1.033/1.57

Mechanics of Material Systems
(Mechanics and Durability of Solids I)

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Lecture: MWF1 // Recitation: F3:00-4:30

Part I: Deformation and Strain

2. Infinitesimal Deformation

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Linearized Strain Tensor

$$E \approx \varepsilon = \frac{1}{2}\left(\text{grad } \xi + {}^t\text{grad } \xi\right)$$

$t=u_{II}$

$\varepsilon \cdot n = \varepsilon_I u_I$

$n=u_I$

$\varepsilon_{nt}t = \gamma(n,t)t$

$\varepsilon \cdot n$

$\varepsilon_{nn}n = \lambda(n)n$

n

t
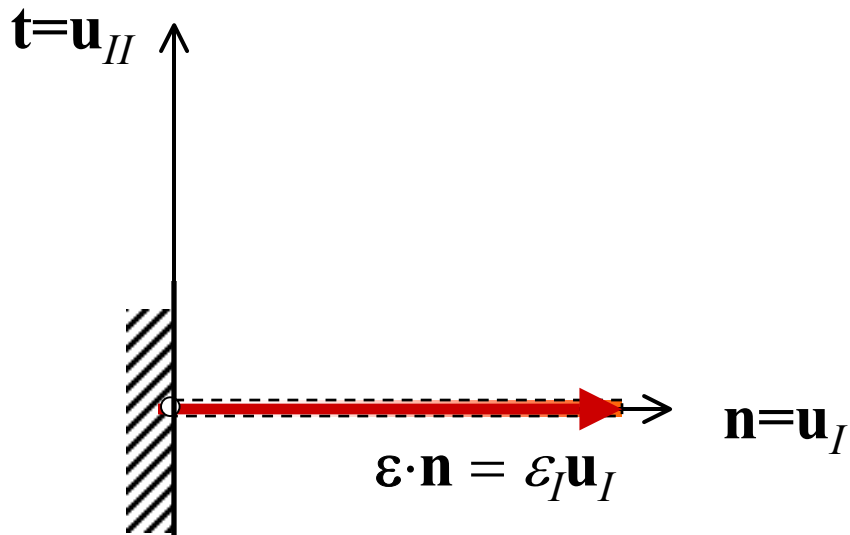
Principal Strain
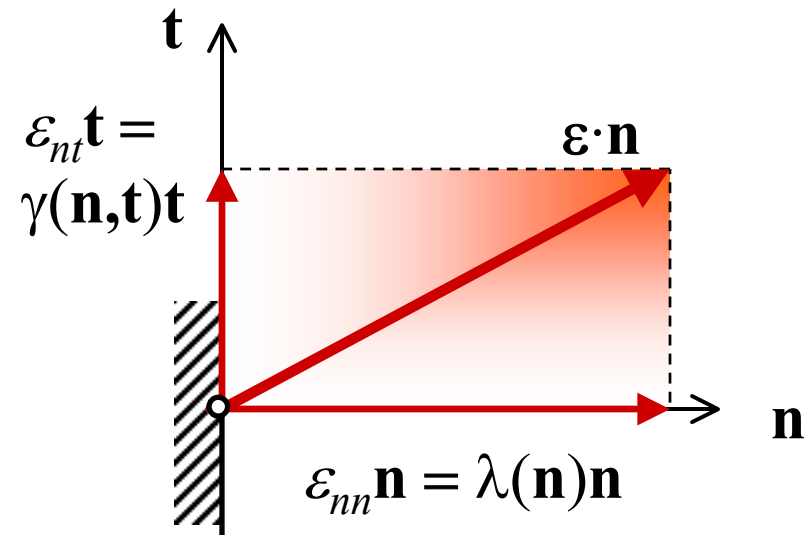
Dilatation + Distortion

