1.050 Solid Mechanics, Fall 2004
Problem Set 9 Solution

Problem 9.1

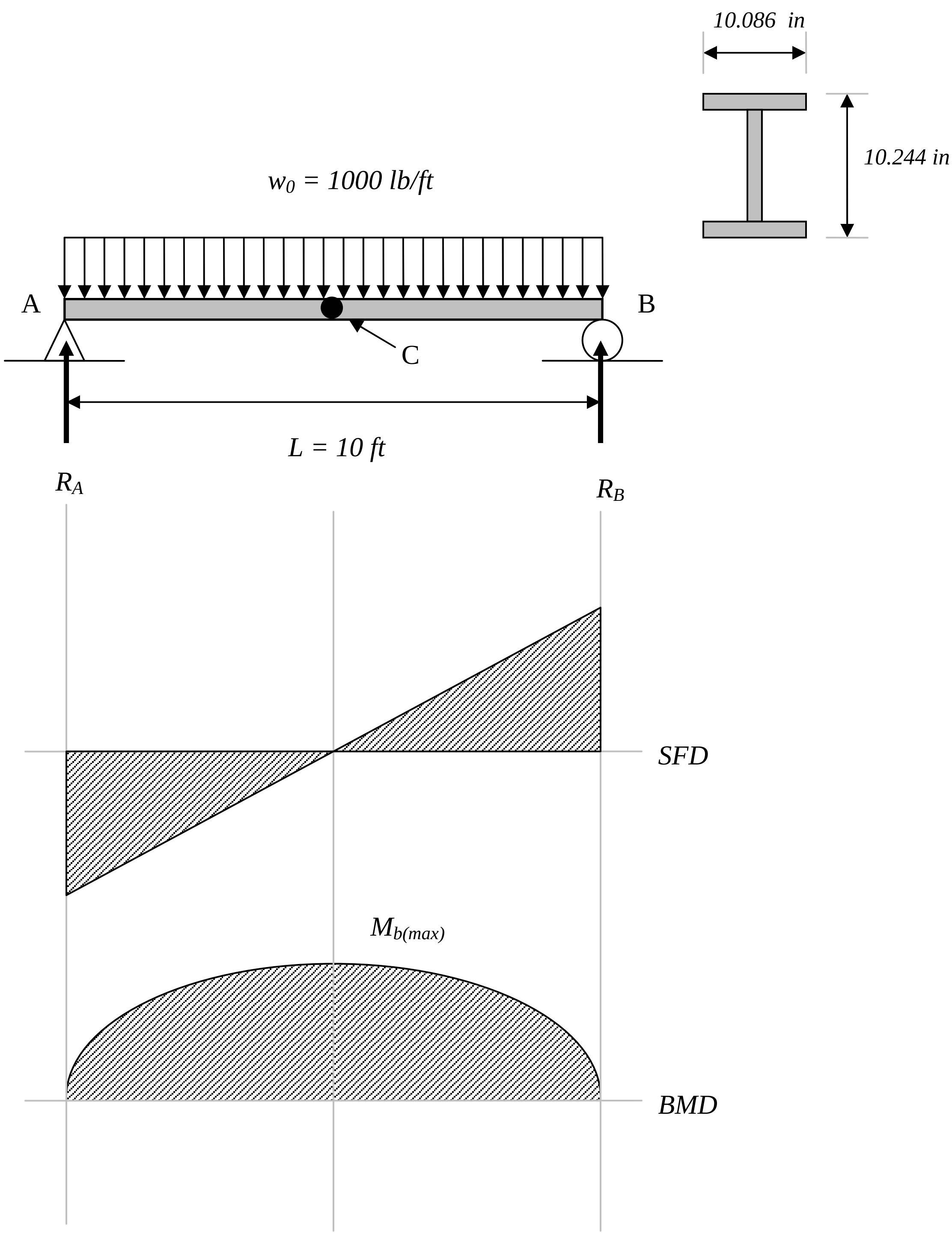

Figure 1.1

$$\sum F_Y = 0; \quad R_A + R_B - 1000(10) = 0$$
$$R_A = R_B$$
$$R_A = \frac{1000(10)}{2} = R_B = 5000 \text{ lbs}$$
$$M_{b(\max)} = 5000(5) - 5000(2.5) = 12500 \text{ lbs - ft}$$
$$\sigma_x = \frac{M_b \cdot y}{I} = \frac{12500 \cdot 10.244 \cdot 12}{2 \cdot 344.6} = 2229.5 \ \frac{lbs}{in^2}$$

