1.203J/6.281J/15.073J/16.76J
Logistical and Transportation Planning Methods
Massachusetts Institute of Technology
Quiz 1
October 25, 2004

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1. Total time allotted is 90 minutes.
2. Open book.
3. Please print your name clearly on each page you turn in.
4. Please answer each problem on a separate page

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"Help is on the way!"
>   --Dick Cheney (2000), John Kerry (2004)

True, but please answer all questions on your own, showing all work. Good luck.

Problem 1 (20 points)

Mendel arrives at a bus stop at which busses appear under a Poisson process with rate λ per minute. Let random variable h be the length in minutes of the interval in which he arrives. What is the mean of h? Explain. (There is more than one way to do this problem.)

Problem 2 (20 points)

Suppose that x_1 and x_2 are U(0,a) and independent, and let $z = x_1 x_2$. Use Crofton's method to find the mean of z. (Hint: To what power of 'a' do you think z would be proportional?

Problem 3 (30 points)

Consider a square city of side 1, with (0,0) as its lower left corner. The city has a barrier at $x = 1/2$ that extends across the city, and another at $y = 2/5$. At the intersection of these two barriers is a point through which vehicles can pass. All travel must be on North/South and East/West grids, and all travel must take place within the city (i.e., there can be no shortcuts involving the suburbs).

If two points are picked at random in the city:

(i)   What is the probability that the barriers will increase the shortest grid distance between them?

(ii)   What is the maximum possible *increase* in grid distance caused by the barriers? Briefly explain.

(iii)   What is the mean travel distance between two points in the city for which the grid distance is greater because of the barriers?

Problem 4 (30 points)

Consider a queueing system with two parallel servers and no space for queueing.

This facility serves two types of customers. Type 1 customers arrive in a Poisson manner at a rate of λ_1 per minute. The service time to these customers has a negative exponential pdf with a rate of μ_1 per minute for each server. Note that any Type 1 customers who find both servers busy are lost.

Type 2 customers are more unusual. They too arrive in a Poisson manner at a rate of λ_2 per minute. However, each one of these customers occupies simultaneously TWO servers (i.e., both of the servers). The servers work together on each of these Type 2 customers. The service time to these customers also has a negative exponential pdf with a rate of μ_2 per minute. (Note that this means that the two servers begin and end service to each Type 2 customer simultaneously and, together, can serve μ_2 Type 2 customers per minute if working continuously on Type 2 customers.) Type 2 customers who do not find two available servers upon arrival are lost.

(a) (6 points) What is the condition for this system to reach steady state?

(b) (12 points) Please draw a state transition diagram that describes this queueing system. Please make sure to define clearly the states.

(c) (6 points) Write expressions for the fraction of Type 1 customers *and* for the fraction of Type 2 customers who are lost by this system, assuming that the system is in steady state. Your expressions should be in terms of λ_1, μ_1, λ_2 and μ_2.

(d) (6 points) What is L, the expected number of Type 1 customers in this queueing system in steady state? Once again your expression should be in terms of λ_1, μ_1, λ_2 and μ_2.