Polar Decomposition

$d\mathbf{x}'$

$d\mathbf{x}$

$d\mathbf{X}$

$\Omega \times d\mathbf{X}$

$d\mathbf{x}$

$d\mathbf{x}'$

$d\mathbf{X}$

θ
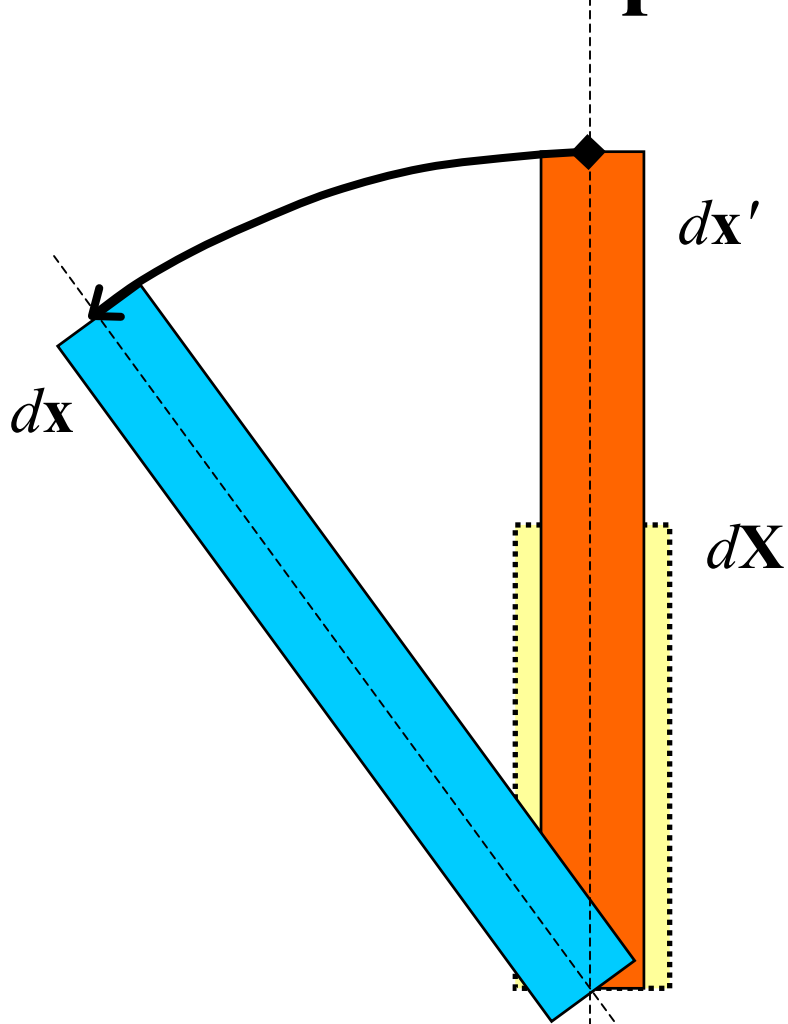
Finite Deformation:
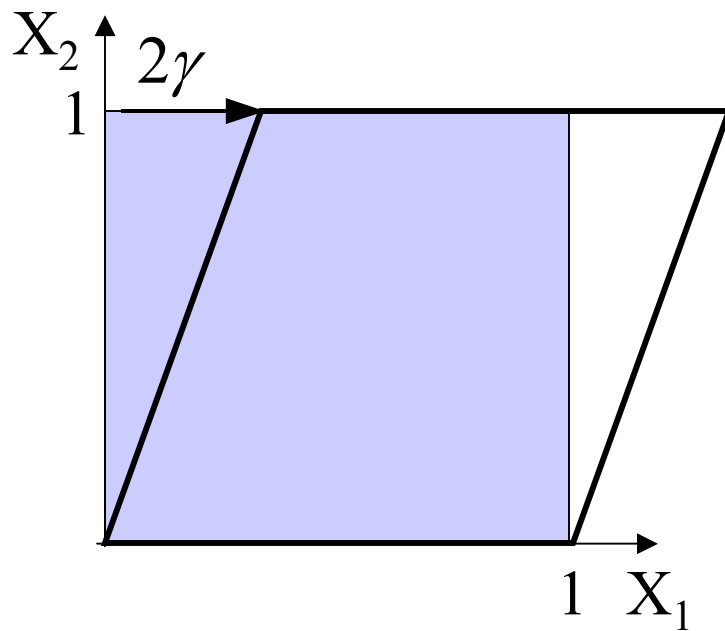$$d\mathbf{x} = (\mathbf{R} \cdot \mathbf{D}) \cdot d\mathbf{X}$$
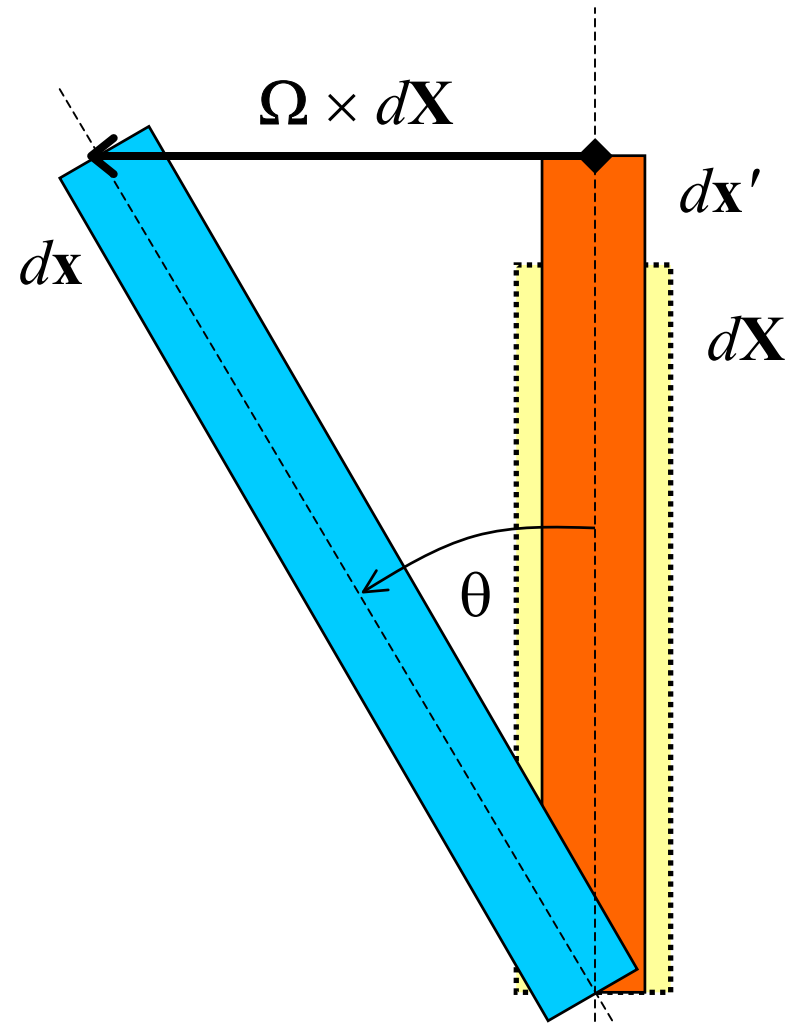
Infinitesimal Deformation:
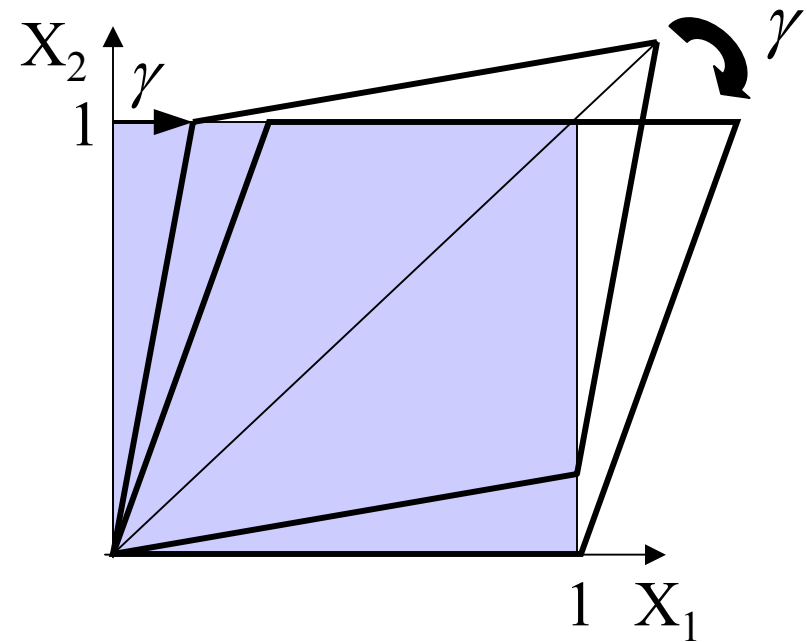$$d\mathbf{x} = (\mathbf{1} + \varepsilon) \cdot d\mathbf{X} + \Omega \times d\mathbf{X}$$

