# 1.033/1.57

# Mechanics of Material Systems

**(Mechanics and Durability of Solids I)**

Franz-Josef Ulm

*Lecture: MWF1 // Recitation: F 3:00-4:30*

# Part IV: Plasticity and Yield Design

## 8. Plasticity Models

# Content 1.033/1.57

# 1D →

Stress σ, Strain ε

Plastic Strain $\varepsilon^p$

Hardening Variable χ

Hardening Force ζ

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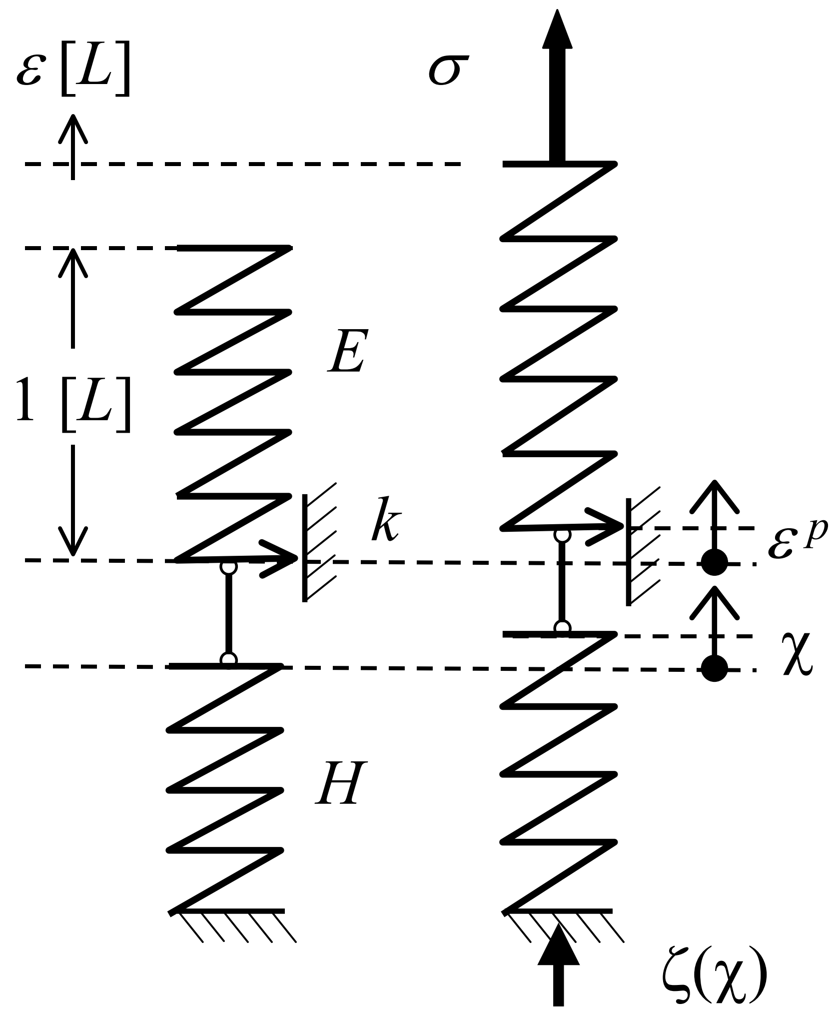
$f = |\sigma+\zeta| - k \leq 0$

