Homework problems on Fluid Dynamics

(1.63J/2.21J)

Chiang C. Mei, 2002

9-slowmud.tex

Ex 9. *Slow mud flow*

Torrential rains in mountainous regions cuase mudeslides and can inflict great damages (as in Southern California). Mud flows due to vocano eruption wiped out a town of 25,000 inhabitants in Colombia in 1984.

Due to the high viscosity, mud behaves like a Bingham plastic fluid where the shear stress depends on strain rate nonlinearly. In nearly parallel flows with $u \gg v$, $u(x, y)$ varies more quickly in y the consitutive relation is approximately

$$\mu \frac{\partial u}{\partial y} = \begin{cases} 0 & \text{if } \tau < \tau_o, \\ \tau - \tau_o & \text{if } \tau > \tau_o \end{cases} \tag{1}$$

where $\tau = \tau_{xy}$ and τ_o is the yield stress.

Consider the slow flow of mud near the end of a long slide on a plane sloe.

1. Write down the approximate equations according to the lubrication theory. State the conditions under which the lubrication approximation is valid.

2. In the limit of a uniform layer, can there be flow at all inclinations? Find the angle of repose.

3. If there is flow what is the velocity profile ? show that the velocity is parabolic when $0 < y < h_o$ and uniform when $h_o < y < h$, where h_o is the depth at which $\tau = \tau_o$.

4. For long-scale slow motion show that mass conservation requires

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{2}$$

where q is the discharge rate.

$$q = \int_0^h u(x, y, t)\, dy \tag{3}$$

Assume the profile obtained for uniform flow with h and h_o depending on x, t. Obtain a set of equations for $h(x, t)$ and $h_o(x, t)$.

5. Consider the head of a stationary mud layer at the threshold of downward flow. Get the profile of the mud head .