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Exercise: Single and Double Shear – The Rotation Makes the Difference
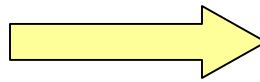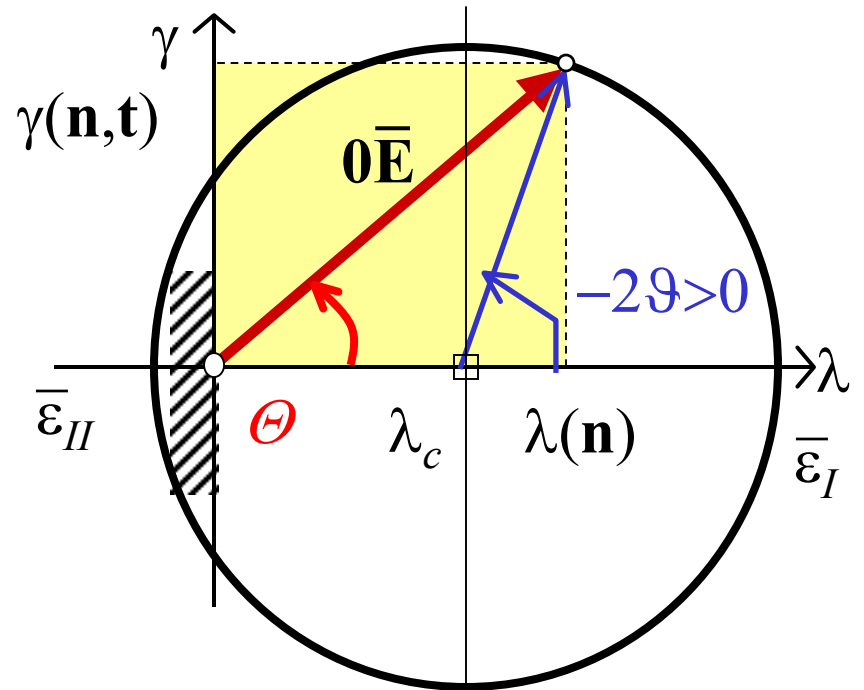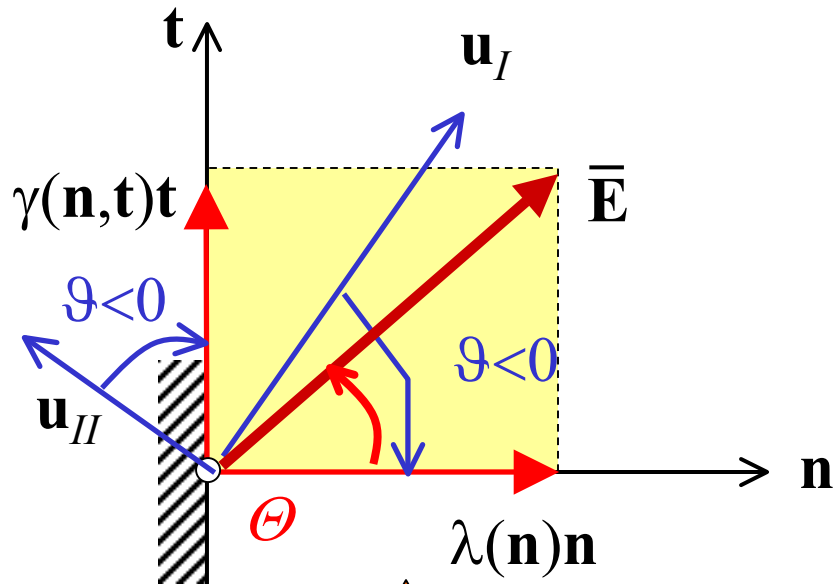