$\varphi dt = \sigma d\varepsilon - d\Psi \geq 0$

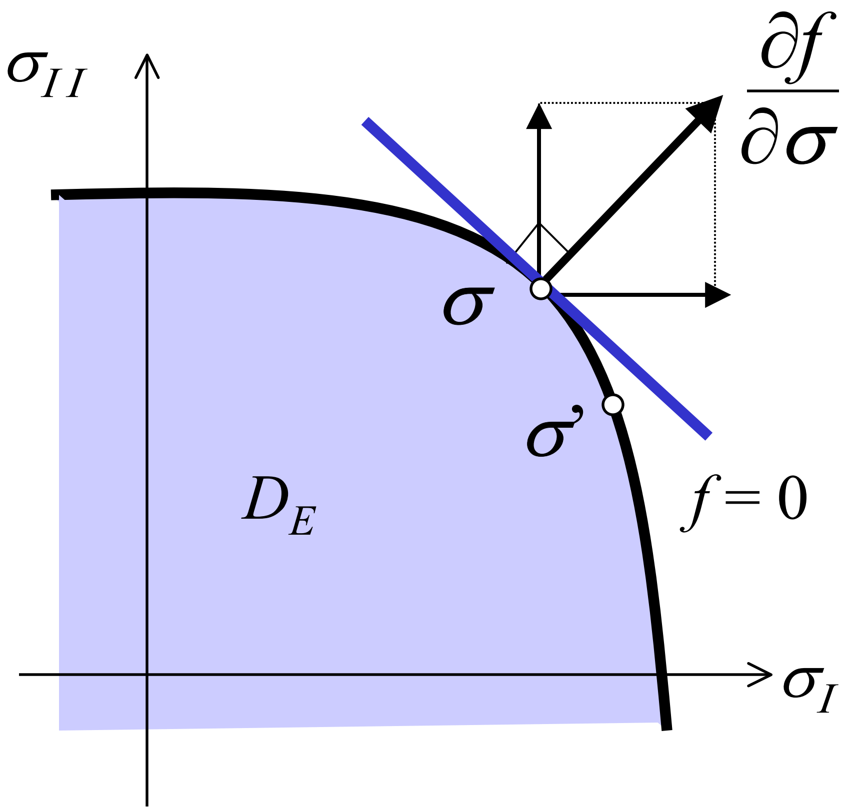
*etc*



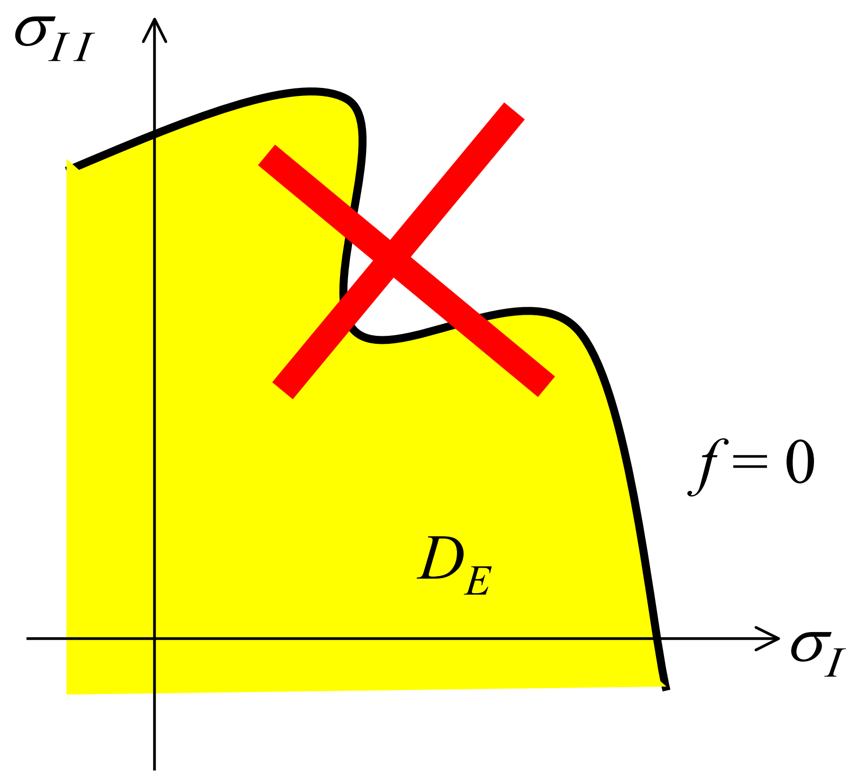
# $1D \rightarrow 3D$ Extension

| 1D | 3D |
|---|---|
| Stress $\sigma$, Strain $\varepsilon$ | Stress Tensor $\boldsymbol{\sigma}$ , Strain $\boldsymbol{\varepsilon}$ |
| Plastic Strain $\varepsilon^p$ | Plastic Strain Tensor $\boldsymbol{\varepsilon}^p$ |
| Hardening Variable $\chi$ | Hardening Variables $\chi$, $\boldsymbol{\chi}$ |
| Hardening Force $\zeta$ | Hardening Forces $\zeta$, $\boldsymbol{\zeta}$ |
| $f = \lvert\sigma+\zeta\rvert - k \leq 0$ | $f = \lvert\mathbf{s} + \boldsymbol{\zeta}\rvert - k \leq 0$ |
| $\varphi dt = \sigma d\varepsilon - d\Psi \geq 0$ | $\varphi dt = \boldsymbol{\sigma}:d\boldsymbol{\varepsilon} - d\Psi \geq 0$ |
| *etc* | *etc* |

# Convexity of Elasticity Domain

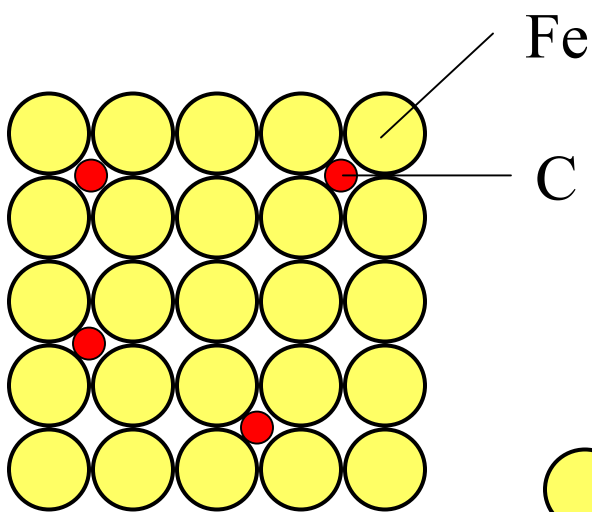



(a)

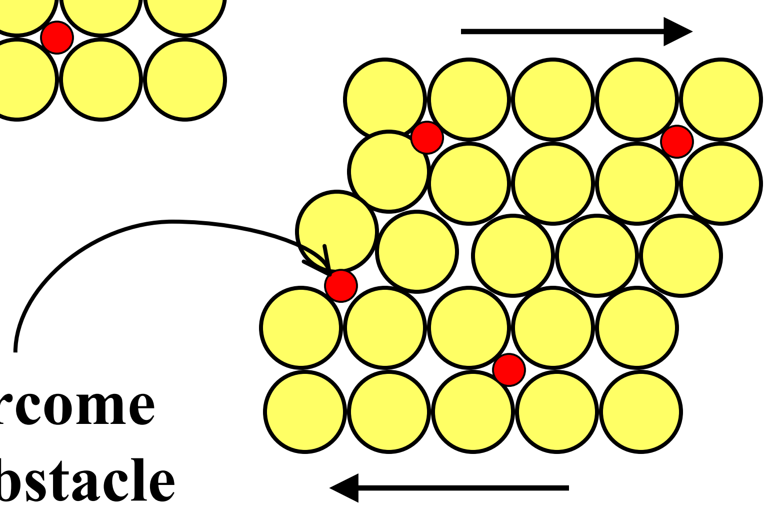



(b)

