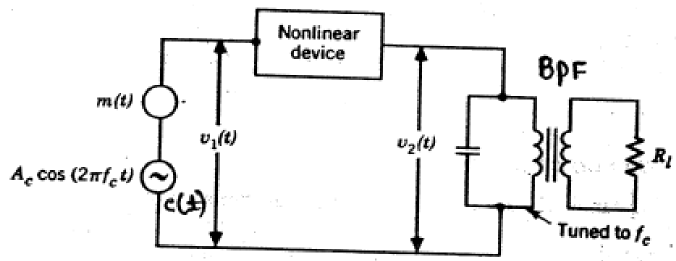



Fig ①: Square-law modulator.

The Square-law modulator consists of three elements:

1) Summer: It adds the carrier and modulating Signal.

2) Non-linear device: A device with non-linear I/p - o/p relation.

3) Band pass filter (BPF): It extract desired Signal (term) from modulator product.

* The Semiconductor diodes or transistor can be used as non-linear element and Single or double tuned circuit can be

used as the filter.

* When a non-linear element such as diode is suitably biased and the signal applied is relatively weak, it is possible to approximate the transfer characteristics as:

$$\boxed{V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)} \longrightarrow ①$$

Where $a_1$ and $a_2$ are constants.

* The I/p voltage '$V_1(t)$' is the sum of carrier signal and modulating signal.

i.e. $$\boxed{V_1(t) = A_c \cos 2\pi f_c t + m(t)} \longrightarrow ②$$

Substituting equation ② in equation ①

$$V_2(t) = a_1 \left[A_c \cos 2\pi f_c t + m(t)\right] + a_2 \left[A_c \cos 2\pi f_c t + m(t)\right]^2$$

W.K.T $\boxed{(a+b)^2 = a^2 + b^2 + 2ab}$

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + a_1 m(t) + a_2 \left[A_c^2 \cos^2 2\pi f_c t + m^2(t) + 2 m(t) . A_c \cos 2\pi f_c t\right]$$

$$V_2(t) = \underline{a_1 A_c \cos 2\pi f_c t} + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t) + \underline{2 a_2 m(t) . A_c \cos 2\pi f_c t} .$$

$$V_2(t) = a_1 A_c \cos 2\pi f_c t + 2 a_2 m(t) A_c \cos 2\pi f_c t + a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t).$$

$$= \underbrace{a_1 A_c \left[1 + \frac{2 a_2}{a_1} m(t)\right] \cos 2\pi f_c t}_{\text{AM Wave}} + \underbrace{a_1 m(t) + a_2 A_c^2 \cos^2 2\pi f_c t + a_2 m^2(t)}_{\text{unwanted terms}} \rightarrow ③$$

❖ **With a neat block diagram, relevant waveforms and expressions explain generation of AM wave using SWITCHING MODULATOR**

**Jan-08,10M**

❖ **Explain the generation of AM wave using SWITCHING MODULATOR with relevant equations waveforms and spectrum before and after filtering process.** **Jan-07,10M Jan-05,6M July-07,10M July-08,6M July-09,8M Jan-10,10M June-107M July-09,8M**

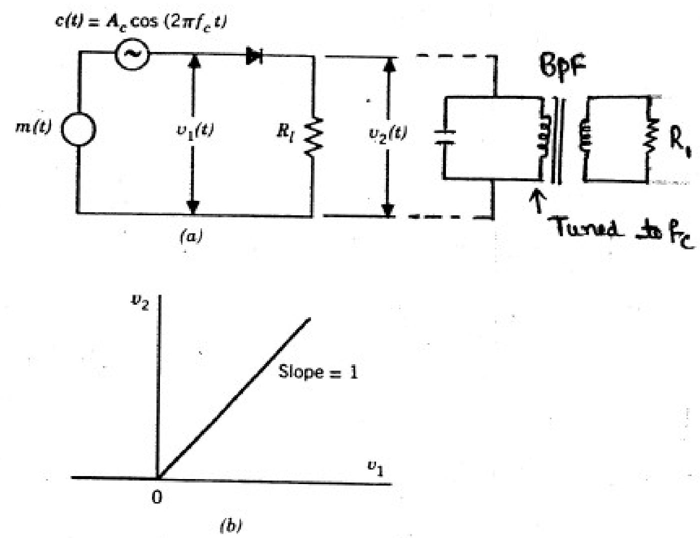



fig ①

*Switching modulator.* (a) *Circuit diagram.* (b) *Idealized input–output relation.*

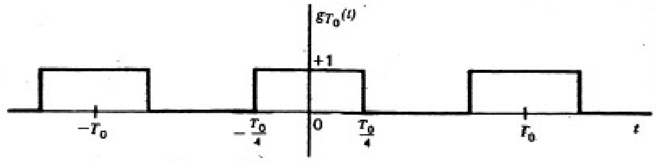



Fig ② Periodic pulse train.

* Consider a Semiconductor diode used as an ideal Switch to which a carrier wave $c(t)$ and an message Signal $m(t)$ are Simultaneously applied as Shown in fig ①.

* It is assumed that the carrier wave $c(t)$ applied to the diode is large in amplitude.

The total I/p '$V_1(t)$' to the diode is given by

$$V_1(t) = m(t) + c(t)$$

$$\boxed{V_1(t) = m(t) + A_c \cos 2\pi f_c t} \longrightarrow ①$$

Where $|m(t)| \ll A_c$.

* The o/p of the diode is

$$V_2(t) = \begin{cases} V_1(t), & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$$

i.e. the o/p of the diode varies between 0 & $V_1$ at a rate equal to carrier frequency $T_0 = \frac{1}{f_c}$.

* The non-linear behavior of the diode can be replaced by assuming the weak modulating Signal compared with the carrier wave. Thus the o/p of the diode is approximately equivalent to linear-time varying operation.

* The first term of equation (3) is the desired AM wave with $K_a = \frac{2a_2}{a_1}$, amplitude sensitivity of the AM wave.

* The remaining three terms are unwanted and are removed by appropriate filtering.

$$\therefore \; S(t) = a_1 A_c [1 + K_a m(t)] \cos 2\pi f_c t.$$

* The o/p of the BPF is

$$V_2'(t) = \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t.$$

$$V_2'(t) = \frac{A_c}{2} \cos 2\pi f_c t \left[ 1 + \frac{2 \cdot 2}{\pi A_c} m(t) \right]$$

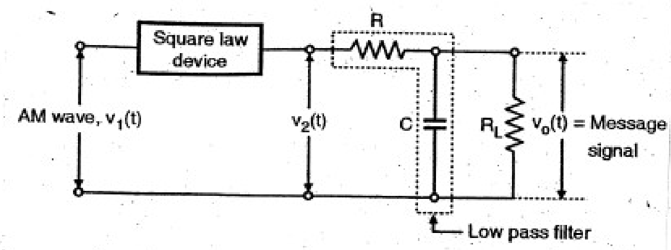
$$= \frac{A_c}{2} \cos 2\pi f_c t \left[ 1 + \frac{4}{\pi A_c} m(t) \right]$$

Where $K_a = \frac{4}{\pi A_c}$ amplitude sensitivity

$$\boxed{V_2'(t) = \frac{A_c}{2} \cos 2\pi f_c t [1 + K_a m(t)]}$$

❖ **Show that a SQUARE LAW device can be used for the detection of an AM wave.** **Jan-07,6M**

❖ **Show that a SQUARE LAW can be used for the detection of an AM wave.** **June-10,6M**




A square law detector

* A Square - law detector is essentially obtained by using a Square - law modulator for the purpose of detection.
* An AM Signal can be demodulated by Squaring it and then passing the Squared Signal through a Low pass filter (LPF)

The transfer characteristics of a non - linear device is given by :

$$\boxed{V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)} \longrightarrow ①$$

Where,

$V_1(t) \rightarrow$ I/p voltage

$V_2(t) \rightarrow$ O/p voltage

$a_1$ and $a_2 \rightarrow$ are Constants.