single shear

=

double shear + rotation

Mohr Strain Plane

$$\overline{\mathbf{E}} = \lambda \mathbf{n} + \gamma \mathbf{t}$$

$$\overline{\mathbf{OE}} = (\lambda , \gamma)$$

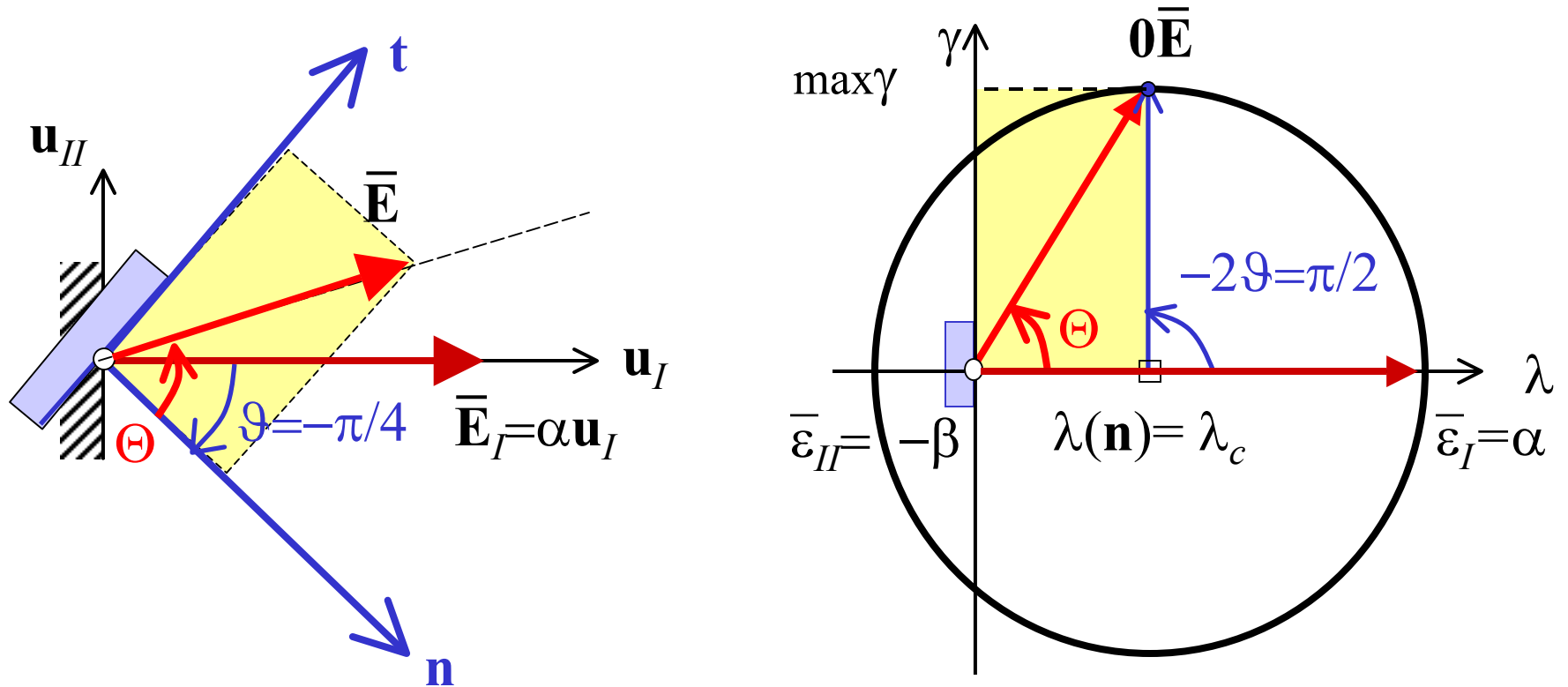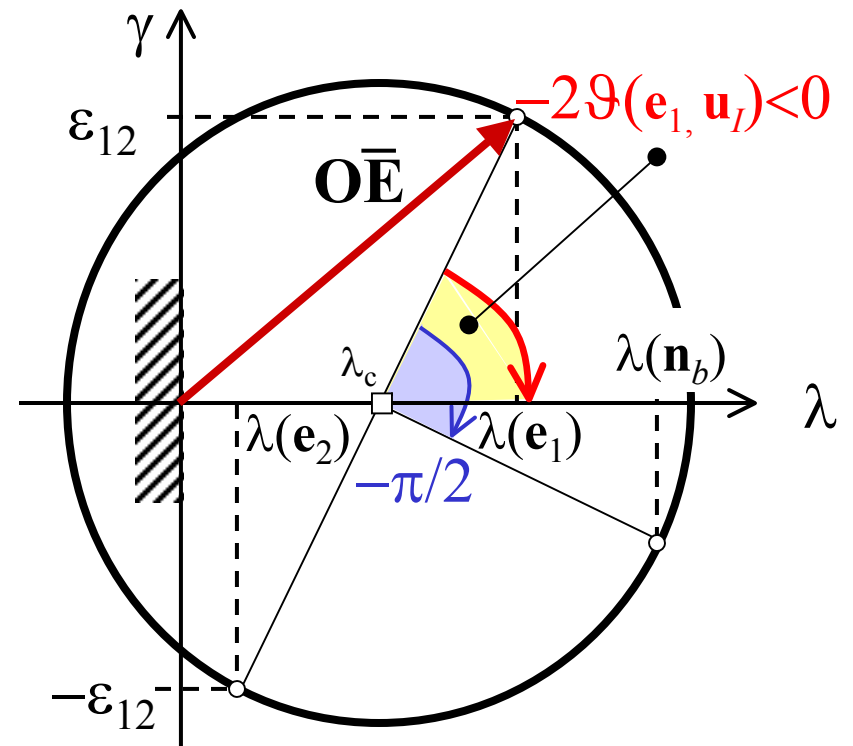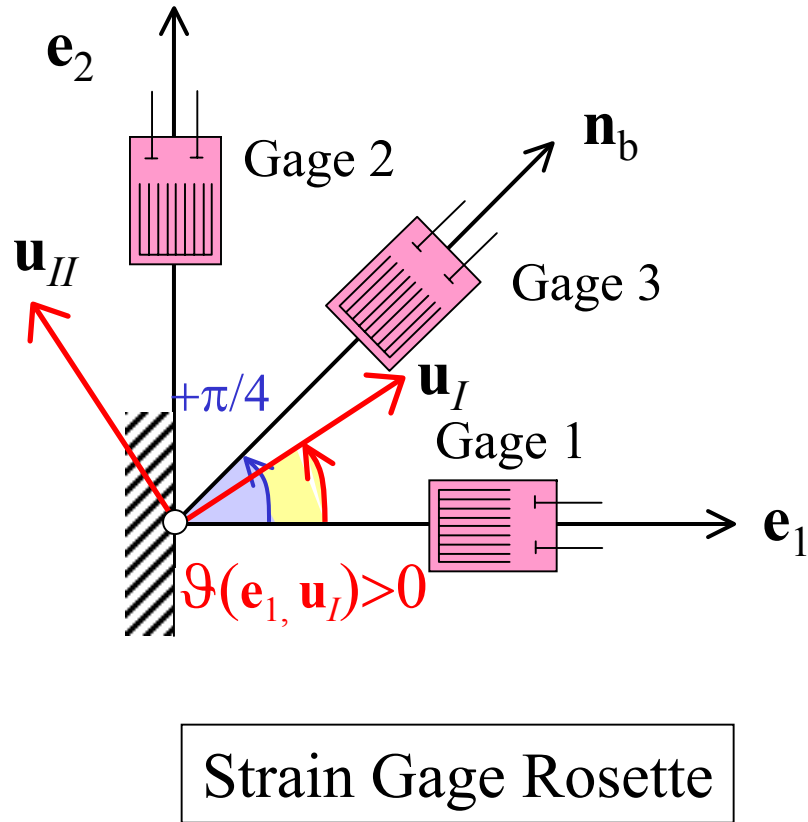$$\vartheta(\mathbf{u}_I, \mathbf{n})$$

