1.020 Ecology II: Engineering for Sustainability
Spring 2008

Practice Questions and Solutions for Exam 1 on March 17, 2008
Exam Review Friday March 14th, 2008 9-11am

1. Mass balance

A deep reservoir with vertical sides has a surface area of A_r m^2. Water (density ρ_w) enters the reservoir at a rate of q_{in}. The depth of water at a given time is h. An outlet of cross sectional area A_{pipe} m^2 is situated at the bottom of the reservoir, and the flow out of the pipe can be expressed as

$$q_{out} = \sqrt{2gh}A_{pipe}$$

   a) Write a conservation equation for the mass of the water in the reservoir:
        (i)   as a difference equation

$$M_{t+\Delta t} - M_t = \rho_w q_{in} - \rho_w q_{out} = \rho_w q_{in}\Delta t - \rho_w \sqrt{2gh_t}A_{pipe}\Delta t$$

        (ii)   as a differential equation

$$\frac{dM}{dt} = \rho_w q_{in} - \rho_w \sqrt{2gh}A_{pipe}$$

      Leave your answer in terms of the water mass, ρ_w, q_{in}, A_r and $A_{pipep.}$

   b) Using the relations given above rewrite the differential conservation equation to give the change of water depth with time.

$$M = \rho_w A_r h$$
$$\frac{dM}{dt} = \rho_w A_r \frac{dh}{dt} = \rho_w q_{in} - \rho_w \sqrt{2gh}A_{pipe} \qquad \text{since } \rho_w \text{ and } A_r \text{ are constant.}$$
$$\frac{dh}{dt} = \frac{q_{in}}{A_r} - \sqrt{2gh}\frac{A_{pipe}}{A_r}$$

2. Chemical kinetics

In the absence of oxygen, certain bacteria are able to respire anaerobically, using acetic acid as a substrate. In addition to CO_2, the bacteria give off methane (CH_4) in the process, and so the respiration process is also known as methanogenesis. In a closed volume V Liters (e.g. an anoxic region fairly deep under a lake's bottom) the following reaction occurs:

$$CH_3COOH \rightarrow CH_4 + CO_2$$
$$\text{Acetic acid} \quad \text{methane}$$
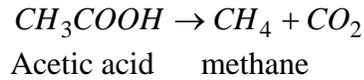
The rate (per second) at which the number of moles of acetic acid is used is $= k_1 V C_{CH_3COOH}$, with C_{CH_3COOH} in mol/L. The molecular masses of the three species are: CH_3COOH 60 g/mol, CH_4 16 g/mol, CO_2 44 g/mol. The system starts off with a concentration of 1 mole/L of CH_3COOH.

a) Using the mass balance (stoichiometry of the reaction) write differential equations for the rate change in **mass concentration** (**g/L**) of acetic acid, methane and CO_2. Leave your answer in terms of k_1.

b) Draw a rough plot of the changes in masses of the three species with time.

$$\frac{V dC_{CH_3COOH}}{dt} = -k_1 V C_{CH_3COOH}$$

Acetic acid: $\quad \frac{dC_{CH_3COOH}}{dt} \left[\frac{mol}{Ls} \right] = -k_1 C_{CH_3COOH}$

$$\frac{dC_{CH_3COOH}}{dt} \left[\frac{g}{Ls} \right] = -k_1 C_{CH_3COOH} \, 60 \frac{g}{mol}$$

Methane: $\frac{dC_{CH_4}}{dt} \left[\frac{g}{Ls} \right] = k_1 C_{CH_3COOH} \, 16 \frac{g}{mol}$ $\qquad$ CO_2: $\quad \frac{dC_{CO_2}}{dt} \left[\frac{g}{Ls} \right] = k_1 C_{CH_3COOH} \, 44 \frac{g}{mol}$

Plot of concentration evolution with time:
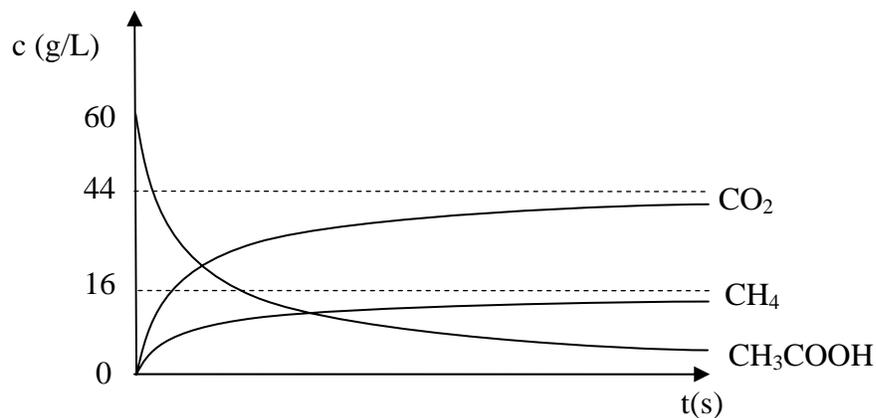
3. Closed system, nutrient cycle: benthic layer nitrogen

The nutrient modeling example given in the Lecture08_5 outline turns out to far underestimate the measured levels of nitrogen on the bottom of the ecosystem (benthic layer).