* The I/p voltage of the AM wave is given by

$$\boxed{V_1(t) = A_c\left[1 + K_a m(t)\right] \cos 2\pi f_c t} \longrightarrow ②$$

Substituting equation (2) in equation (1), we get

$$V_2(t) = a_1\{A_c[1+K_a m(t)]\cos 2\pi f_c t\} + a_2\{A_c[1+K_a m(t)]\cos 2\pi f_c t\}^2$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + a_2\{A_c^2[1+K_a m(t)]^2\cos^2 2\pi f_c t\}$$

W.K.T

$$\boxed{(a+b)^2 = a^2+b^2+2ab} \quad \text{and} \quad \boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + a_2 A_c^2 \cos^2 2\pi f_c t\,[1^2+K_a^2 m^2(t)+2K_a m(t)]$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + a_2 A_c^2\left[\frac{1+\cos 2(2\pi f_c t)}{2}\right]\left[1+K_a^2 m^2(t) + 2K_a m(t)\right]$$

$$V_2(t) = a_1 A_c[1+K_a m(t)]\cos 2\pi f_c t + \frac{a_2 A_c^2}{2}\left[1+K_a^2 m^2(t)+2K_a m(t)\right](1+\cos 4\pi f_c t) \rightarrow (3)$$

* In eq (3) $\frac{a_2 A_c^2}{\cancel{2}}\cancel{2}K_a m(t)$ is the desired term which is due to the $a_2 V_1^2$ term. Hence the name of this detector is - Square law detector.

* The desired term is extracted by using a LPF (fig ). Thus the o/p of LPF is $\boxed{V_0(t) = a_2 A_c^2 K_a m(t)}$

Thus the message signal $m(t)$ is recovered at the o/p of the message signal.

<u>Distortion in the detector o/p :-</u>

* The other term which passes through the LPF to the load resistance $R_L$ is as follows : $\frac{1}{2}a_2 A_c^2 K_a^2 m^2(t)$.

* This is an unwanted Signal & gives rise to a Signal distortion. The ratio of desired Signal to the undesired one is given by:

$$D = \frac{\cancel{\tfrac{1}{2}}\cancel{A_c^2}\cancel{K_a}\cancel{m(t)}}{\frac{1}{2}\cancel{\tfrac{1}{2}}\cancel{A_c^2}K_a^{\cancel{2}}m^{\cancel{2}}(t)} = \frac{1}{\frac{1}{2}K_a m(t)} = \frac{2}{K_a m(t)}$$

{

We Should maximize this ratio in order to minimize the distortion. To achieve this we Should Choose $|K_a m(t)|$ Small as Compared to unity for all values of $t$. If $K_a$ is Small then the AM wave is weak.

}

## Envelope Detector:

❖ **How a modulating signal can be detected using a AM detector? Use a envelope detector and explain.** **July-05,8M**

❖ **Explain the detection of message signal from amplitude modulated signal using an envelope detector & bring out the significance of RC time constant**

**July-09,6M July-07,5M June-09,6M July-06,5M**

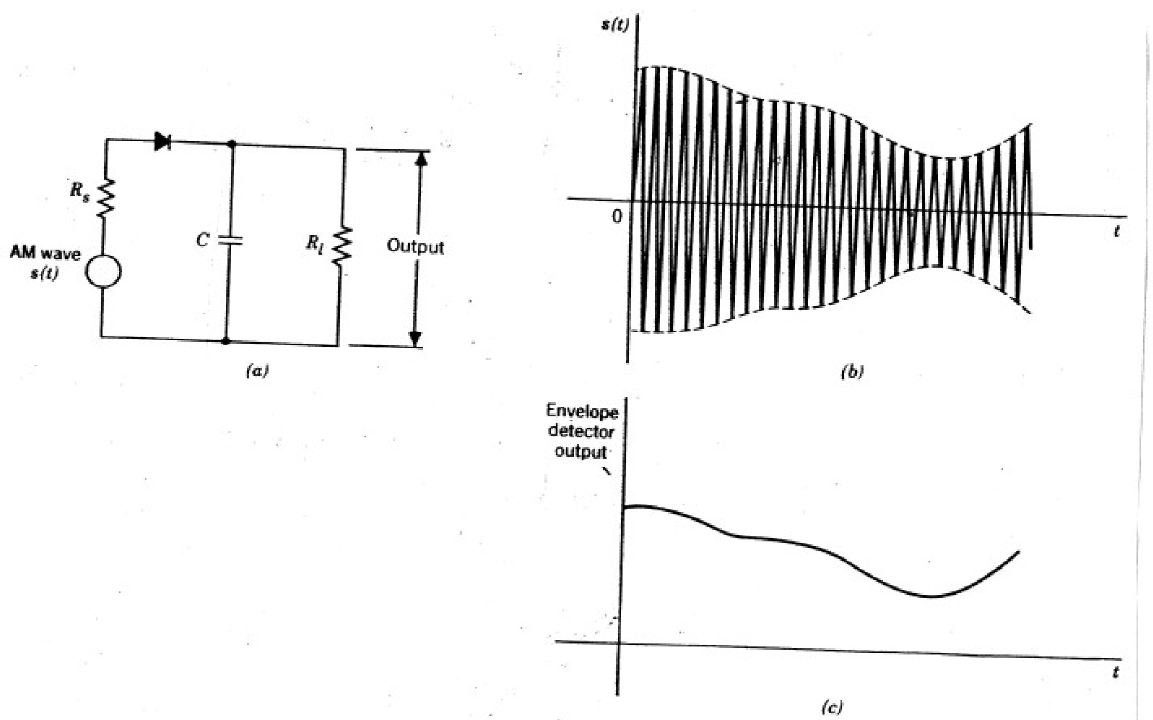



**Figure**
*Envelope detector. (a) Circuit diagram. (b) AM wave input. (c) Envelope detector output.*

* Envelope detector is a Simple and highly effective device used to demodulate AM Wave. It Consists of a diode and a resistor capacitor (RC) filter.

## Operation :-

During positive half cycle of the I/p Signal, diode is forward-biased and the Capacitor 'C' charges upto the peak-value of the I/p Signal. When the I/p voltage falls below this value the diode becomes reverse biased and capacitor 'C' discharges slowly through the load resistor $R_L$. As a result only positive half cycle of AM wave appears across $R_L$.

The discharging process continues until the next positive half cycle. When the I/p Signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

## Selection of the RC time Constant :-

* The Capacitor charges through 'D' & $R_s$ when the diode is 'ON' & it discharges through '$R_L$' when diode is OFF.
* The Charging time Constant $R_sC$ should be short as compared to the carrier period $1/f_c$ $\therefore$ $\boxed{R_sC \ll \frac{1}{f_c}}$ So capacitor 'c' charges rapidly.
* on the other hand the discharging time Constant $R_LC$ should be long enough to ensure that the Capacitor discharges slowly through the load resistance '$R_L$' b/w +ve peak of the carrier wave

i.e. $\boxed{\frac{1}{f_c} \ll R_LC \ll \frac{1}{W}}$, Where W = Maximum modulating frequency.

Result is that the capacitor voltage or detector o/p is very nearly the Same as the envelope of AM wave. The detector

## ❖ What is DSB-SC modulation? Explain the time and frequency domain expression of DSB-SC wave.

To overcome the drawback of power wastage in AM wave (DSB-FC) an DSB-SC method is used.

* DSB-SC is a method of transmission where only the TWO Sidebands are transmitted without the Carrier (Suppressing Carrier)

OR

The Conventional AM wave in which the carrier is Suppressed is called DSB-SC modulation.

Time domain representation of DSB-SC Wave :-

* Let $m(t)$ be the message Signal having a bandwidth equal to 'W' Hz and

$c(t) = A_c \cos 2\pi f_c t$ represents the carrier, then the time-domain expression for DSB-SC wave is

$$S(t) = m(t)\, c(t)$$

$$\boxed{S(t) = A_c \cos(2\pi f_c t)\, m(t)} \longrightarrow (1)$$

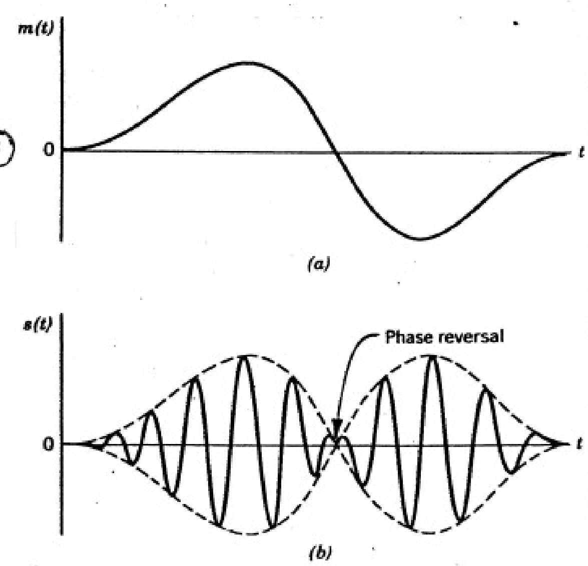



**Figure**
(a) *Message signal.* (b) *DSBSC-modulated wave* s(t).

* The $s(t)$ Signal undergoes a phase reversal whenever the message Signal crosses Zero.