$$-2\vartheta(\lambda, \gamma)$$
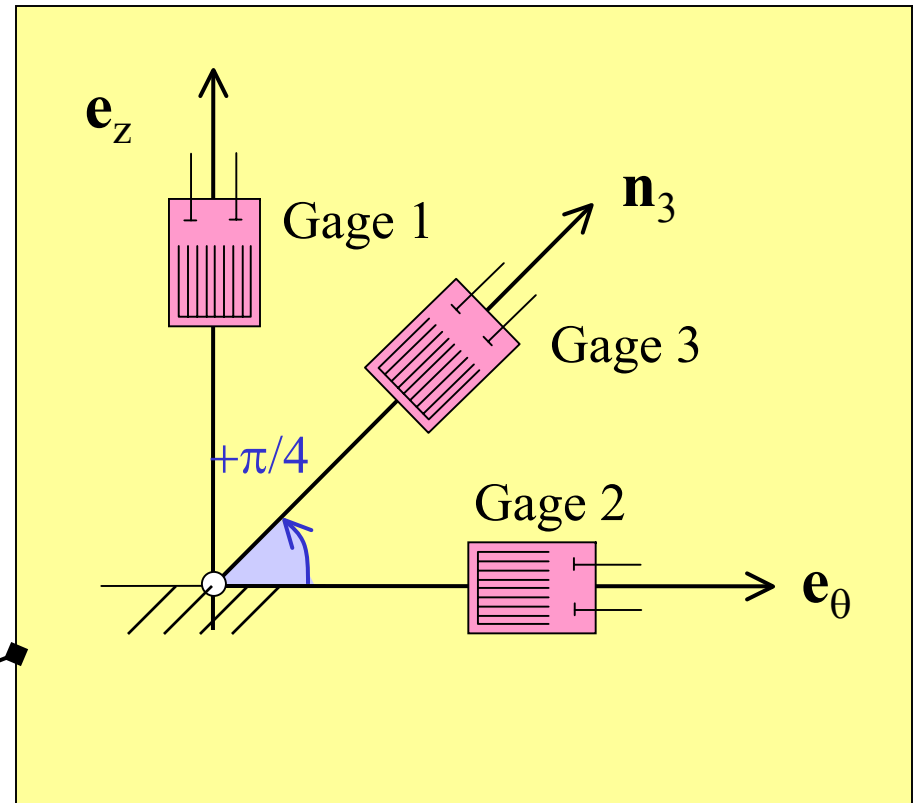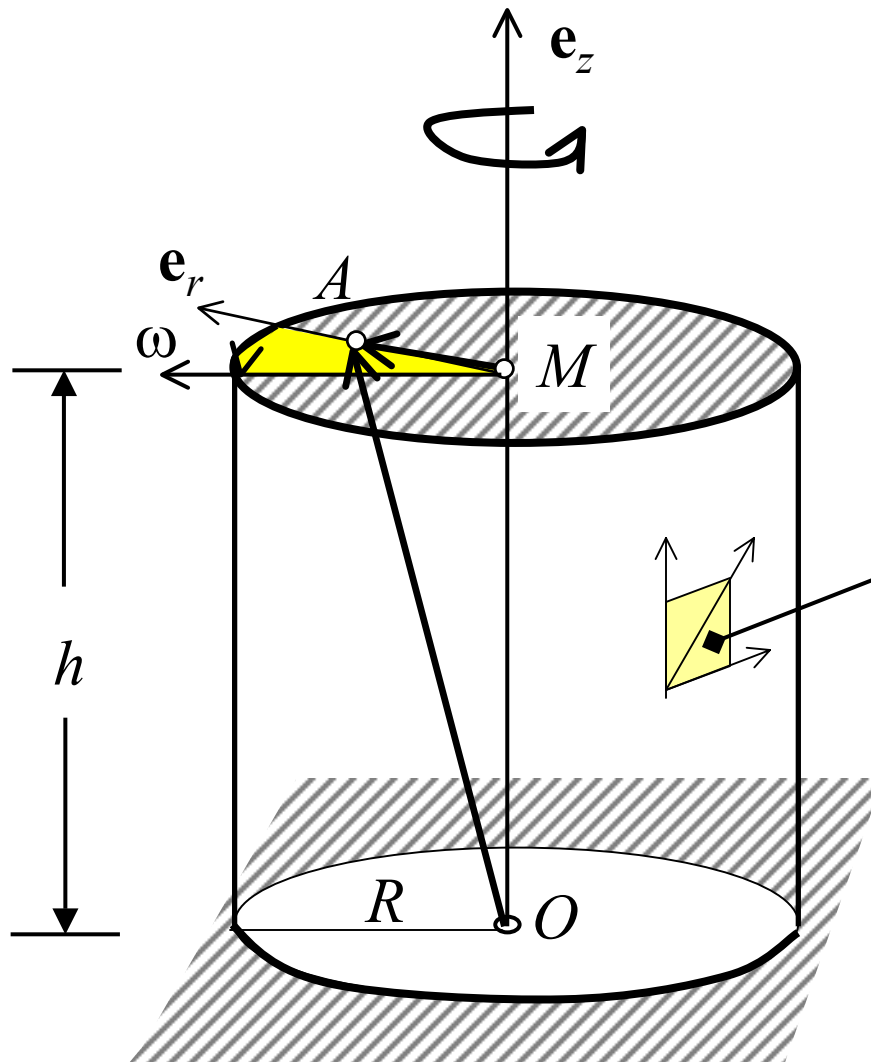
Exercise: Max. Distortion in Direct Tension