It is found that nitrogen fixing bacteria are present on particles in suspension. They uptake the dissolved nitrogen in a first order process (rate of uptake directly proportional to dissolved nitrogen mass), and then they settle to the bottom. In addition, a small fraction of the dead phytoplankton and zooplankton (denoted by f), instead of becoming dissolved nitrogen, directly enters the sediment.

The original model is shown below. Subscripts 1=dissolved N mass, 2 = N mass in phytoplankton, 3 = N mass in zooplankton.

$$\frac{dM_1}{dt} = \dot{m}_{boundary,1} + d_2 M_2 + d_3 M_3 - g_2\left(\frac{M_1}{M_{1h} + M_1}\right)M_2 + (1-\varepsilon)g_3\left(\frac{M_2}{M_{2h} + M_2}\right)M_3$$

$\qquad\qquad\qquad$ Plankton death $\qquad\qquad$ Phyto uptake $\qquad\qquad$ Grazing residue

$$\frac{dM_2}{dt} = g_2\left(\frac{M_1}{M_{1h} + M_1}\right)M_2 - d_2 M_2 - g_3\left(\frac{M_2}{M_{2h} + M_2}\right)M_3$$

$\qquad\qquad\quad$ Phyto uptake $\qquad$ Phyto death $\qquad$ Grazing

$$\frac{dM_3}{dt} = \varepsilon\, g_3\left(\frac{M_2}{M_{2h} + M_2}\right)M_3 - d_3 M_3$$

$\qquad\qquad\quad$ Zoo uptake $\qquad$ Zoo death

If we introduce a fourth compartment 4 = N mass in sediment, write down the rate equation for the sediment nitrogen mass. Also, update the above three equations of this closed system model.

Solution: Let the first order removal of dissolved N be equal sM_1. Since we have a closed system, the sink terms in one compartment must enter as source terms in the other compartments. The four new equations, with the added terms in bold, are:

$$\frac{dM_1}{dt} = \dot{m}_{boundary,1} + (1-f)(d_2 M_2 + d_3 M_3) - g_2\left(\frac{M_1}{M_{1h} + M_1}\right)M_2 + (1-\varepsilon)g_3\left(\frac{M_2}{M_{2h} + M_2}\right)M_3 - sM_1$$

$$\frac{dM_2}{dt} = g_2\left(\frac{M_1}{M_{1h} + M_1}\right)M_2 - d_2 M_2 - g_3\left(\frac{M_2}{M_{2h} + M_2}\right)M_3$$

$$\frac{dM_3}{dt} = \varepsilon g_3\left(\frac{M_2}{M_{2h} + M_2}\right)M_3 - d_3 M_3$$

$$\frac{dM_4}{dt} = f(d_2 M_2 + d_3 M_3) + sM_1$$

4. Transportation

The traffic network shown below consists of four links (1 to 4) and four nodes (1 to 4)
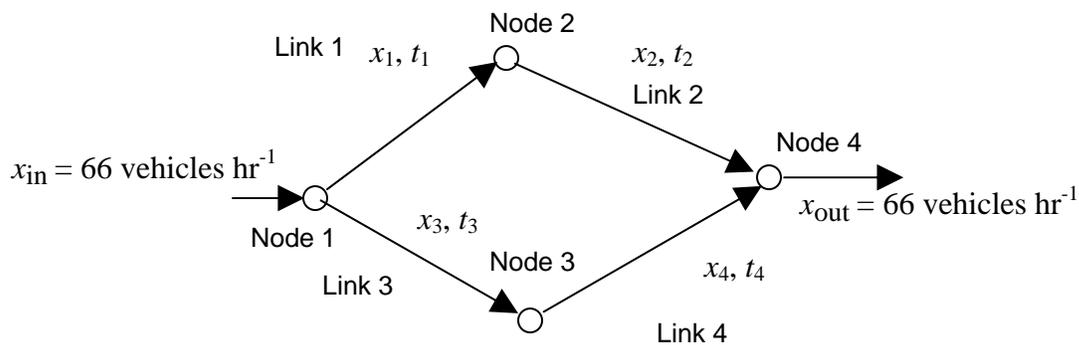
You are given the following information:

i) The travel time/flow equations for each link must be satisfied:

Link 1: $t_1 = 15$; t_1 = Link 1 travel time (hrs) , x_1 = Link 1 traffic flow (vehicles hr^{-1})
Link 2: $t_2 = 1 + 0.2x_2$; t_2 = Link 2 travel time (hrs) , x_2 = Link 2 traffic flow (vehicles hr^{-1})
Link 2: $t_3 = ?$; t_3 = Link 3 travel time (hrs) , x_3 = Link 3 traffic flow (vehicles hr^{-1})
Link 2: $t_4 = 2 + 0.1x_4$; t_4 = Link 4 travel time (hrs) , x_4 = Link 4 traffic flow (vehicles hr^{-1})

The equilibrium time taken for both routes = 24.4 hours.

a) Find t_3, the travel time for Link 3.