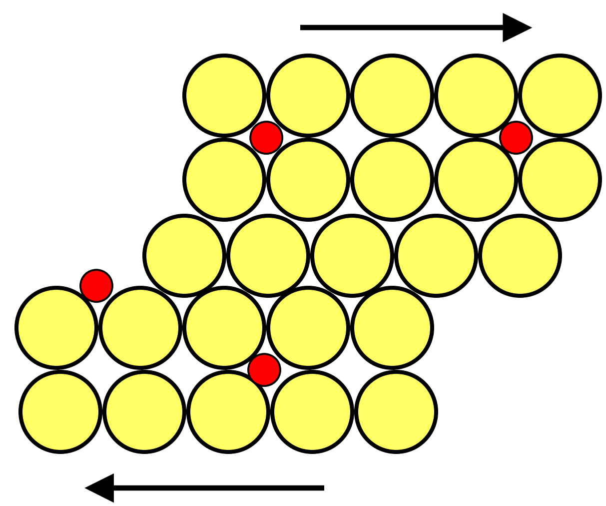
# Example: Crystal Structure of Steel




Impurity = defects in the crystal structure / network

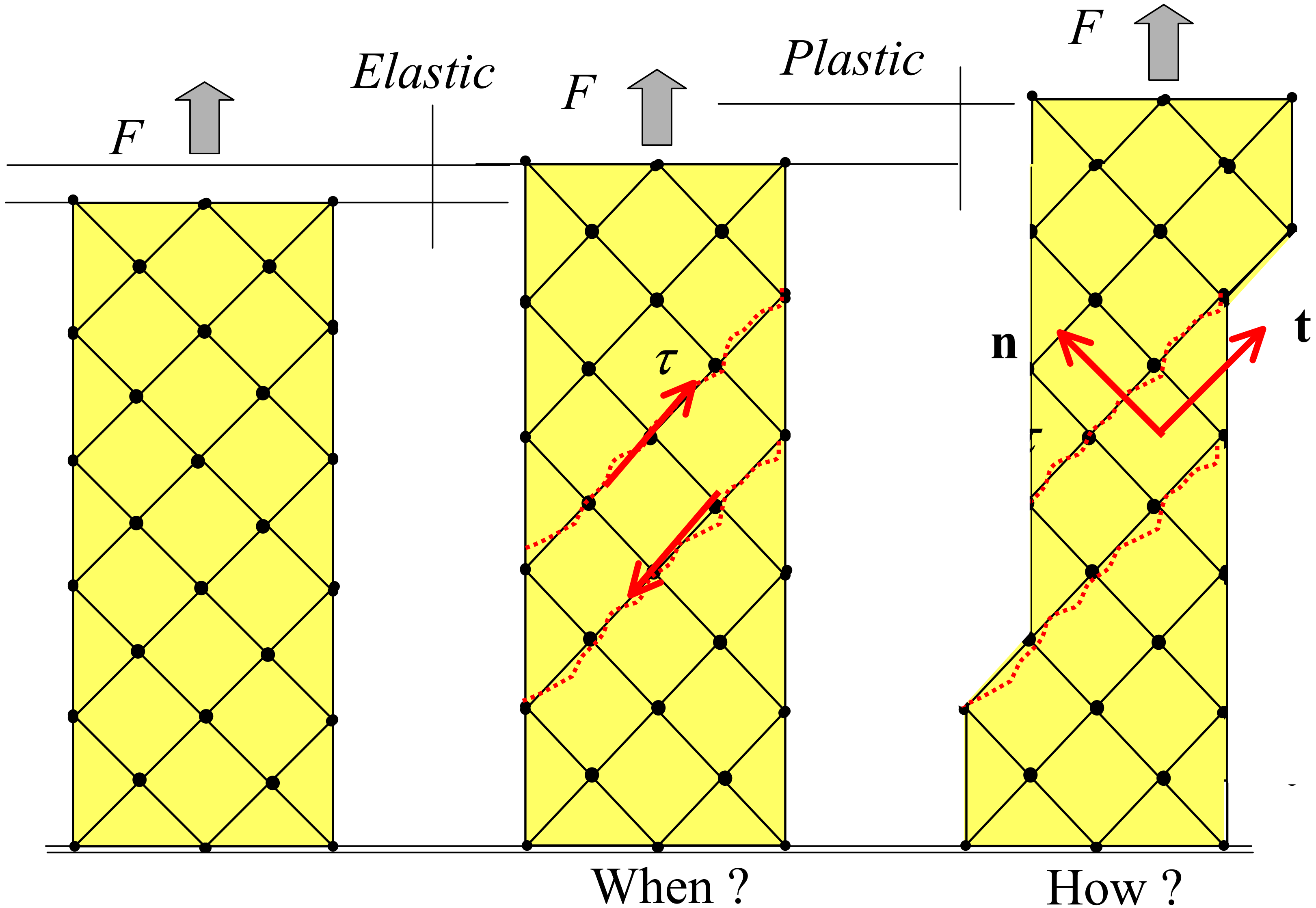