Extensometer Measurements

Strain Gage Rosette

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Training Set:

e$_z$

ω

e$_r$

A

M

ω

h

R

O

e$_z$

n$_3$

Gage 1

Gage 3

$+\pi/4$

Gage 2

e$_\theta$

Torsion of a Cylinder

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$\mathbf{e}_z$

b

$\xi_\theta = \theta z$

$z\mathbf{e}_z$

$\mathbf{e}_\theta$

b

θ

$\xi_\theta = \theta z$

$\omega z / h$

a

$r\mathbf{e}_r$

a

$\mathbf{e}_\theta$

$\mathbf{e}_r$

$\xi_\theta = \omega r z / h$
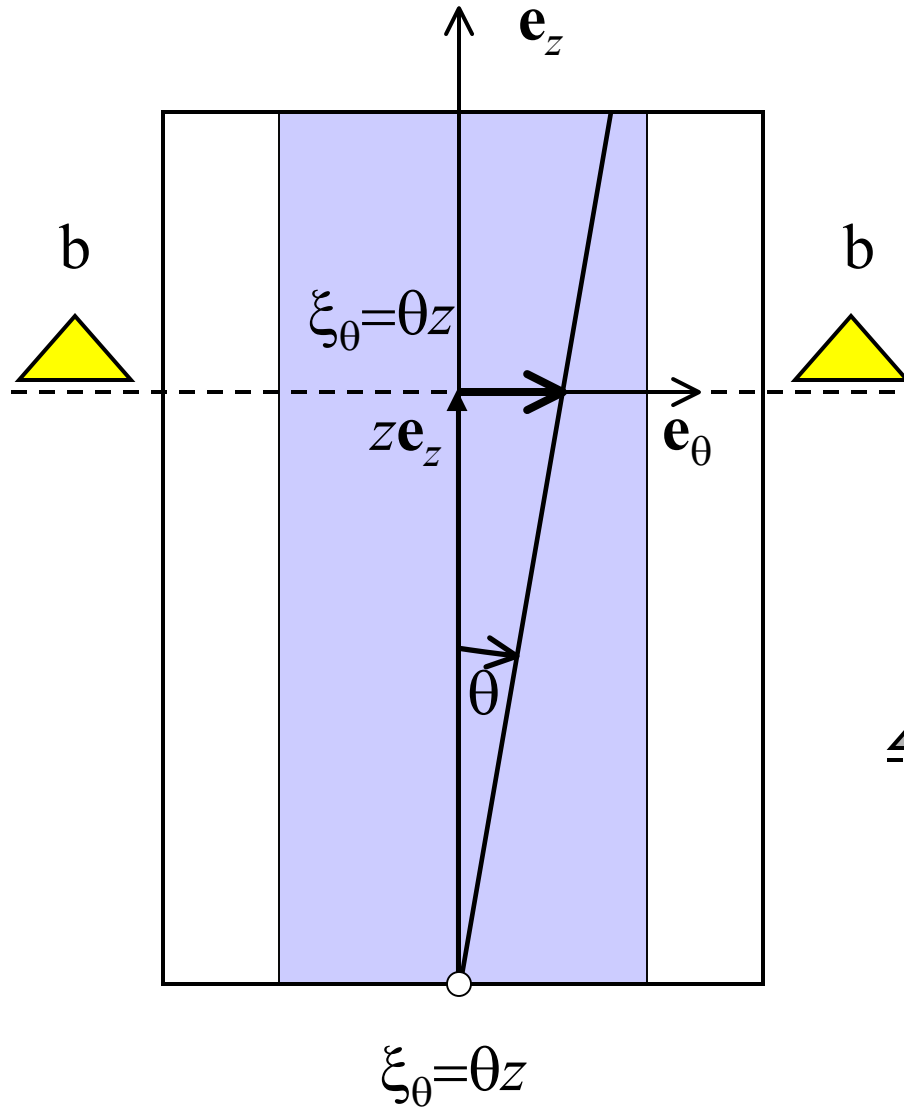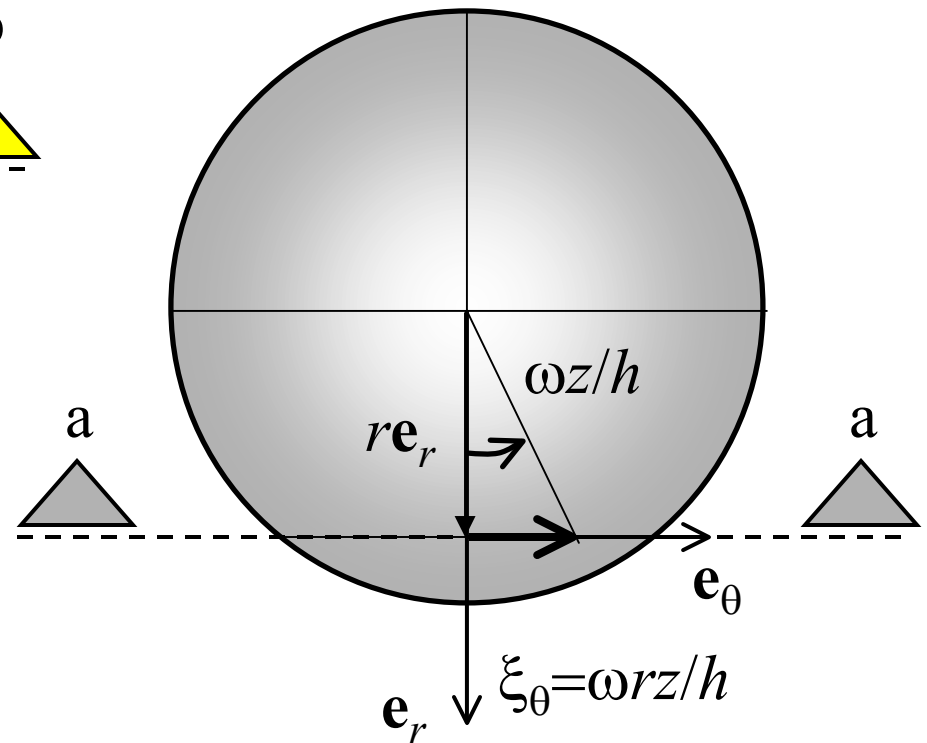
Geometrical Considerations