Frequency - Domain Description :-

Taking Fourier transform on both Sides of equation ②, we get

$$\boxed{S(f) = \frac{A_c}{2}\left[M(f-f_c)+M(f+f_c)\right]} \longrightarrow (3)$$

Where $S(f)$ is the Fourier transform of the modulated wave $s(t)$

$M(f)$ is the Fourier transform of the message Signal $m(t)$.

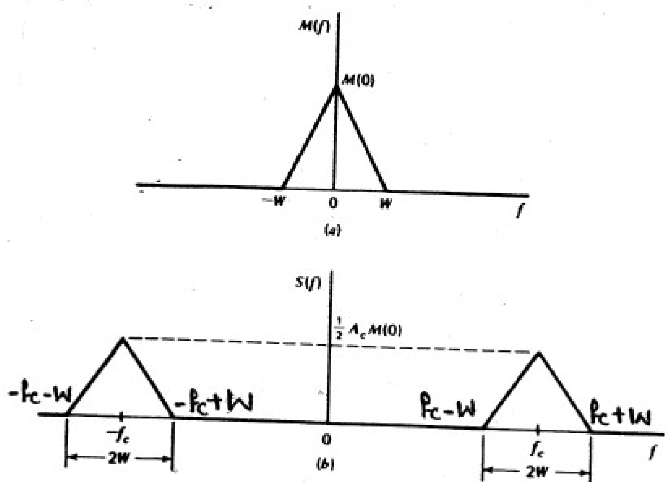



**Figure**
(a) *Spectrum of message signal.* (b) *Spectrum of DSBSC modulated wave.*

The amplitude Spectrum drawn above exhibits the following factors

i) on either Sides of $\pm f_c$, we have two Sidebands designated as Lower and upper Sidebands.

$$s(t) = A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t$$

W.K.T.

$$\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$$

$$\boxed{s(t) = \frac{A_m A_c}{2}\cos 2\pi(f_c - f_m)t + \frac{A_m A_c}{2}\cos 2\pi(f_c + f_m)t} \rightarrow (1)$$

Taking Fourier transform on both sides of the equation (1)

$$S(f) = \frac{A_m A_c}{4}\left\{\delta[f-(f_c-f_m)] + \delta[f+(f_c-f_m)]\right\}$$
$$+ \frac{A_m A_c}{4}\left\{\delta[f-(f_c+f_m)] + \delta[f+(f_c+f_m)]\right\}.$$

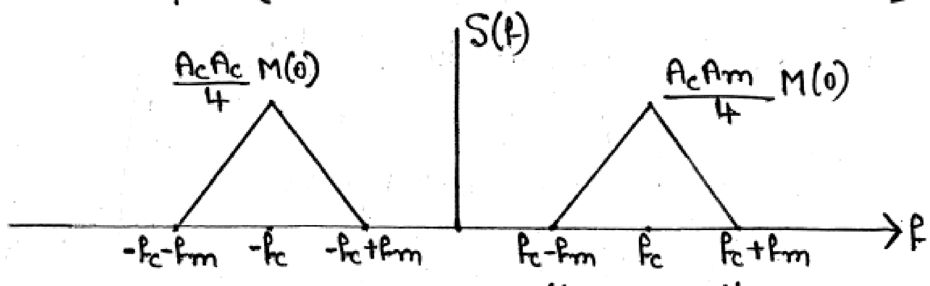



Fig (2)

Fig (2) shows amplitude spectrum of a DSB-SC signal. We observe that either side of $\pm f_c$, we have lower and upper sideband also the carrier term is suppressed in the spectrum as there are no impulses at $\pm f_c$.

* The minimum transmission bandwidth in DSB-SC is '$2f_m$'.

## Balanced Modulator:

❖ **With a neat block diagram, explain the balanced modulator method of generating DSB-SC wave.** **June-10,6M**

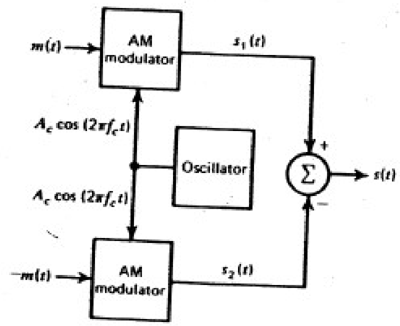



**Figure**
*Balanced modulator.*

Fig ① Shows the block diagram of a balanced modulator used for generating a DSB-SC Signal.

* It Consist of two amplitude modulators that are interconnected in Such a way as to Suppress the Carrier.

* one I/p to the amplitude modulator is from an oscillator that generates a Carrier wave. The Second I/p to the amplitude modulator in the top path is the modulating Signal $m(t)$ while in the bottom path is $-m(t)$.

. The o/p of the two AM modulator are as follows:

$$S_1(t) = A_c[1+K_a m(t)]\cos 2\pi f_c t \quad \text{and}$$

$$S_2(t) = A_c[1-K_a m(t)]\cos 2\pi f_c t.$$

The o/p of the Summer is

$$S(t) = S_1(t) - S_2(t)$$

$$S(t) = A_c[1+K_a m(t)]\cos 2\pi f_c t - [A_c(1-K_a m(t))\cos 2\pi f_c t]$$

$$= A_c\cos 2\pi f_c t + A_c K_a m(t)\cos 2\pi f_c t - [A_c\cos 2\pi f_c t - A_c K_a m(t)\cos 2\pi f_c t]$$

$$= \cancel{A_c\cos 2\pi f_c t} + A_c K_a m(t)\cos 2\pi f_c t - \cancel{A_c\cos 2\pi f_c t} + A_c K_a m(t)\cos 2\pi f_c t.$$

$$\boxed{S(t) = 2A_c K_a m(t)\cos 2\pi f_c t} \rightarrow ①$$

* The balanced modulator o/p is equal to the product of the modulating Signal $m(t)$ & Carrier $C(t)$ except the Scaling factor $2K_a$.

Taking Fourier Transform on both Side of equation ①, we get

$$S(f) = \frac{\cancel{2}A_c K_a}{\cancel{2}}[M(f-f_c) + M(f+f_c)]$$

$$\boxed{S(f) = A_c K_a[M(f-f_c) + M(f+f_c)]}$$

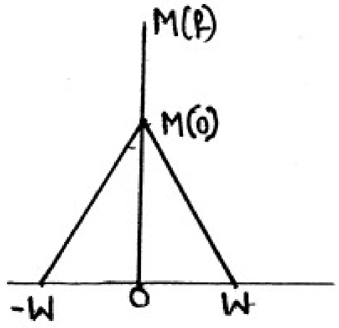



fig ⓐ: Message Spectrum

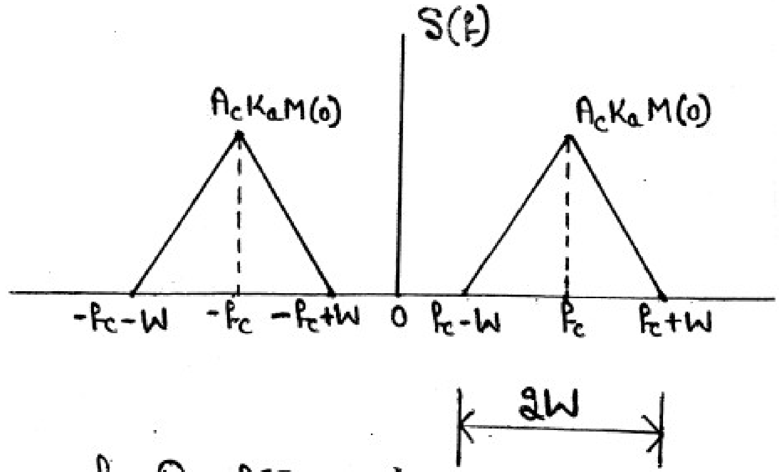


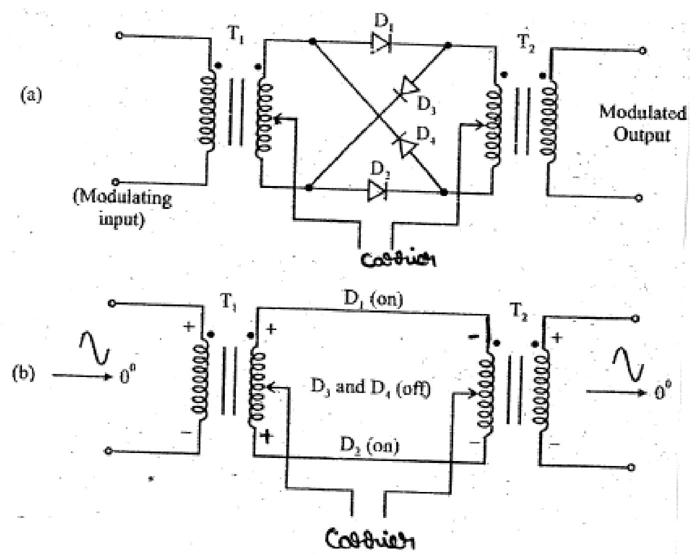

fig ⓑ: DSB-SC Spectrum

Since the carrier component is eliminated, the o/p is called DSB-SC Signal.