From the equilibrium condition,
1. $t_1 + t_2 = t_3 + t_4$ → 15 + (1+0.2x_2) = t_3+(2+0.1x_4) = 24.4 hours
     This gives 16+0.2x_2 = 24.4 → x_2 = 42 vehicles hr^{-1} = x_1.
For nodes
2.   Node 3: $x_3 = x_4$;

3.   Node 2: $x_1=x_2$
4.   Node 4: $x_2+x_4 = 66$ → $x_4 = 66-42 = 24 = x_3$
Therefore from (1), $t_3 = 24.4 - (2+0.1*24) = 20$ hours

b) If the link lengths are equal and all vehicles are identical, which link (1-2 or 3-4) do you think has the greater CO emissions rate (in gm CO km^{-1} hr^{-1})?

Since $x_2>x_4$, everything else being equal there is more traffic volume in the top route and therefore has greater CO emissions.

5. Heat Transfer

A 1 kg aluminum pan (specific heat $c_p = 900$ J kg^{-1}K^{-1}), of surface area 0.01 m^2, is being heated on a butane burner in a room of fixed temperature $T_a=20$ degrees C.

a) Write a differential equation describing the time change in temperature of the aluminum pan. Assume that the pan's temperature is uniform.

We are taking the pan as the control volume, and assume that the temperature is uniform throughout. This means that we can neglect any internal conduction effects of the pan.

Energy balance:

$$\frac{dU}{dt} = \dot{Q}_{radin} - \dot{Q}_{radout} - \dot{Q}_{conv} + \dot{Q}_{burner}$$

The change of internal energy can be expressed as $\dfrac{dU}{dt} = C_v \dfrac{dT}{dt} = C_p \dfrac{dT}{dt} = mc_p \dfrac{dT}{dt}$

Cv and Cp are virtually identical for solids. We are also only dealing with one material (aluminum) for which we can look up a specific heat value.

Radiative heat out $= \dot{Q}_{radin} = A\varepsilon\sigma T_a^4$;   Radiative heat in $= \dot{Q}_{radout} = A\varepsilon\sigma T^4$

Convective heat out $= \dot{Q}_{conv} = Ah(T - T_a)$

Subsituting into the balance equation of internal energy and heat flow rates:

$$mc_p \frac{dT}{dt} = A\varepsilon\sigma T_a^4 - A\varepsilon\sigma T^4 - Ah(T - T_a) + \dot{Q}_{burner}$$

$$\text{Or}\quad \boxed{\frac{dT}{dt} = \frac{1}{mc_p}\left(A\varepsilon\sigma T_a^4 - A\varepsilon\sigma T^4 - Ah(T - T_a) + \dot{Q}_{burner}\right)}$$

b) Say that the pan is now at a steady 100 degrees C. What is the mass of butane required per second to maintain this temperature? The heat of combustion of butane is roughly 50kJ/gram

Take $h_{conv} = 10$ Wm^{-2}K^{-1}, Emissivities $\varepsilon = 1.0$.

From part a, $mc_p \dfrac{dT}{dt} = A\varepsilon\sigma T_a^4 - A\varepsilon\sigma T^4 - Ah(T - T_a) + \dot{Q}_{burner} = 0$ at steady state.

Rearranging, $-A\varepsilon\sigma T_a^4 + A\varepsilon\sigma T^4 + Ah(T - T_a) = \dot{Q}_{burner}$

Radiative heat in $= Q_{radin} = A\varepsilon\sigma T_a^4 = 0.01m^2 \times 1.0 \times 5.7 \times 10^{-8} \frac{W}{m^2 K} \times (293K)^4 = 4.2W$

Radiative heat out $= Q_{radout} = A\varepsilon\sigma T^4 = 0.01m^2 \times 1.0 \times 5.7 \times 10^{-8} \frac{W}{m^2 K} \times (373K)^4 = 11.0W$

Convective heat out $= Q_{conv} = Ah(T - T_a) = 0.01m^2 \times 10 \frac{W}{m^2 K} (373 - 293K) = 8.0W$

$\dot{Q}_{burner} = -4.2 + 11.0 + 8.0W = 14.8$W

(Note that the net radiative loss is nearly half of the net heat loss by the pan)

The equivalent amount of fuel per second used is

$=14.814.8 \dfrac{J}{s} \times \dfrac{g}{50000J} = 2.96 \times 10^{-4} \dfrac{g}{s}$

This is ~1 gram per hour.