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Pure Torsion
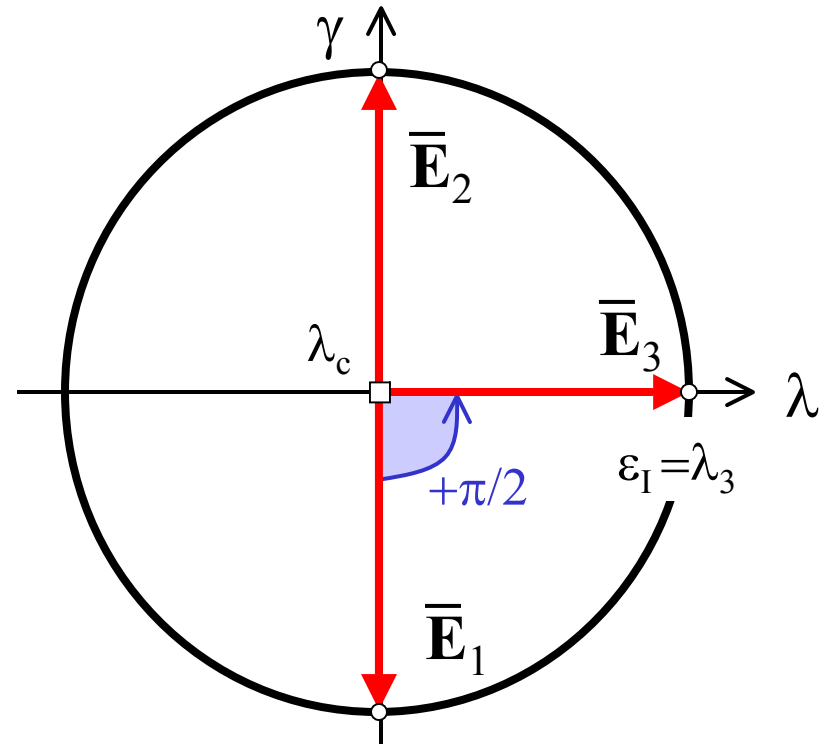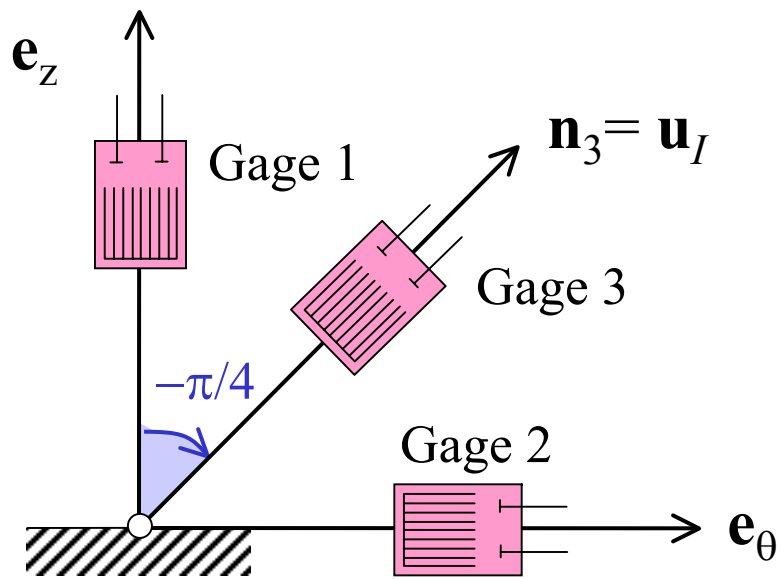
Torsion + Axial Displacement