**Overcome an Obstacle “when?”**

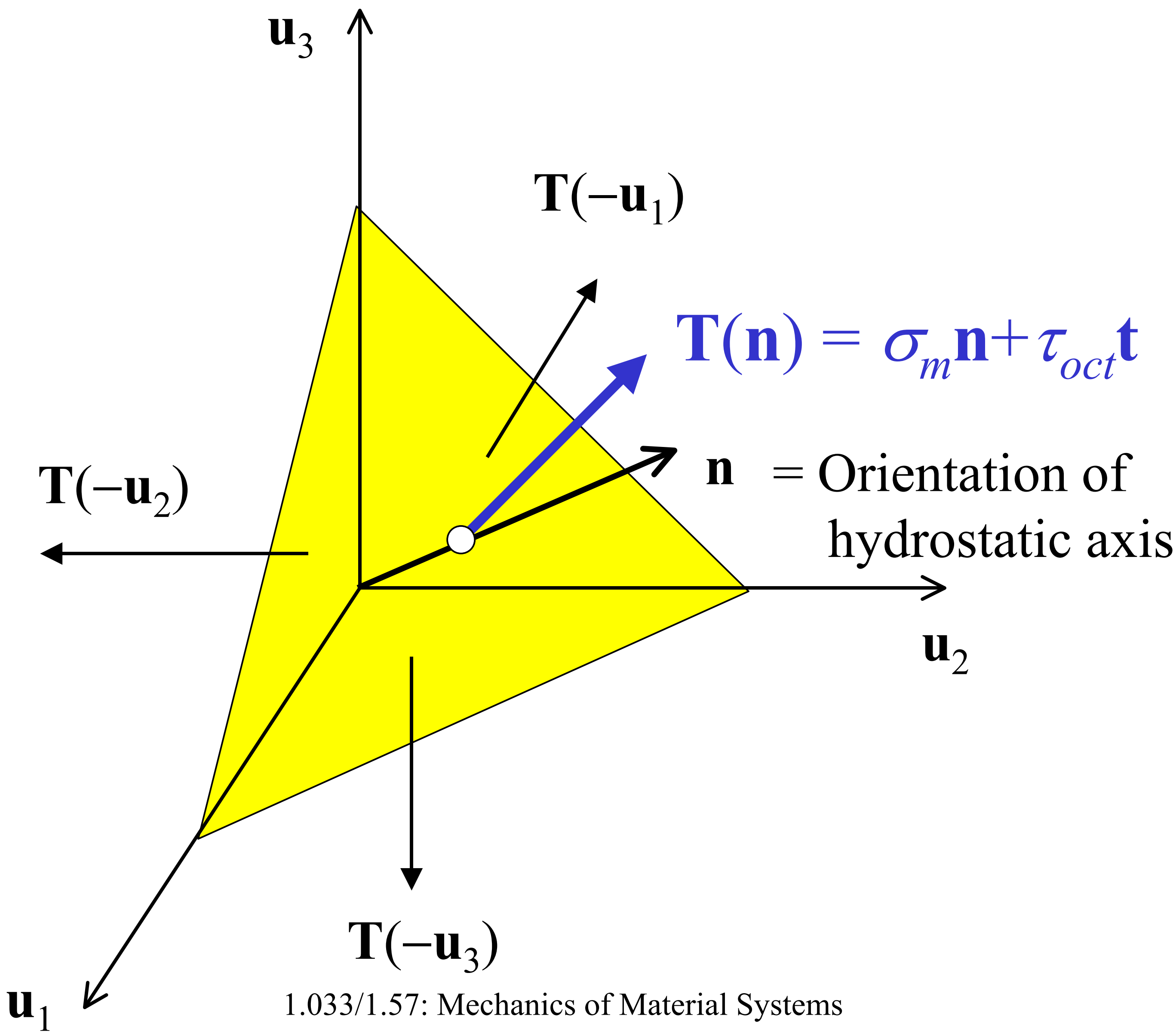


**Slippage planes = Direction of permanent deformation “how?”**

# Sliding in a Monocrystal ("Kinematics")