❖ **Explain how RING modulator can be used to generate DSB-SC modulation** **Jan-05,9M**

❖ **Briefly explain generation of DSB-SC modulated wave using RING modulator. Give relevant mathematical expressions and waveforms.**

**Jan-08,10M Jan-07,8M Jan-09,6M July-09,10M June-10,10m**



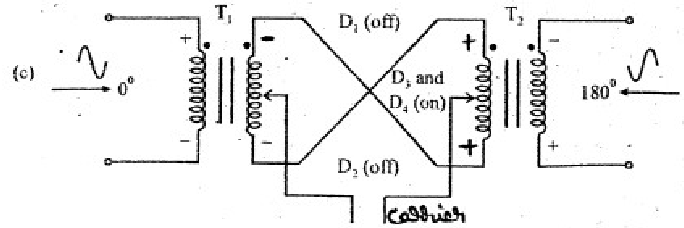



*Fig. : (a) Balanced Ring Modulator*
*(b) Equivalent Circuit when square wave carrier positive*
*(c) Equivalence circuit when square wave carrier negative*

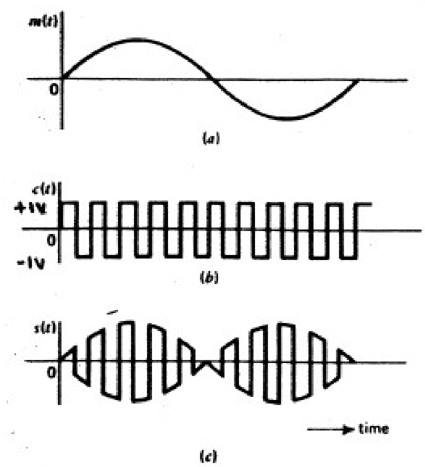



**Figure**
*Waveforms illustrating the operation of the ring modulator for a sinusoidal modulating wave. (a) Modulating wave. (b) Square-wave carrier. (c) Modulated wave.*

Ring modulator is a product modulator used for generating DSB-SC modulated wave. The ring modulator consists of :-

1) I/p transformer '$T_1$'
2) O/p transformer '$T_2$'
3) Four diodes connected in a bridge circuit (Ring)

The carrier amplitude '$A_c$' is greater than the modulating signal amplitude '$A_m$' i.e. $A_c > A_m$ and carrier frequency '$f_c$' is greater than modulating signal '$f_m$' $= W$ i.e. $f_c > W$. These conditions ensures that the diode operation is controlled by $c(t)$ only.

* The diodes are controlled by a square wave carrier $c(t)$

of frequency '$f_c$' which is applied by means of two center-tapped transformer.

* The modulating signal $m(t)$ is applied to the I/p transformer '$T_1$'. The o/p appears across the secondary of the transformer '$T_2$'.

Operation :-

i) When the carrier is +ve, the diodes $D_1$ & $D_2$ are forward-biased and diodes $D_3$ & $D_4$ are reverse biased. Hence the modulator multiplies the message signal $m(t)$ by +1 i.e. $V_0(t) = m(t)$.

ii) When the carrier is -ve, the diodes $D_3$ & $D_4$ are forward biased whereas $D_1$ & $D_2$ are reverse biased. Thus the modulator multiplies the message signal $m(t)$ by -1 i.e. $V_0(t) = -m(t)$.

* The square wave carrier $c(t)$ can be represented by a Fourier series as:

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t (2n-1)]$$

$$c(t) = \frac{4}{\pi} \Big[ \underbrace{\cos 2\pi f_c t}_{n=1} - \underbrace{\frac{1}{3} \cos 6\pi f_c t}_{n=2} + \cdots\cdots \Big] \longrightarrow (1)$$

The ring modulator o/p is

$$s(t) = c(t) \cdot m(t) \longrightarrow (2)$$

Substituting equation (1) in equation (2), we get

$$s(t) = \Big[\frac{4}{\pi} \cos 2\pi f_c t - \frac{4}{3\pi} \cos 6\pi f_c t + \cdots\cdots\Big] m(t)$$

$$s(t) = \frac{4}{\pi} m(t) \cos 2\pi f_c t - \frac{4}{3\pi} m(t) \cos 6\pi f_c t + \cdots\cdots \Big] \longrightarrow (3)$$

{ Taking Fourier transform on both sides of equation (3), we get

$$S(f) = \frac{2\cancel{4}}{\cancel{2}\pi}\left[M(f-f_c) + M(f+f_c)\right] - \frac{2\cancel{4}}{\cancel{2}\times 3\pi}\left[M(f-3f_c) + M(f+3f_c)\right]$$

$$\boxed{S(f) = \frac{2}{\pi}\left[M(f-f_c)+M(f+f_c)\right] - \frac{2}{3\pi}\left[M(f-3f_c)+M(f+3f_c)\right]}$$

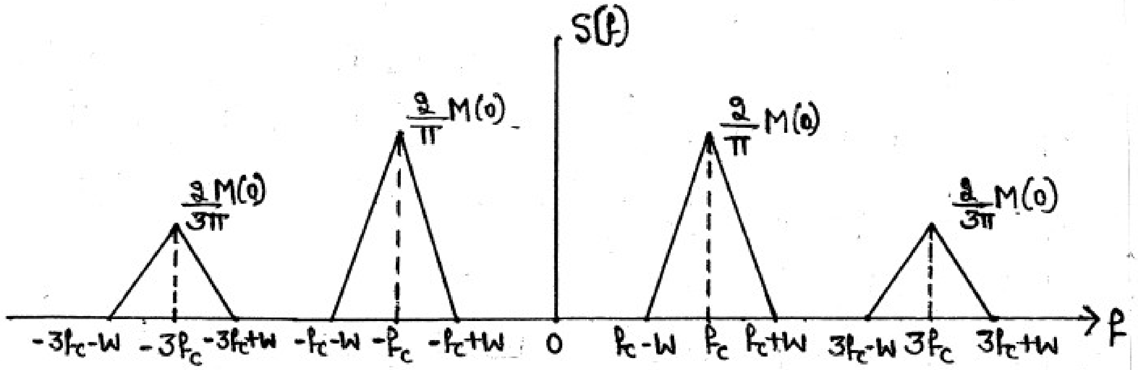



Fig : Amplitude Spectrum of S(f).

}

* The DSB-SC Wave is extracted from S(t) by passing equation (3) (S(t)) through an Ideal BPF having centre frequency '$f_c$' and bandwidth equal to 2WHz.

The o/p of the BPF is

$$\boxed{S(t) = \frac{4}{\pi}m(t)\cos 2\pi f_c t}$$

## COHERENT Detection of DSB-SC wave:-

❖ **With block diagram and related equations explain coherent detection of a DSB-SC wave. What are its disadvantages? Explain the synchronous receiving system(COSTAS Loop)** **June-10,8M July-08,10M**

❖ **Write a note on how coherent detection is used in DSB-SC receiver** **July-06,7M**

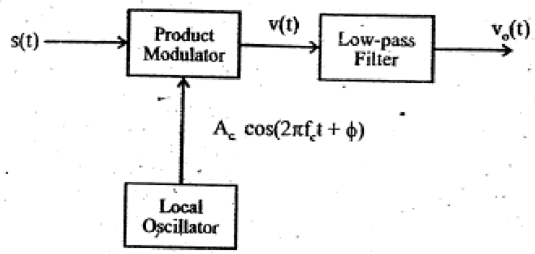



*Fig. : Coherent detective for DSBSC*

* The modulating Signal $m(t)$ is recovered from a DSB-SC wave $S(t)$ by first multiplying $S(t)$ with a locally generated carrier wave and then low pass filtering the product as shown in fig ①.

* For faithful recovery of modulating signal $m(t)$, the local oscillator o/p should be exactly coherent or synchronized in both frequency and phase with the carrier wave $C(t)$ used in the product modulator to generate $V_o(t)$ with the local oscillator o/p equal to $\cos(2\pi f_c t + \phi)$.

