Lecture 18 - summary

Topic: Linear deformation theory
Key assumption: **Small deformation**

$$\text{Linear: } \left\| \text{Grad}\,\vec{\xi} \right\| \ll 1 \Rightarrow \text{Grad}\,(.) \simeq \text{grad}\,(.)$$

Small strain tensor $$\underline{\underline{\varepsilon}} = \frac{1}{2}\left(\text{grad}\,\vec{\xi} + \left(\text{grad}\,\vec{\xi}\right)^T\right)$$

Distortion $$\frac{1}{2}\theta(\vec{e}_\alpha,\vec{e}_\beta) = \theta_{\alpha\beta} = \varepsilon_{\alpha\beta} \qquad \frac{1}{2}\theta_{\vec{m},\vec{n}} = \vec{m}\cdot\underline{\underline{\varepsilon}}\cdot\vec{n}$$ (general)

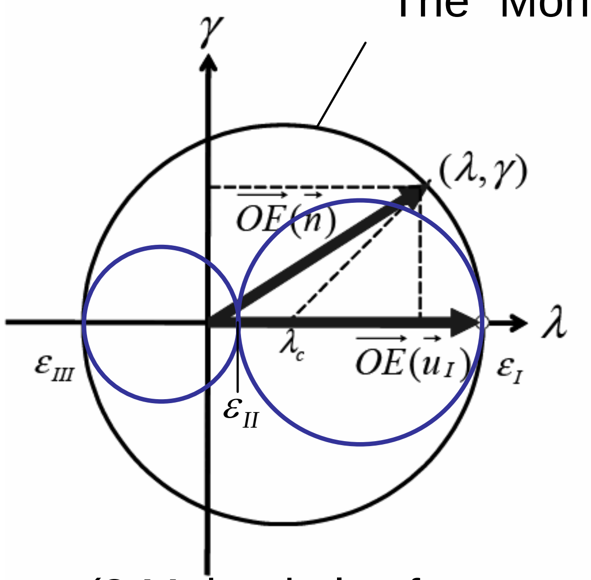
Dilatation $$\lambda(\vec{e}_\alpha) = \varepsilon_{\alpha\alpha} \qquad \lambda_{\vec{n}} = \vec{n}\cdot\underline{\underline{\varepsilon}}\cdot\vec{n}$$ (general)

Volume change $$J - 1 = \frac{d\Omega_t - d\Omega_0}{d\Omega_0} \simeq \text{tr}\,\boldsymbol{\varepsilon} = \lambda(\vec{e}_1) + \lambda(\vec{e}_3) + \lambda(\vec{e}_3)$$

Surface change $$\vec{n}\,da \simeq (1 + \text{tr}\,\boldsymbol{\varepsilon})\left(\mathbf{1} - \left(\text{grad}\,\vec{\xi}\right)^T\right)\cdot\vec{N}\,dA$$

Strain Mohr circles $$\vec{E}(\vec{n}) = \underline{\underline{\varepsilon}}\cdot\vec{n}$$ (strain vector)

$$\vec{E}(\vec{n}) = \lambda\vec{n} + \gamma\vec{t}\begin{cases}\lambda = \vec{n}\cdot\vec{E}(\vec{n}) = \dfrac{\epsilon_I + \epsilon_{III}}{2} + \dfrac{\epsilon_I - \epsilon_{III}}{2}\cos 2\vartheta \\[2ex] \gamma = \vec{t}\cdot\vec{E}(\vec{n}) = \dfrac{\epsilon_I - \epsilon_{III}}{2}\sin(-2\vartheta)\end{cases}$$



Concept: Decompose deformation into dilatation λ and distortion γ (3 Mohr circles for general $\underline{\underline{\varepsilon}}$)