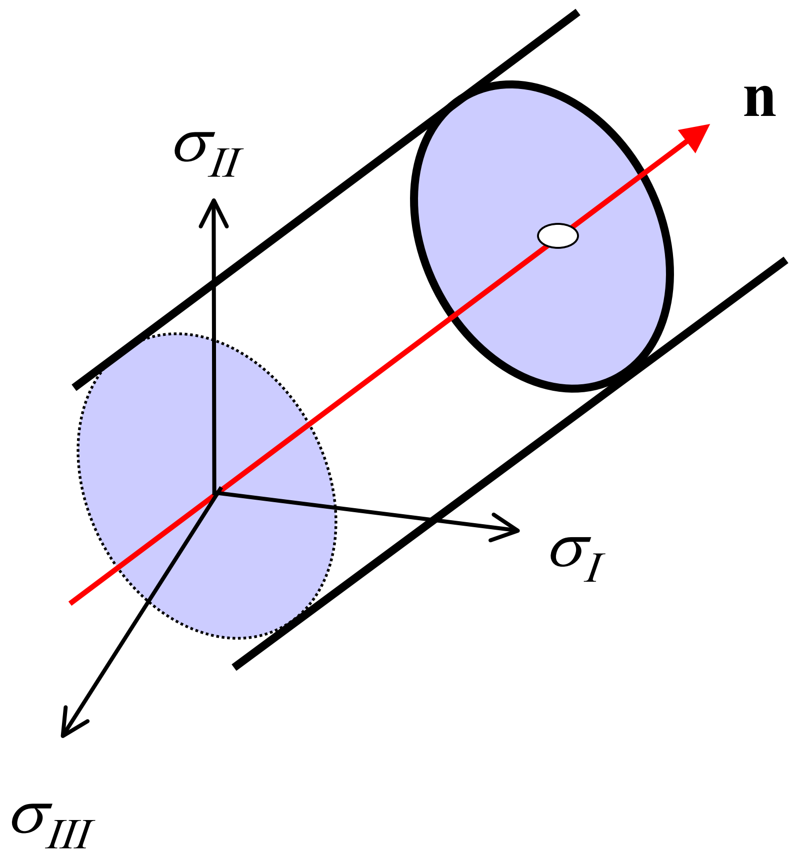


# Stress Vector on Deviator Plane

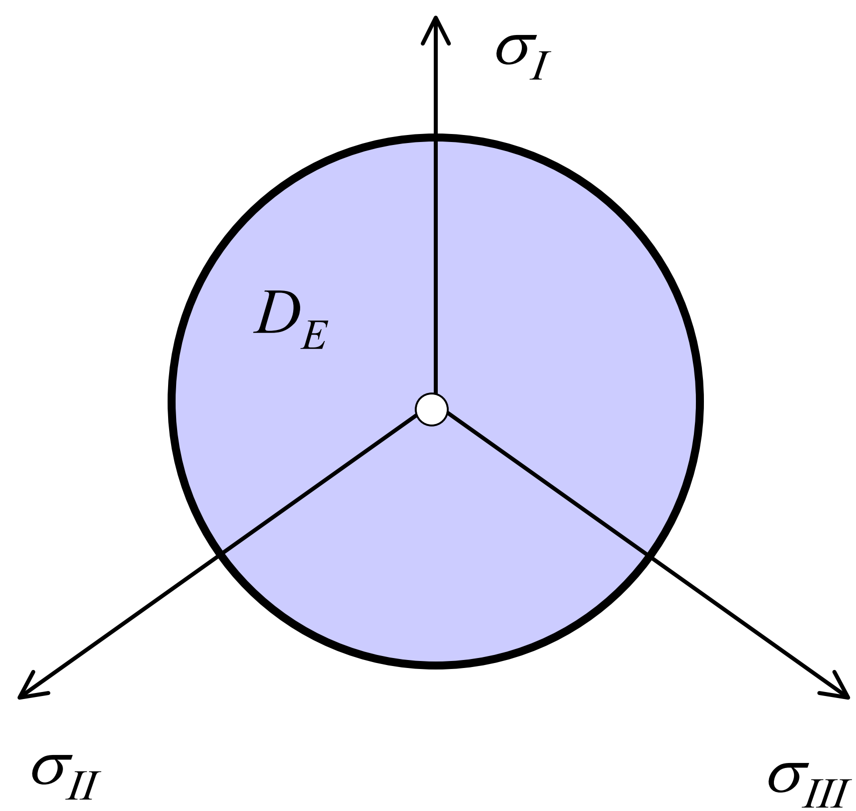


# Von-Mises Plasticity: Yield Criterion

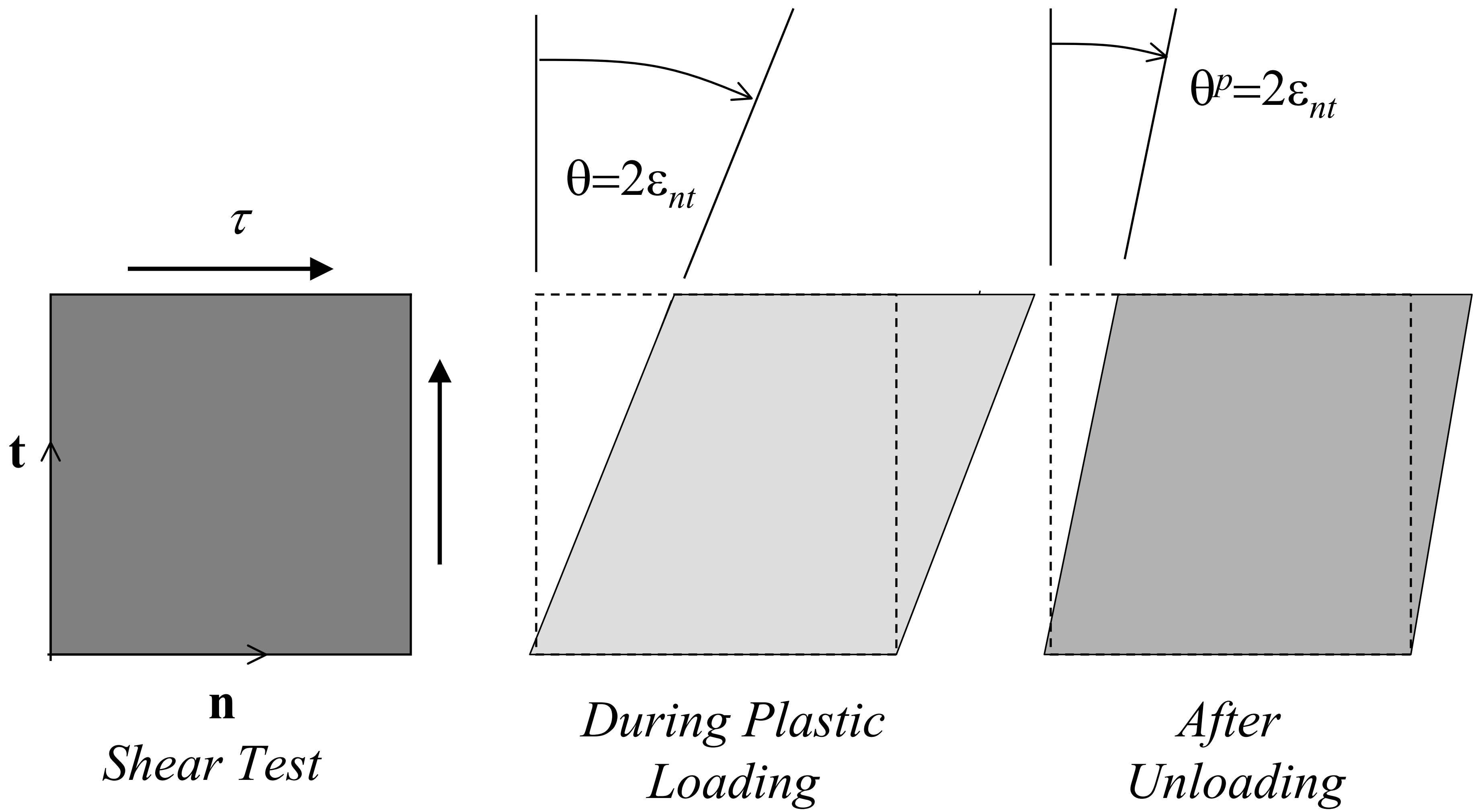
