Lecture 5 - summary

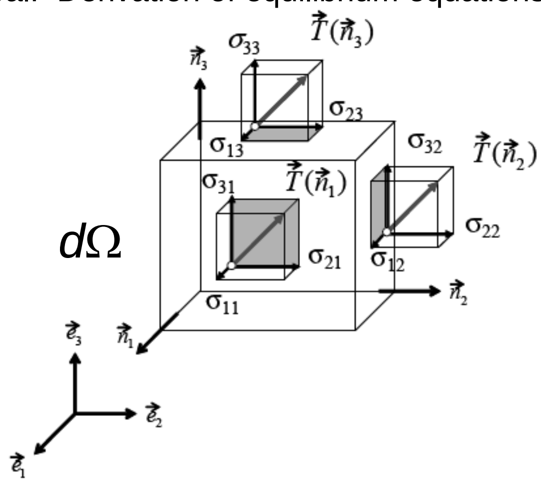
Introduction of the continuum model

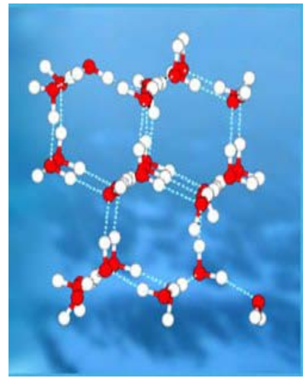
Three scales:

Structural scale (H,B,D..) >> REV >> molecular scale

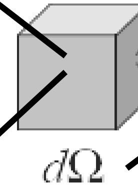
The three scales are separated (“>>” operator)

Goal: Derivation of equilibrium equations for REV $d\Omega$





Atomic bonds O(Angstrom=1E-10m)



Continuum representative volume element REV

Equilibrium:

$$\frac{d\vec{\wp}}{dt} = \frac{d}{dt}\left(\rho \vec{V}\, d\Omega\right) \stackrel{def}{=} \vec{F}^{ext}$$

External forces:

$$\vec{F}^{ext} = \rho \vec{g}\, d\Omega + \sum_i \vec{T}_i da_i$$

Stress vector

$$\vec{T} \stackrel{def}{=} \vec{T}\left(\vec{x}, \vec{n}\right)$$

Definition of stress tensor (description of material forces only as function of position, not normal):

$$\boldsymbol{\sigma} = \sigma_{ij}\, \vec{e}_i \otimes \vec{e}_j$$

$$\vec{T}\left(\vec{n}\right) \stackrel{def}{=} \boldsymbol{\sigma} \cdot \vec{n}$$

Integration over entire material/structure volume:

$$\frac{d\vec{\wp}}{dt} = \int_\Omega \rho \vec{a}\, d\Omega \stackrel{def}{=} \vec{F}^{ext} = \int_\Omega \rho \vec{g}\, d\Omega + \int_{\partial\Omega} \boldsymbol{\sigma} \cdot \vec{n}\, da$$

$$\vec{F}^{ext} - \frac{d\vec{\wp}}{dt} = \int_\Omega \left[\mathrm{div}\, \boldsymbol{\sigma} + \rho\left(\vec{g} - \vec{a}\right)\right] d\Omega = 0$$

Local equilibrium: $\mathrm{div}\, \boldsymbol{\sigma} + \rho\left(\vec{g} - \vec{a}\right) = 0$

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} + \rho(g_1 - a_1) = 0$$

$$\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} + \rho(g_2 - a_2) = 0$$

$$\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} + \rho(g_3 - a_3) = 0$$

Complete problem (Dynamic Resultant + Moment Theorems):

on S: $\vec{T}\left(\vec{n}\right) + \vec{T}\left(-\vec{n}\right) = 0$

on $\partial\Omega$: $\vec{T}^d = \vec{T}(\vec{n})$

$$\text{in } \Omega: \begin{cases} \vec{T}\left(\vec{n}\right) = \boldsymbol{\sigma} \cdot \vec{n} \\ \mathrm{div}\, \boldsymbol{\sigma} + \rho\left(\vec{g} - \vec{a}\right) = 0 \\ \sigma_{ij} = \sigma_{ji} \end{cases}$$

$\vec{a}$ =0 (static)

Skyscraper photograph courtesy of jochemberends on Flickr.