1.033/1.57 H#2: Stress & Strength

Due: October 6, 2003

MIT – 1.033/1.57
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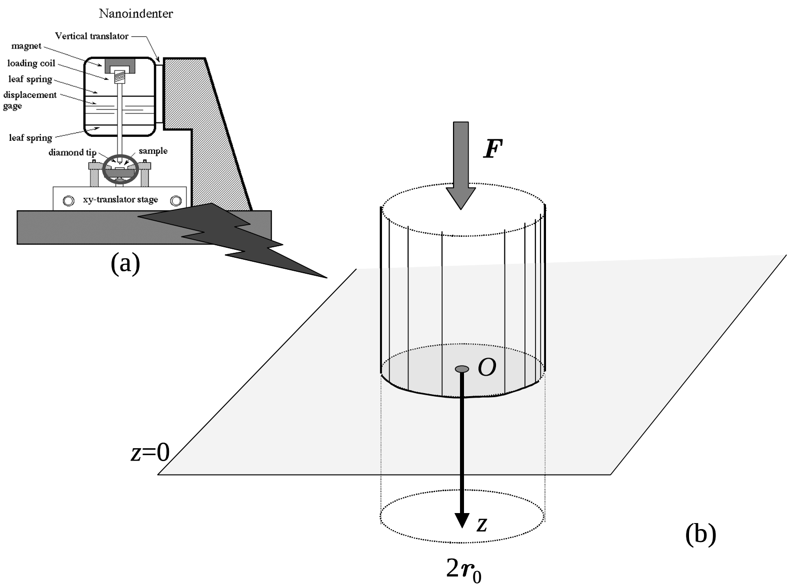
Instrumented Nano-Indentation: Instrumented nano-indentation is a new technique in materials science and engineering to determine material strengths at very fine scales. The test consists in a penetration of a needle-type indenter in a continuous material system (see experimental setup in figure (a) below). The force required to penetrate is then related to the strength of the material – by means of mechanical modeling.

In this exercise, we propose to develop a simplified triaxial stress–strength model of the nano-indentation test. To simplify the problem, we consider that the indenter is a rigid cylinder of radius r_0, situated on the surface of a horizontal half-space composed of a homogeneous material, as sketched in figure (b) below. A vertical force F is exerted on the cylinder in the direction of the cylinder axis Oz, until it penetrates into the half-space. The value of the force F at this moment is noted $\max F$, and the material property that is reported from the test is known as micro-hardness:

$$H = \frac{\max F}{A}$$

where A is the contact area of the indenter with the material. We suppose that the contact of the cylinder with the half-space (at $z = 0; r \leq r_0$) is without friction. Aim of this exercise is to relate the micro-hardness measurement to the strength properties of the material composing the half-space.

Throughout this exercise we will assume quasi-static conditions (inertia effects neglected), and we will neglect body forces.



Nano-Indentation test: (a) Experimental Setup; (b) Simplified Mechanical Model.

1. **Statically Admissible Stress Field**: For purpose of analysis, we separate the half-space Ω in two subdomains, noted respectively Ω_1 and Ω_2. In these domains, we consider the following stress fields:

   - in Ω_1 defined by $z > 0$ and $r < r_0$:

   $$\sigma'_{rr} = q'; \quad \sigma'_{\theta\theta} = q''; \quad \sigma'_{zz} = \sigma \text{ (other } \sigma'_{ij} = 0)$$

   - in Ω_2 defined by $z > 0$ and $r > r_0$:

   $$\sigma'_{rr} = -q(r_0/r)^2; \quad \sigma'_{\theta\theta} = q(r_0/r)^2 \text{ (other } \sigma'_{ij} = 0)$$

   (a) Specify precisely ALL conditions which statically admissible stress fields in Ω_1 and Ω_2 need to satisfy.

   (b) Determine the constants q', q'', q and σ, so that the stress field $\boldsymbol{\sigma}'$ is statically admissible in $\Omega = \Omega_1 \cup \Omega_2$.

   (c) In the Mohr Plane ($\sigma \times \tau$), give a graphical representation of the stress field $\boldsymbol{\sigma}'$ for Ω_1 and Ω_2, by considering that $F > q\pi r_0^2$. In both Mohr Plane and material plane, determine the surface and the corresponding stress vector, where the shear stress is maximum in Ω.

2. **Mohr-Coulomb Strength Criterion**: The material we consider is a Mohr-Coulomb material, for which the strength domain is defined by:

   $$f(\boldsymbol{\sigma}) = |\tau| + \sigma \tan\varphi - c \leq 0$$

   where $|\tau| = \sqrt{\mathbf{T}^2 - \sigma^2}, \sigma = \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n}$; $\tan\varphi$ is the friction coefficient, and c is the cohesion. Alternatively, the Mohr-Coulomb criterion can be written in terms of the principal stresses $\sigma_I \geq \sigma_{II} \geq \sigma_{III}$:

   $$f(\boldsymbol{\sigma}) = \sigma_I(1 + \sin\varphi) - \sigma_{III}(1 - \sin\varphi) - 2c\cos\varphi \leq 0$$

   (a) Display the Mohr-Coulomb criterion in the Mohr Plane ($\sigma \times \tau$);

   (b) Determine the relation between micro-hardness H and the strength material properties of the Mohr-Coulomb criterion.

   (c) In the material plane, represent the orientation of the critical material surfaces, on which the Mohr-Coulomb criterion is reached.

3. **Refined Approach:** By considering that the stress field in Ω_2 was constant, determine a second relation between the micro-hardness H and the Mohr-Coulomb model parameters. Which of the two solutions is closer to the 'real' maximum micro-hardness value at failure of the Mohr-Coulomb material system. Say why (HINT: Sketch your response in the Mohr-Plane)?