Principal Stress Space



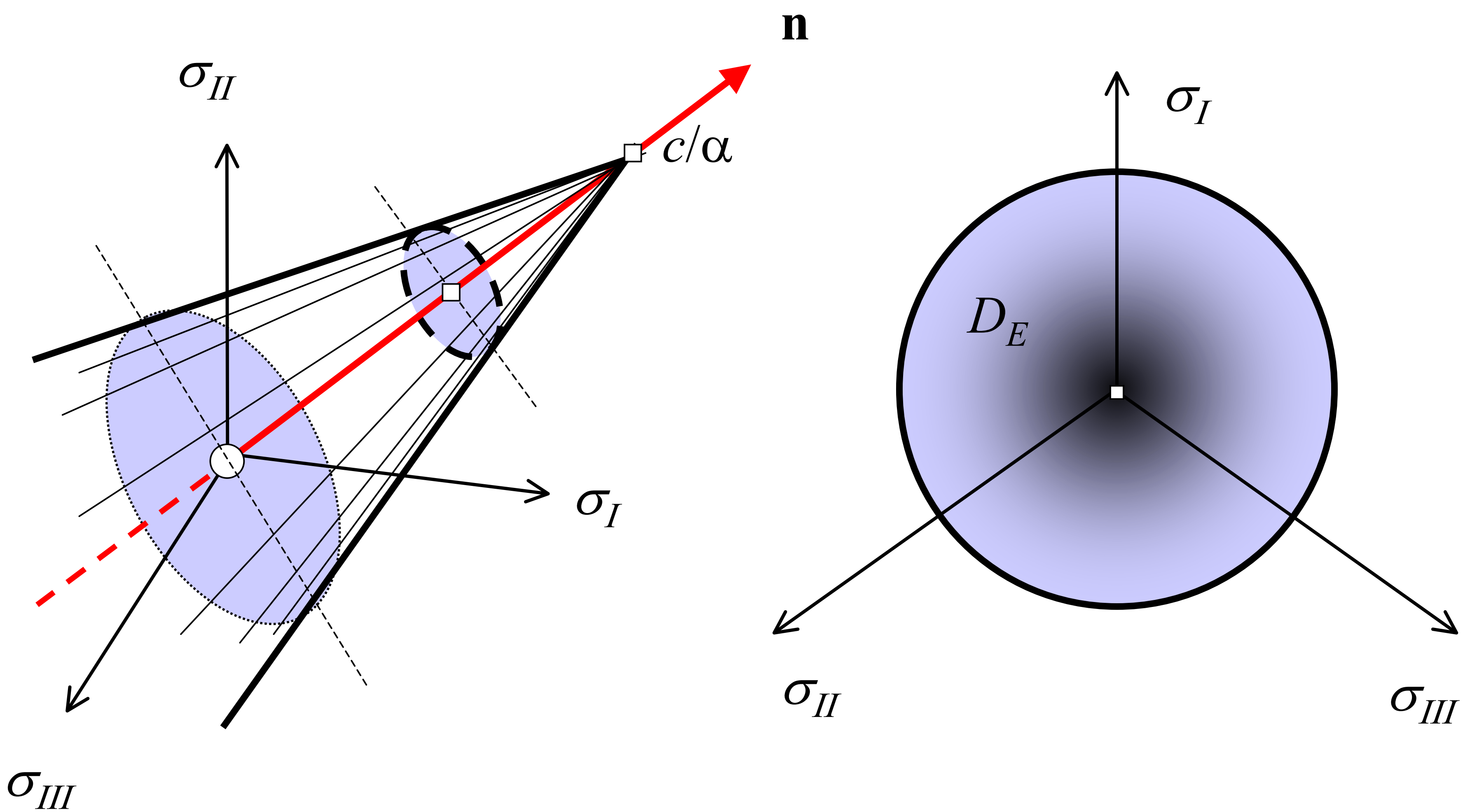

Deviator Plane



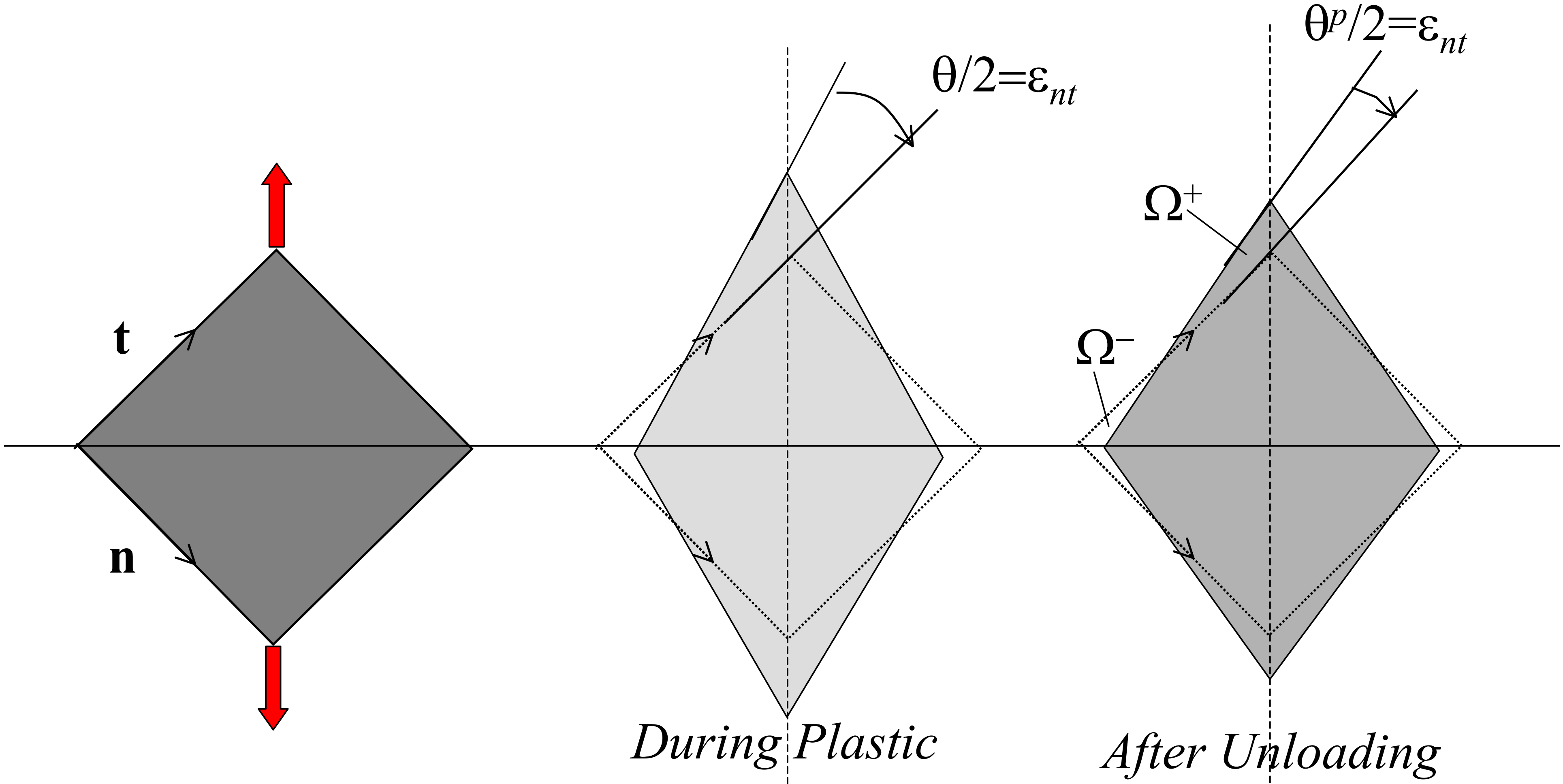
# Von-Mises Plasticity: "Kinematics"