The product modulator o/p can be given as:

$$V(t) = S(t) \cdot \cos(2\pi f_c t + \phi) \longrightarrow ①$$

W.K.T $\quad S(t) = A_c \cos 2\pi f_c t \cdot m(t) \longrightarrow ②$

Substituting equation ② in equation ①, we get

$$V(t) = A_c \cos(2\pi f_c t + \phi) \cos(2\pi f_c t) \cdot m(t)$$

W.K.T $\boxed{\cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)}$

$$V(t) = \frac{A_c m(t)}{2}\left[\cos(\cancel{2\pi f_c t} + \phi - \cancel{2\pi f_c t})\right] + \frac{A_c m(t)}{2}\left[\cos(2\pi f_c t + \phi + 2\pi f_c t)\right]$$

$$V(t) = \frac{A_c m(t)}{2}\cos\phi + \frac{A_c m(t)}{2}\cos(4\pi f_c t + \phi) \longrightarrow (2)$$

{ Taking Fourier transform on both Sides of equation (2), we get

$$V(f) = \frac{A_c}{2}M(f)\cos\phi + \frac{A_c}{4}\left[M(f-2f_c) + M(f+2f_c)\right]$$

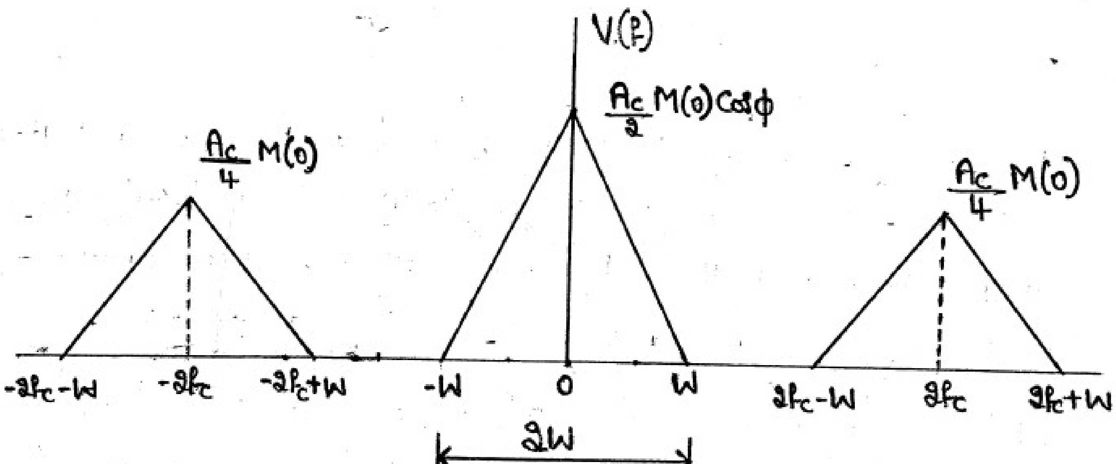



Fig (2) Amplitude Spectrum of v(f).

}

* The desired message Signal is obtained by passing V(t) through a LPF having the bandwidth greater than 'W' Hz but less than '$2f_c - W$' Hz.

* The o/p of the LPF is

$$\boxed{V_o(t) = \frac{A_c}{2}\cos\phi \, m(t)}$$

The demodulated Signal $V_o(t)$ is therefore proportional to m(t).

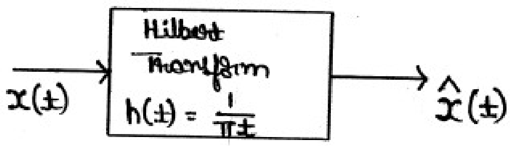
Where, $\phi \rightarrow$ phase error.

## Hilbert Transform:-

* The device which produces a phase shift of $-90°$ for all +ve frequencies & a phase shift of $+90°$ for all -ve frequencies.

The amplitude of all frequency components of the I/p signal are unaffected by transmission through device.

Such an ideal device is called a Hilbert transform.



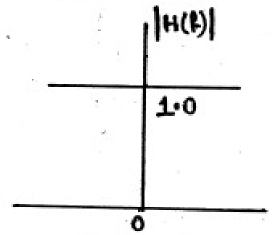




Fig ①: Amplitude response

$$\hat{x}(t) = x(t) * h(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\boxed{\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{(t-\tau)}\, d\tau}$$

Where, $\hat{x}(t)$ is the hilbert transform of $x(t)$

## INVERSE Hilbert transform :-

We can recover back the original signal $x(t)$ back from $\hat{x}(t)$ by taking the inverse hilbert transform as follows:

$$\boxed{x(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\hat{x}(\tau)}{(t-\tau)}\, d\tau}$$

## Interpretation of Hilbert Transform :-

The Fourier transform of $x(t)$ & $\frac{1}{\pi t}$ are as follows:

$$\boxed{\begin{array}{l} x(t) \xrightarrow{FT} X(f) \\[6pt] \frac{1}{\pi t} \xrightarrow{FT} -j\,\mathrm{Sgn}(f) \end{array}}$$

Where Sgn is the Signum function defined as

$$\mathrm{Sgn} = \begin{cases} 1 & , f>0 \\ 0 & , f=0 \\ -1 & , f<0 \end{cases}$$

$$\therefore \boxed{\hat{x}(t) = x(t) * \frac{1}{(\pi t)}} \longrightarrow ①$$

Taking Fourier Transform on both Side of eq①, we get

$$\hat{X}(f) = X(f)\left[-j\,\mathrm{Sgn}(f)\right]$$

$$\therefore \boxed{\hat{X}(f) = -j\,\mathrm{Sgn}(f).\,X(f)} \longrightarrow ②$$

Thus the hilbert transform $\hat{x}(t)$ of Signal $x(t)$ is obtained by passing $x(t)$ through a linear two port device whose transfer function is equal to $-j\mathrm{Sgn}(f)$ as shown below.

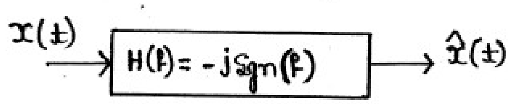



Fig ①: Two port device

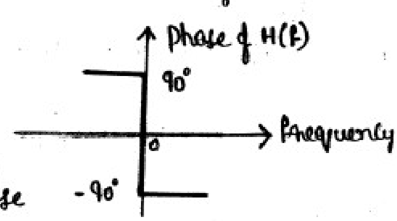



Fig ② phase response

## Canonical representation of Band pass Signals:-

* Let the pre-envelope of a Narrow band Signal $x(t)$, with its FT $X(f)$ centered about some frequency $\pm f_c$, be expressed in the form

$$x_+(t) = \tilde{x}(t)\exp(j2\pi f_c t)$$

$$\boxed{x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t}} \longrightarrow (1)$$

Where $\tilde{x}(t)$ is the complex envelope of the Signal.

Eq ① is the definition for the Complex envelope $\tilde{x}(t)$ in terms of the pre-envelope $x_+(t)$.

* The Spectrum of $x_+(t)$ is limited to the frequency band $f_c - W \leq f \leq f_c + W$ as Shown in fig ①.

* Applying frequency - Shifting property of the Fourier transform to eq ①, then the Spectrum of the Complex envelope $\tilde{x}(t)$ is limited to the band $-W \leq f \leq W$ & Centered at the origin as Shown in fig ©.

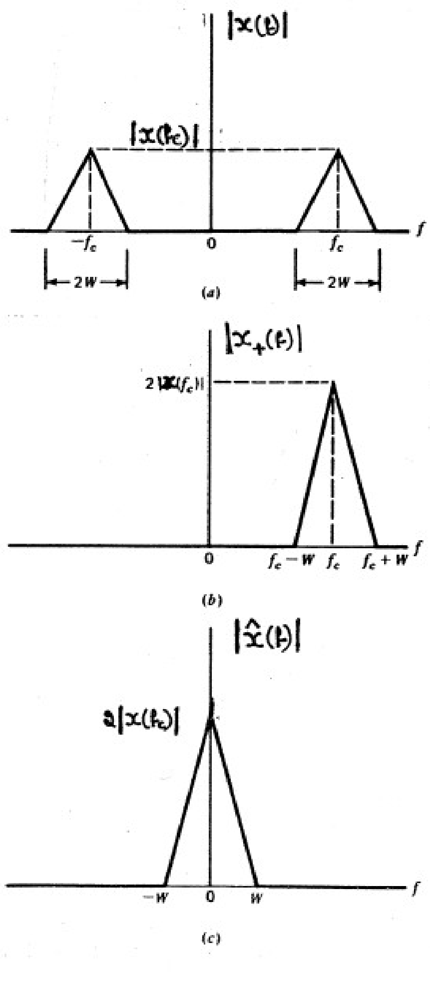



Figure . (a) Amplitude spectrum of band-pass signal $x(t)$. (b) Amplitude spectrum of pre-envelope $x_+(t)$. (c) Amplitude spectrum of complex envelope $\tilde{x}(t)$.

