Note on the problem solved in the class

1 October 2007

In the class today, we solved the problem of a skier (represented using a point mass) sliding on a semicircular mountain using Newton's law and the work-energy principle. Using the latter method we also found the angle at which the skier separates from the platform. Here we do the same using the equations of motion obtained using Newton's law.

We obtained the following equations:

$$-mg\sin\theta + N = -m\rho\dot{\theta}^2 \qquad \text{(EQ 1)}$$

$$-mg\cos\theta = m\rho\ddot{\theta} \qquad \text{(EQ 2)}$$

In order to find the angle at which the skier separates from the mountain, we set $N = 0$. Let θ_o be that angle.

Thus,

$$g\sin\theta_o = \rho\dot{\theta}_o^2 \qquad \text{(EQ 3)}$$

$$-g\cos\theta_o = \rho\ddot{\theta}_o \qquad \text{(EQ 4)}$$

The above two equations don't tell us the value of θ_o. We will integrate Equation 2 (which is valid for any θ as long as the skier is in contact with the surface) to obtain the value of $\dot{\theta}_o$ and substitute it in Equation 3. Integrating Equation 2,

$$-\int_{\frac{\pi}{2}}^{\theta_o} g\,\mathrm{co}\;\theta d\theta = \int_{\frac{\pi}{2}}^{\theta_o} \rho\ddot{\theta}d\theta \qquad \text{(EQ 5)}$$

$$-\int_{\frac{\pi}{2}}^{\theta_o} g\cos\theta d\theta = \int_{\frac{\pi}{2}}^{\theta_o} \rho\frac{d}{dt}\dot{\theta}\cdot d\theta = \int_{\frac{\pi}{2}}^{\theta_o} \rho\frac{d}{d\theta}\dot{\theta}\cdot\dot{\theta}d\theta = \int_{\frac{\pi}{2}}^{\theta_o} \rho\frac{d}{d\theta}\left(\frac{\dot{\theta}^2}{2}\right)d\theta = \rho\int_{0}^{\dot{\theta}_o} d\left(\frac{\dot{\theta}^2}{2}\right)$$

$$[-g\sin\theta]_{\frac{\pi}{2}}^{\theta_o} = \rho\left[\frac{\dot{\theta}^2}{2}\right]_0^{\theta_o}$$

$$1 - g\sin\theta_o = \rho\frac{\dot{\theta}_o^2}{2}$$

Substituting the above epression in Equation 3,

$$g\sin\theta_o = 2(1 - g\sin\theta_o)$$

Thus,

$$\sin\theta_o = \frac{2}{3} \qquad \text{(EQ 6)}$$