# Drucker-Prager Plasticity: Yield Criterion

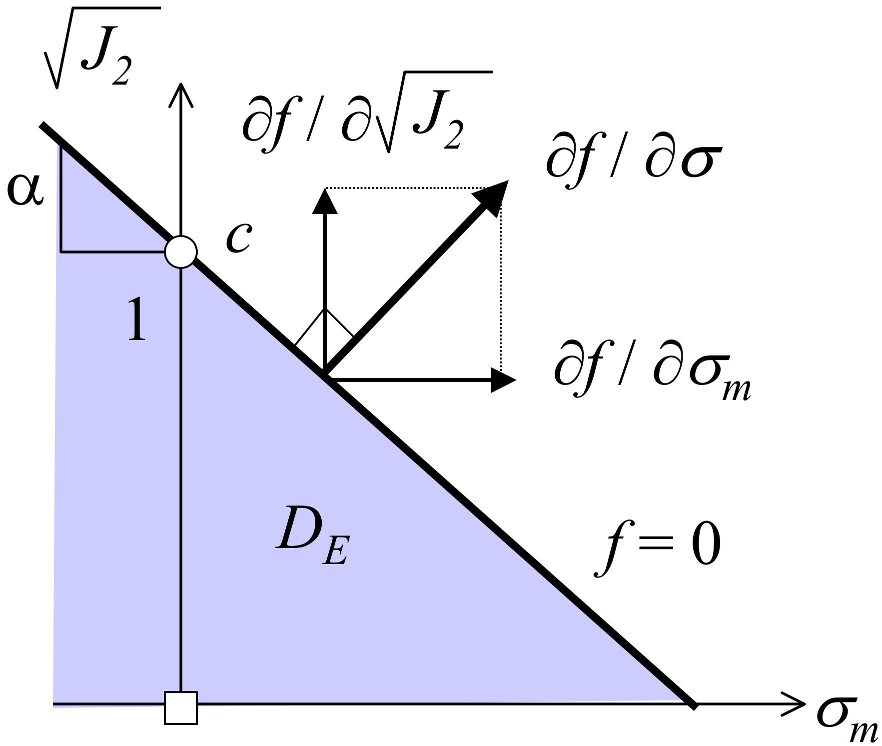


# Drucker-Prager Plasticity: "Kinematics"

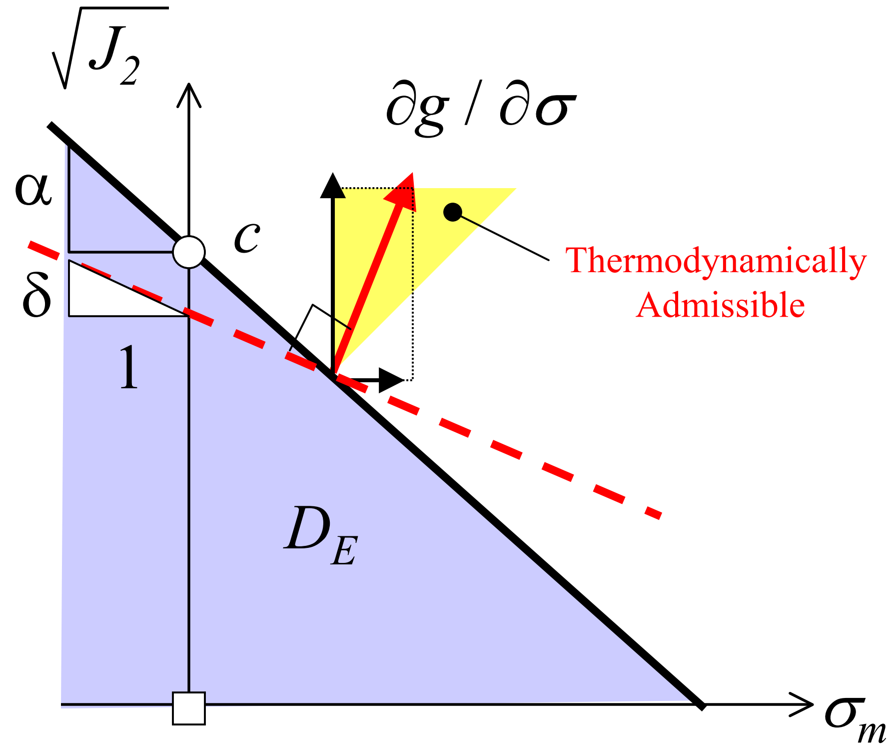



| | |
|---|---|
| $\Omega^{+} > \Omega^{-}$ | Plastic Dilatation |

# Drucker-Prager Plasticity: Thermodynamic Restrictions




Associated Plasticity