* The Signal $x(t)$ is the real part of the pre-envelope $x_+(t)$. Hence the given bandpass Signal $x(t)$ can be expressed interms of the Complex envelope as:

$$\boxed{x(t) = R_e\left[\hat{x}(t)\, e^{j2\pi f_c t}\right]} \longrightarrow (1)$$

* In general, $\hat{x}(t)$ is a Complex quantity, we can express it as:

$$\boxed{\hat{x}(t) = x_I(t) + j\,x_Q(t)} \longrightarrow (2)$$

Substituting eq (2) in eq (1), we get

$$x(t) = R_e\left[x_I(t) + j\,x_Q(t)\, \underline{e}^{j2\pi f_c t}\right] \longrightarrow (3)$$

W.K.T $\quad e^{j\theta} = \cos\theta + j\sin\theta \qquad\qquad \theta = 2\pi f_c t$

$$\boxed{e^{j2\pi f_c t} = \cos(2\pi f_c t) + j\sin(2\pi f_c t)} \longrightarrow (4)$$

Substituting eq (4) in eq (3), we get

$$x(t) = R_e\left\{\left[x_I(t) + j x_Q(t)\right]\left(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right)\right\}$$

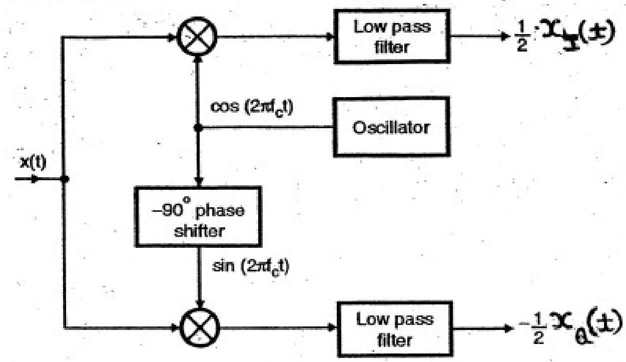
$$x(t) = R_e\left\{x_I(t)\cos(2\pi f_c t) + j x_I(t)\sin(2\pi f_c t) + j x_Q(t)\cos(2\pi f_c t) + \underline{j}^2 x_Q(t)\sin(2\pi f_c t)\right\}$$

$$x(t) = R_e\left\{x_I(t)\,\underline{\cos}(2\pi f_c t) + j x_I(t)\sin(2\pi f_c t) + j x_Q(t)\cos(2\pi f_c t) + (-1)\, x_Q(t)\,\underline{\sin}(2\pi f_c t)\right\}$$

$$\boxed{x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin(2\pi f_c t)} \longrightarrow (5)$$

* In eq (5) $x_I(t)$ is the in-phase Component of the band pass Signal & $x_Q(t)$ is the Quadrature of the Signal $x(t)$.

## Generation of In-Phase and Quadrature phase components:-

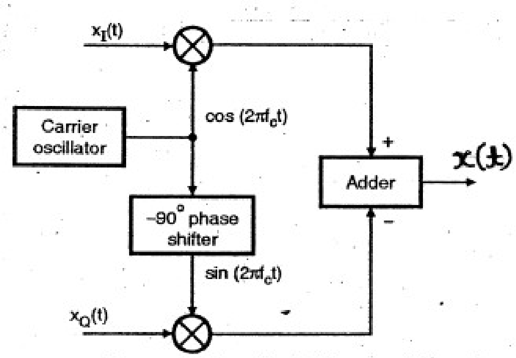



**Scheme to generate the in phase and quadrature components of bandpass signal x ( t )**

* The $x_I(t)$ & $x_Q(t)$ are low pass Signals limited to the band $-W \leq f \leq W$. The bandwidth of each filter is 'W'.

* The in-phase component $x_I(t)$ is produced by multiplying $x(t)$ with $\cos(2\pi f_c t)$ & passing the product through a LPF.

* The Quadrature component $x_Q(t)$ is obtained by multiplying $x(t)$ with $\sin(2\pi f_c t)$ & passing the product through an Identical LPF.




**Reconstruction of x ( t ) from $x_I$ ( t ) and $x_Q$ ( t )**

* The in-phase low pass Signal $x_I(t)$ & $x_Q(t)$ are multiplied with the $\cos(2\pi f_c t)$ & $\sin(2\pi f_c t)$ respectively.

* The resultant product terms are then Subtracted to get the bandpass Signal $x(t)$.

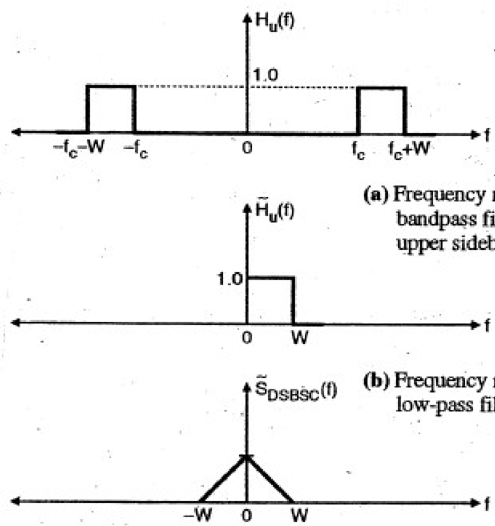
* The multiplication process of $x_I(t)$ & $x_Q(t)$ with the carriers is a - linear modulation process.

## Time-domain description of SSB wave :-

- **Using Hilbert transform, derive the equations for SSB signals. Specify the advantages of SSB over DSB-SC.** **July-05,12M**
- **Derive an expression for SSB modulated wave for which upper sideband is retained.** **Jan-09,8M Jan-05,8M**

Sol :-

The SSB Signal may be generated by passing a DSB-SC modulated wave through a BPF of transfer function $H_u(f)$.




(a) Frequency response of ideal bandpass filter to select the upper sideband.

(b) Frequency response of equivalent low-pass filter.

(c) Spectrum of complex envelope of DSBSC.

* The DSB-SC modulated wave is defined mathematically as

$$\boxed{S_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t)}$$

Where, $m(t) \rightarrow$ Message Signal

$A_c \cos 2\pi f_c t \rightarrow$ Carrier Signal

* The Low pass Complex envelope of the DSB-SC modulated wave is expressed as:

$$\boxed{\tilde{S}_{DSBSC}(t) = A_c m(t)}$$

* Consider the SSB modulated wave $S_u(t)$, in which only the USB is retained. It has quadrature as well as in-phase Component.

Then $\tilde{S}_u(t)$ is the Complex envelope of $S_u(t)$ & we can write

$$S_u(t) = R_e\left[\tilde{S}_u(t) \exp(j2\pi f_c t)\right]$$

$$\boxed{S_u(t) = R_e\left[\tilde{S}_u(t)\, e^{j2\pi f_c t}\right]} \longrightarrow (1)$$

Where, $R_e \longrightarrow$ real part.

* To determine $\tilde{S}_u(t)$, we proceed as follows :

i) The BPF transfer function $H_u(f)$ is replaced by an equivalent LPF of transfer function $\tilde{H}_u(f)$ as shown in fig (b).