For the shear stress at the NA (at the left end of the beam), we get that

$$\sigma_{xy} @ NA = \frac{V}{Ib}\int ydA = \frac{\mathrm{V}}{\mathrm{Ib}}\left(\bar{y}A'\right)$$
$$\sigma_{xy} @ NA = \frac{5000}{(344.6)(0.422)}\left[(10.086)(0.682)(4.781) + (4.494)(0.422)(2.247)\right]$$
$$\sigma_{xy} @ NA = 1277.3 \ \frac{lbs}{in^2}$$

From the stress transformation equations and the σ'_{xy} must be zero on the plane that have maximum and minimum normal stresses, we get that

$$\sigma'_{xy} = -\left[\frac{\sigma_x - \sigma_y}{2}\right](\sin 2\phi) + \sigma_{xy}(\cos 2\phi) = 0$$
$$1277.3(\cos 2\phi) = 0$$
$$\cos 2\phi = 0$$
$$2\phi = 90^\circ$$
$$\phi = 45^\circ$$

At the left end of the beam, the normal stress caused by the bending moment (σ_x) is zero. Therefore, we get that

$$\sigma'_x = \left[\frac{\sigma_x + \sigma_y}{2}\right] + \left[\frac{\sigma_x - \sigma_y}{2}\right](\cos 2\phi) + \sigma_{xy}(\sin 2\phi)$$
$$\sigma'_x = 1277.3(\sin 90^\circ)$$
$$\sigma'_x = 1277.3 \ \frac{lbs}{in^2}$$
$$\sigma'_y = \left[\frac{\sigma_x + \sigma_y}{2}\right] - \left[\frac{\sigma_x - \sigma_y}{2}\right](\cos 2\phi) - \sigma_{xy}(\sin 2\phi)$$

$$\sigma_y^{'} = -1277.3\left(\sin 90^\circ\right)$$

$$\sigma_y^{'} = -1277.3 \ \frac{lbs}{in^2}$$

For the shear stress at the point where the web meets the flange (at the left end of the beam), we get that

$$\sigma_{xy} = \frac{V}{Ib}\int ydA = \frac{\mathrm{V}}{\mathrm{Ib}}\left(\bar{y}A'\right)$$

$$\sigma_{xy} = \frac{5000}{(344.6)(0.422)}\left[(10.086)(0.682)(4.781)\right]$$

$$\sigma_{xy} = 1130.7 \ \frac{lbs}{in^2}$$

Note that the maximum normal stress caused by the shear force at the left end of the beam is much less than the maximum normal stress caused by the beam bending at the middle (pt.C) of the beam (1277.3 psi vs. 2229.5 psi). Therefore, the maximum normal stress that you should use in the design of the beam is still that of the flexure formula at the maximum bending moment (2229.5 psi). Also note that the shear force at the middle of the beam is zero and hence have no effect on the maximum normal stress at that point.

Problem 9.2

The main differences between the σ_x from the three section are from the y/I factor. Let's look at the y/I factor of each case. Case 1, 2, and 3 represent the I section wood chip, 2x10, and 4x10 section, respectively. Also note that the moment of inertia, I, for a rectangle section is $bh^3/12$.

Case 1: $\left.\frac{y}{I}\right|_1 = \frac{7}{320} = 0.0219 \ \text{in}^{-3}$

Case 2: $\left.\frac{y}{I}\right|_2 = \frac{5\cdot 12}{2\cdot 10^3} = 0.030 \ \text{in}^{-3}$

Case 3: $\left.\frac{y}{I}\right|_2 = \frac{5\cdot 12}{4\cdot 10^3} = 0.015 \ \text{in}^{-3}$

Therefore, we get that $\sigma_{x3} < \sigma_{x1} < \sigma_{x2}$ and $\frac{\sigma_{x1}}{\sigma_{x2}} = 0.73$ and $\frac{\sigma_{x1}}{\sigma_{x3}} = 0.1.46$.

Problem 9.3

In this solution, I show you how to derive the moment of inertia, I, using the direct integration method which is different from the method used in the class (in the class you use J to calculate I). Figure 3.1 shows the sketch of the solid and hollow circular section with relevant dimensions for the calculation of the moment of inertia.