Non-Associated Plasticity

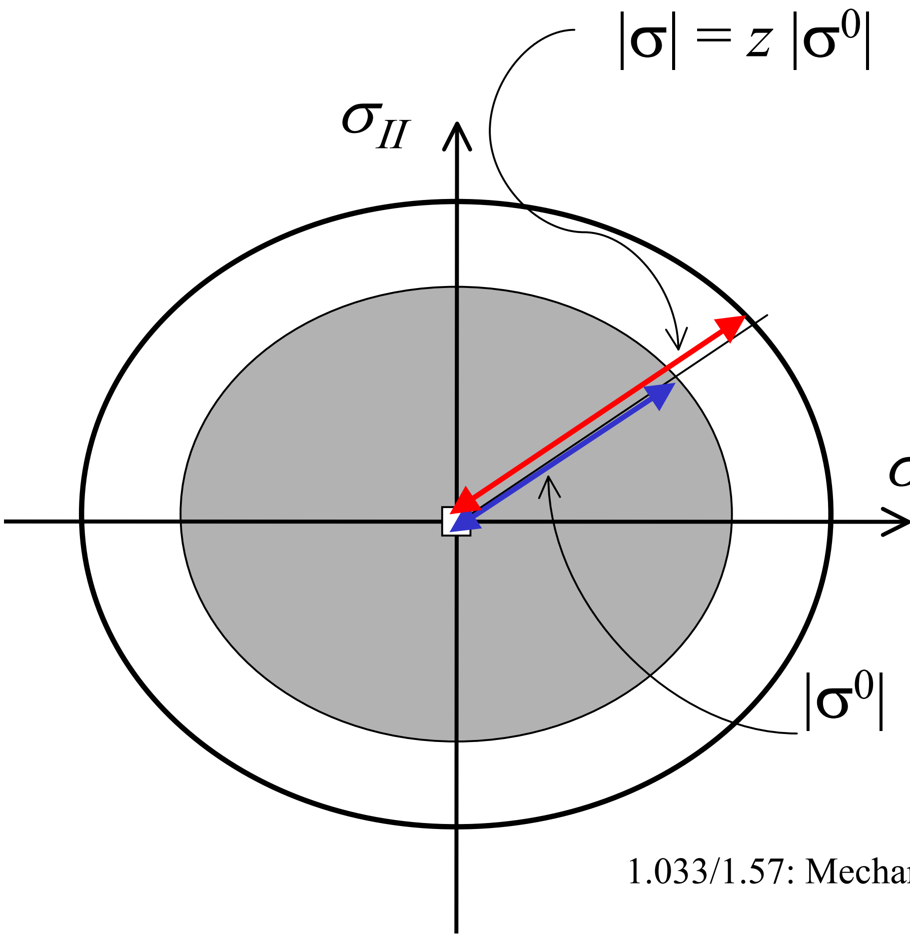
# Plastic Hardening Models

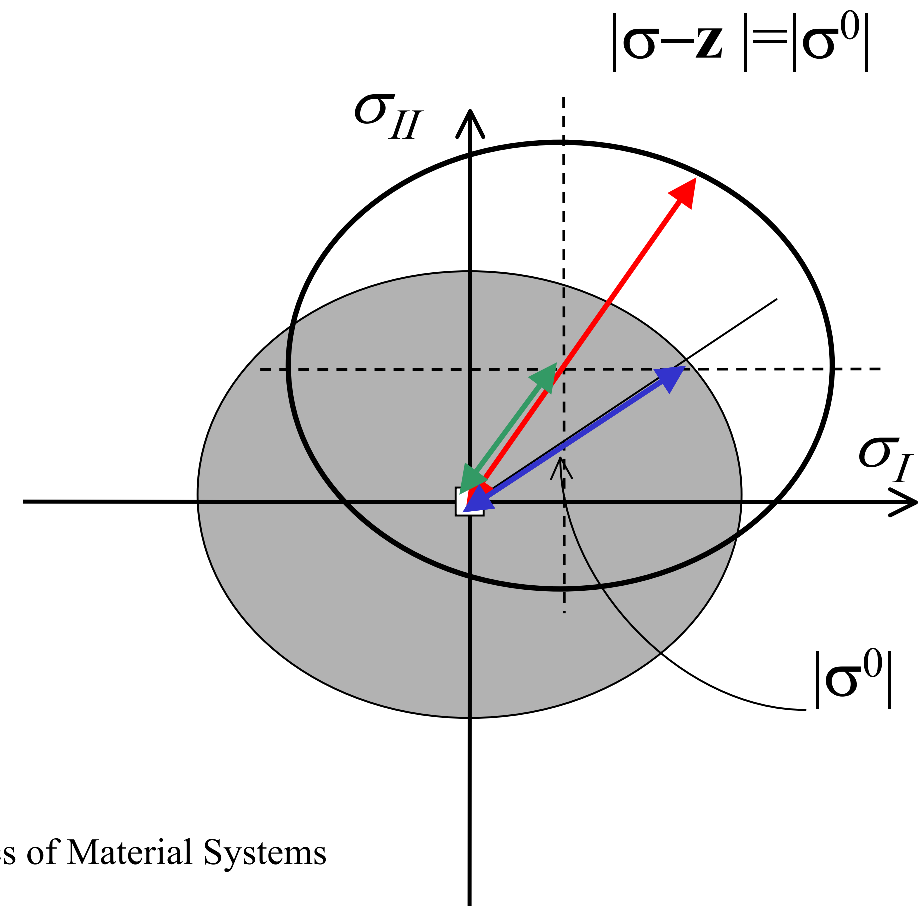
**Isotropic Hardening**

$f(\sigma, \zeta) \leq 0$

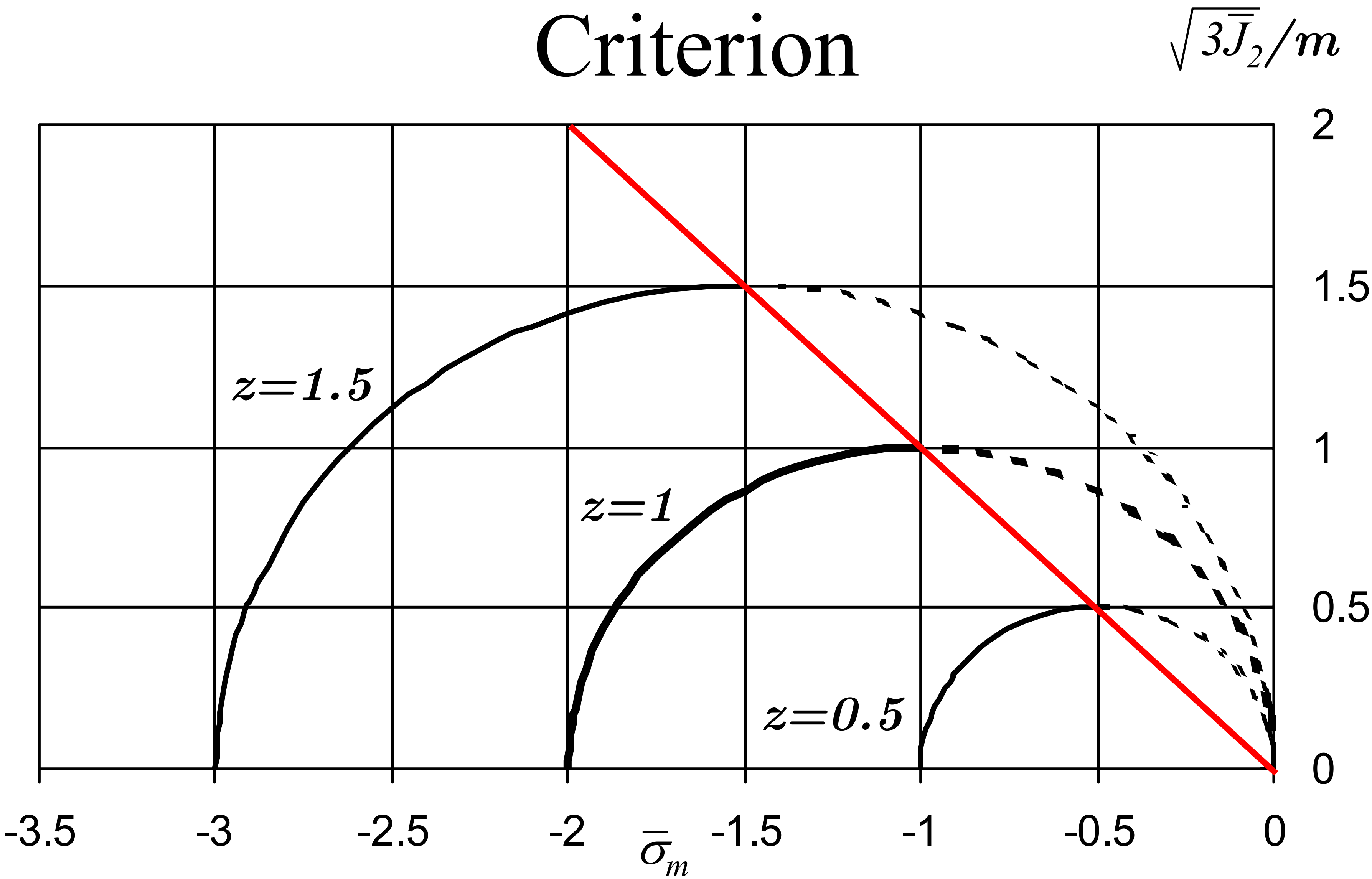
**Kinematical Hardening**

$f(\sigma, \zeta) \leq 0$



# Cam-Clay Model: Yield Criterion



# Cam-Clay Model: "Kinematics"