We can express $\tilde{H}_u(f)$ as follows:

$$\tilde{H}_u(f) = \begin{cases} \frac{1}{2}[1+\mathrm{Sgn}(f)] & , \quad 0<f<W \\ 0 & , \quad \text{otherwise} \end{cases} \longrightarrow (2)$$

Where, $\mathrm{Sgn}(f)$ is the Signum function.

ii) The DSB-SC modulated wave is replaced by its Complex-envelope. The Spectrum of this envelope is as shown in fig (c). i.e. $\boxed{\tilde{S}_{DSBSC}(f) = A_c M(f)} \longrightarrow (3)$

iii) The desired Complex envelope $\tilde{S}_u(t)$ is determined by evaluating the IFT of the product $\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)$

i.e. $\tilde{S}_u(t) = \mathrm{IFT}\left[\tilde{H}_u(f) \cdot \tilde{S}_{DSBSC}(f)\right] \longrightarrow (4)$

Substituting eq ② & eq ③ in eq ④, we get

$$\tilde{S}_u(t) = \text{IFT}\left[\frac{1}{2}\left[1+\text{Sgn}(f)\right]\cdot A_c M(f)\right]$$

$$= \text{IFT}\left\{\frac{A_c}{2}\left[M(f) + \text{Sgn}(f)M(f)\right]\right\}$$

$$\boxed{\tilde{S}_u(t) = \frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right]} \longrightarrow ⑤$$

Substituting eq ⑤ in eq ①, we get

i.e. $S_u(t) = R_e\left[\underline{\tilde{S}_u(t)}\, e^{j2\pi f_c t}\right] \longrightarrow ①$

$$S_u(t) = R_e\left\{\frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right]\cdot e^{j2\pi f_c t}\right\}$$

$$S_u(t) = R_e\left\{\frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right]\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right\}$$

$$= R_e\left\{\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + jm(t)\sin(2\pi f_c t) + j\hat{m}(t)\cos(2\pi f_c t) + \underline{j^2}\,\hat{m}(t)\sin(2\pi f_c t)\right]\right\}$$

$$= R_e\left\{\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + jm(t)\sin(2\pi f_c t) + j\hat{m}(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]\right\}$$

$$\boxed{S_u(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]} \longrightarrow ⑥$$

In-phase Component　　　　Quadrature Component

Equation ⑥ Shows that the SSB modulated wave Contains only USB With an In-phase Component & a quadrature Component.

## Single Tone SSB Modulation

**Explain single tone modulation for transmitting only upper side (USB) frequency of SSB modulation.**

* Let the modulating signal $m(t)$ is represented as

$$\boxed{m(t) = A_m \cos(2\pi f_m t)} \longrightarrow (1)$$

* The hilbert transform of the modulating signal $m(t)$ is obtained by passing it through a -90° phase shifter. So the hilbert transform is given by

$$\boxed{\hat{m}(t) = A_m \sin(2\pi f_c t)} \longrightarrow (2)$$

* WKT the SSB wave with only USB is given by

$$S_u(t) = \frac{A_c}{2}\left[\underline{m(t)} \cos(2\pi f_c t) - \underline{\hat{m}(t)} \sin(2\pi f_c t)\right] \longrightarrow (3)$$

Substituting eq (1) & eq (2) in eq (3), we get

$$S_u(t) = \frac{A_c}{2}\left[A_m \cos(2\pi f_m t) \cdot \cos(2\pi f_c t) - A_m \sin(2\pi f_m t) \cdot \sin(2\pi f_c t)\right]$$

W.K.T

$$\boxed{\cos(A+B) = \cos(A) \cdot \cos(B) - \sin(A) \cdot \sin(B)}$$

$$S_u(t) = \frac{A_c A_m}{2}\left[\underset{\cos(A)}{\cos(2\pi f_c t)} \cdot \underset{\cos(B)}{\cos(2\pi f_m t)} \underset{-}{-} \underset{\sin(A)\cdot}{\sin(2\pi f_c t)} \cdot \underset{\sin(B)}{\sin(2\pi f_m t)}\right]$$

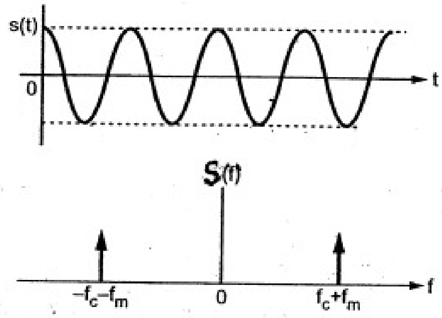
$$S_u(t) = \frac{A_c A_m}{2}\left[\cos(2\pi f_c t + 2\pi f_m t)\right]$$

$$\boxed{S_u(t) = \frac{A_c A_m}{2} \cos 2\pi\left[f_c + f_m\right] t} \longrightarrow (4)$$

Equation (4) Shows that the SSB Wave Consists of only the upper Sideband of frequency $(f_c + f_m)$.

* This is exactly Same as the result obtained by Suppressing the lower Side-frequency $(f_c - f_m)$ of the corresponding DSB-SC Wave.

Spectrum of SSB with lower sideband supressed



**Phase discriminator method** or **Hartley Modulator :-**

- **Explain the generation of SSB-SC wave using Phase discrimination method with the help of a neat functional diagram. Bring out the merits and demerits of this.** **Jan-06,8M**

**With a neat diagram, explain how SSB wave is generated using Phase shift method.**

**June-10,8M June-10,6M(IT) Jan-10,7M Jan-07,5M July-06,7M**

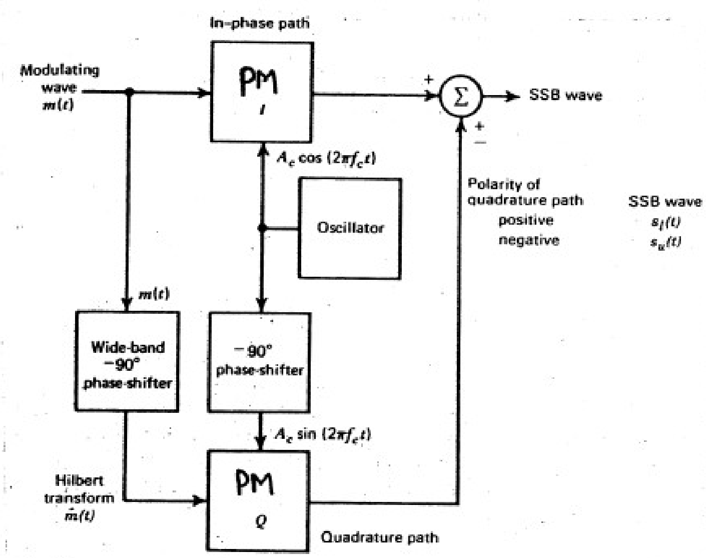



**Figure**
*Block diagram of the phase discrimination method for generating SSB modulated waves.*

Fig ① Shows the block diagram of phase discriminator method of generating SSB.

* The SSB modulator uses two product modulators I & Q, Supplied with carrier waves in phase quadrature to each other.

* The message signal $m(t)$ & a carrier signal $A_c \cos(2\pi f_c t)$ is directly applied to the product modulator I, producing a DSB-SC wave.

* The hilbert transform $\hat{m}(t)$ ($-90°$ phase shift) of $m(t)$ & carrier signal shifted by $90°$ are applied to the product modulator Q, producing DSB-SC wave.

* The o/p of product modulator 'I' is

$$\boxed{S_I(t) = m(t) A_c \cos(2\pi f_c t)}$$

* The o/p of product modulator 'Q' is

$$\boxed{S_Q(t) = \hat{m}(t) \cdot A_c \sin(2\pi f_c t)}$$

These signals $S_I(t)$ & $S_Q(t)$ are fed to a summer.

* The o/p of the summer is

$$S(t) = S_I(t) \pm S_Q(t)$$

$$\boxed{S(t) = A_c m(t) \cos 2\pi f_c t \pm A_c \hat{m}(t) \sin 2\pi f_c t}$$

* The plus sign at the summing junction yields an SSB with only the LSB i.e.

$$\boxed{S_L(t) = A_c m(t) \cos 2\pi f_c t + A_c \hat{m}(t) \sin 2\pi f_c t}$$

* Similarly the minus sign at the summing junction yields an SSB with only the USB i.e.

## Demodulation of SSB Wave:-

❖ **Show that the output of coherent detector of a SSB modulated wave is given by: $V_o(t) = 1/4\, A_c m(t)\cos\phi + 1/4 A_c m(t)\sin\phi$**

**Where $\phi$ is the phase error.**

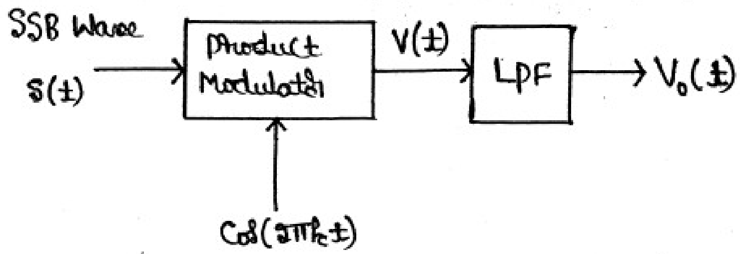



Fig ①: Coherent detection of an SSB modulated wave.

* The baseband Signal $m(t)$ can be recovered from the SSB Wave $S(t)$ by using Coherent detection.