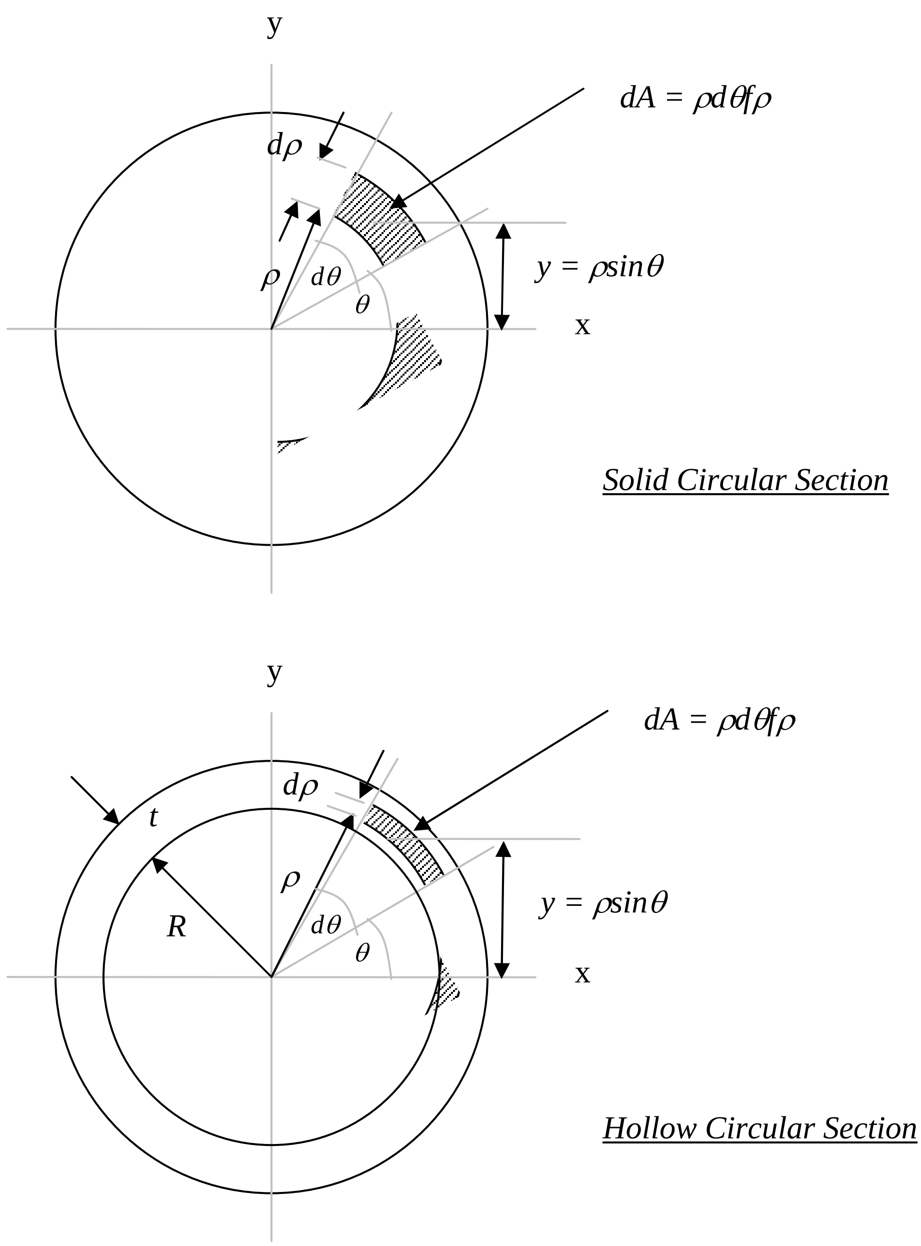

Figure 3.1

For a **solid** circular section with radius of *r*:

$$I_X = \int y^2 dA = \int_0^r \int_0^{2\pi} \rho^2 (\sin^2\theta) d\theta d\rho$$

$$I_X = \frac{r^4}{4}\int_0^{2\pi} \sin^2\theta d\theta$$

$$I_X = \frac{\pi r^4}{4}$$

For a **hollow** circular section with radius of *R* and thickness of *t*:

$$I_X = \int y^2 dA = \int_R^{R+t} \int_0^{2\pi} \rho^2 (\sin^2\theta) d\theta d\rho$$

$$I_X = \left[\frac{\rho^4}{4}\right]\Bigg|_R^{R+t} \int_0^{2\pi} \sin^2\theta d\theta$$

$$I_X = \frac{(R+t)^4 - R^4}{4}(\pi)$$

$$I_X = \frac{\pi}{4}\left(R^4 + 4R^3 t + 6R^2 t^2 + 4Rt^3 + t^4 - R^4\right)$$

$$I_X = \frac{\pi}{4}\left(4R^3 t + 6R^2 t^2 + 4Rt^3 + t^4\right)$$

For a hollow circular section with small *t*, we get that

$$I_X = \frac{\pi}{4}\left(4R^3 t + 6R^2 t^2\right)$$

From $2\pi Rt = \pi r^2$, we get that

$$I_{x(solid)} = \frac{\pi (2Rt)^2}{4} = \frac{\pi}{4}\left(4R^2 t^2\right)$$

Therefore, for the same area, the $I_{x(hollow)}$ is larger than $I_{x(solid)}$.

$$I_{X(hollow)} = \frac{\pi}{4}\left(4R^3 t + 6R^2 t^2\right) = \pi R^3 t + \frac{\pi R^2 t^2}{2} + I_{X(solid)}$$