* The product modulator is having two I/p's. one I/p is the SSB modulated wave $S(t)$ & another I/p is the locally generated carrier $\cos(2\pi f_c t)$ then Low-pass filtering the modulator o/p as shown in above figure.

* Thus product modulator o/p is given by

$$\boxed{V(t) = S(t)\cos(2\pi f_c t)} \longrightarrow ①$$

WKT

$$S(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right] \longrightarrow ②$$

Substituting eq ② in eq ①, we get

$$V(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right]\cos 2\pi f_c t$$

$$V(t) = \frac{A_c}{2} m(t)\cos 2\pi f_c t \cdot \cos 2\pi f_c t \pm \frac{A_c}{2}\hat{m}(t)\cos 2\pi f_c t \cdot \sin 2\pi f_c t.$$

W.K.T $\boxed{\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]}$

$\boxed{\cos A \cdot \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]}$

$$V(t) = \frac{A_c}{4} m(t) \left[ \cos(2\pi f_c + 2\pi f_c)t + \cos(2\pi f_c - 2\pi f_c)t \right] \pm \frac{A_c}{4} \hat{m}(t)$$

$$\left[ \sin(2\pi f_c + 2\pi f_c)t + \sin(2\pi f_c - 2\pi f_c)t \right]$$

$$V(t) = \frac{A_c}{4} m(t) \left[ \cos(4\pi f_c t) + \cos(0) \right] \pm \frac{A_c}{4} \hat{m}(t) \left[ \sin(4\pi f_c t) + \sin(0) \right]$$

W.K.T, $\boxed{\cos(0) = 1, \ \sin(0) = 0}$

$$V(t) = \frac{A_c}{4} m(t) \left[ \cos(4\pi f_c t + 1 \right] \pm \frac{A_c}{4} \hat{m}(t) \left[ \sin(4\pi f_c t) + 0 \right]$$

$$V(t) = \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t) \cos(4\pi f_c t) \pm \frac{A_c}{4} \hat{m}(t) \sin(4\pi f_c t)$$

$$V(t) = \underbrace{\frac{A_c}{4} m(t)}_{\boxed{\text{Scaled message Signal}}} + \underbrace{\frac{A_c}{4} \left[ m(t) \cos(4\pi f_c t) \pm \hat{m}(t) \sin(4\pi f_c t) \right]}_{\boxed{\text{Unwanted terms}}}$$

* When $V(t)$ is passed through the filter, it will allow only the 1st term to pass through & will reject all other unwanted terms.
* Thus at the o/p of the filter we get the scaled message signal & the coherent SSB demodulation is achieved.

$$\therefore \boxed{V_o(t) = \frac{A_c}{4} m(t)}$$

The detection of SSB modulated waves is based on the assumption that there is perfect synchronization between local carrier & that in the transmitter both in <u>frequency</u> & <u>phase</u>.

* But in practice a phase error $\phi$ may arise in the locally - generated carrier wave. Thus the detector o/p is modified due to

phase error as follows:

$$\boxed{V_o(t) = \frac{A_c}{4} m(t) \cos\phi \pm \frac{A_c}{4} \hat{m}(t) \sin\phi}$$

NOTE :-

* The phase distortion is not serious with voice communication because the human ear is relative insensitivity to phase distortion. The presence of phase distortion gives rise to what is called the Donald Duck voice effect.

* The phase distortion cannot be tolerable in the transmission of music & video signal.

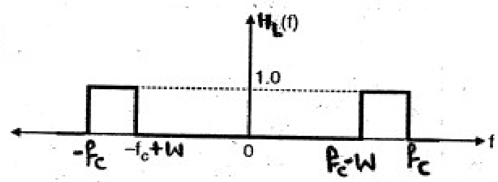
* Derive an expression for SSB modulated wave for which Lower Sideband is retained.

Jan - 2009, 8M

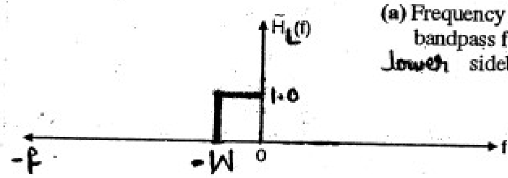
{ Let $S_L(t)$ denote an SSB modulated wave in which only the lower Sideband is retained. }

Sol:-

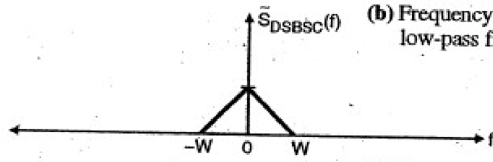
The SSB Signal may be generated by passing a DSB-SC modulated wave through a BPF of transfer function $H_L(f)$




(a) Frequency response of ideal bandpass filter to select the lower sideband.




(b) Frequency response of equivalent low-pass filter.




(c) Spectrum of complex envelope of DSBSC.

* The DSB-SC modulated wave is defined mathematically as:

$$\boxed{S_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t)}$$

Where,

$m(t) \rightarrow$ Message Signal

$A_c \cos(2\pi f_c t) \rightarrow$ Carrier Signal

* The Low pass Complex envelope of the DSB-SC modulated wave is expressed as :

$$\boxed{S_{DSBSC}(t) = A_c m(t)}$$

* Consider the SSB modulated wave $S_L(t)$, in which only the LSB is retained. It has quadrature as well as In-phase Component.

Then $\tilde{S}_L(t)$ is the complex envelope of $S_L(t)$ & we can write

$$S_L(t) = R_e\left[\tilde{S}_L(t)\exp(j2\pi f_c t)\right]$$

$$\boxed{S_L(t) = R_e\left[\tilde{S}_L(t)\, e^{j2\pi f_c t}\right]}$$

Where, $R_e \rightarrow$ real part.

* To determine $\tilde{S}_L(t)$, We proceed as follows :

i) The BPF transfer function $H_L(f)$ is replaced by an equivalent LPF of transfer function $\tilde{H}_L(f)$ as shown in fig (b).

We can express $\tilde{H}_L(f)$ as follows :

$$\tilde{H}_L(f) = \begin{cases} \frac{1}{2}\left[1+\mathrm{sgn}(-f)\right], & 0<f<-W \\ 0, & \text{otherwise} \end{cases} \longrightarrow (2)$$

Where, $\mathrm{Sgn}(-f)$ is the Signum function.

ii) The DSB-SC modulated wave is replaced by its Complex envelope. The Spectrum of this envelope is as shown in fig (c)

i.e.

$$\boxed{\tilde{S}_{DSBSC}(f) = A_c M(f)} \longrightarrow (3)$$

iii) The desired complex envelope $\tilde{S}_L(t)$ is determined by evaluating the IFT of the product $\tilde{H}_L(f) \cdot \tilde{S}_{DSBSC}(f)$

i.e. $\tilde{S}_L(t) = \text{IFT}\left[\tilde{H}_L(f) \cdot \tilde{S}_{DSBSC}(f)\right] \longrightarrow (4)$

Substituting eq (2) & eq (3) in eq (4), we get

$$\tilde{S}_L(t) = \text{IFT}\left\{\frac{1}{2}\left[1 + \text{Sgn}(-f)\right] \cdot A_c M(f)\right\}$$

$$\tilde{S}_L(t) = \text{IFT}\left\{\frac{A_c}{2}\left[M(f) + \text{Sgn}(-f)\, M(f)\right]\right\}$$

$$\boxed{\tilde{S}_L(t) = \frac{A_c}{2}\left[m(t) - j\hat{m}(t)\right]} \longrightarrow (5)$$

Substituting eq (5) in eq (1), we get $\left(S_L(t) = R_e\left[\tilde{S}_L(t)\, e^{j2\pi f_c t}\right] \rightarrow (1)\right)$

$$S_L(t) = R_e\left\{\frac{A_c}{2}\left[m(t) - j\hat{m}(t)\right] e^{j2\pi f_c t}\right\}$$

$$S_L(t) = R_e\left\{\frac{A_c}{2}\left[m(t) - j\hat{m}(t)\right] \cos(2\pi f_c t) + j\sin(2\pi f_c t)\right\}$$

$$= R_e\left\{\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + jm(t)\sin(2\pi f_c t) - j\hat{m}(t)\cos(2\pi f_c t) - j^2\hat{m}(t)\sin(2\pi f_c t)\right]\right\}$$

$$= R_e\left\{\frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + jm(t)\sin(2\pi f_c t) - j\hat{m}(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]\right\}$$

$\because -j^2 = +1$

$$\boxed{S_L(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]} \longrightarrow (6)$$

In-phase Component    Quadrature Component.

Equation (6) shows that the SSB modulated wave contains only LSB with an Inphase Component & a quadrature Component.