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# Problem Set VIII Solution

**Solution:**
Evaluate the impact of changing the longitudinal tendon pitch from 165 mm to 200 mm on the required prestress level to prevent tensile in the concrete upon pressurization.

Material properties and operating conditions:
$E_s = 210GPa; \quad E_c = 21GPa \text{ and } \nu = 0.15$
Internal pressure $P=350\ kPa$, and $R=R_{in}+t/2= 21.19m$

To calculate the required prestress level, we should go through the following steps:
At first, compute some constants that will be used:

$$\chi_{ls} = \frac{2A_s}{tp_l}, \text{where } A_s = \frac{\pi \cdot d_t^2}{4} = 2551.8mm^2$$

$$\chi_{\theta s} = \frac{4A_s}{tp_\theta}$$

$$N_l = \frac{PR}{2} = 3.708MPa \cdot m$$

$$\bar{\sigma}_r = -\frac{P}{2}$$

$$E_l = (1-\chi_{ls})\frac{E_c}{1-\nu^2}$$

$$E_\theta = (1-\chi_{\theta s})\frac{E_c}{1-\nu^2}$$

$$E^* = \chi_{ls}E_s + E_l$$

*and then*

$$\alpha = \frac{t}{R}\left[\chi_{\theta s}E_s + E_\theta(1-\frac{\nu^2 E_l}{E^*})\right]$$

$$\gamma = \frac{\nu E_\theta}{E^*}N_l + \frac{\nu t}{1-\nu}\bar{\sigma}_r(1-\chi_{\theta s}-(1-\chi_{ls})\frac{\nu E_\theta}{E^*})$$

$$D = \frac{E_c}{1-\nu^2}(\frac{t^3}{12}+\chi_{ls}\chi_s{}^2 t) + E_s\chi_{ls}\chi_s{}^2 t \text{ , where} \chi_s = thickness/2 - depth = 0.525m$$

According to notes for problem set L54, we get

$$\bar{\varepsilon}_{\theta c} = \frac{w_p}{R} = \frac{R}{\alpha}(P-\frac{\gamma}{R})$$

$$\bar{\varepsilon}_{lc} = \frac{\frac{N_l}{t} - \left[\nu E_l\bar{\varepsilon}_{\theta c} + (1-\chi_{ls})\frac{\nu}{1-\nu}\bar{\sigma}_r\right]}{E^*}$$

The maximum stress in longitudinal direction occurs at $\chi = -\frac{t}{2}$ for $\bar{\varepsilon}_{\theta c\max}$ *and* $\bar{\varepsilon}_{lc\max}$

$$\sigma_{lc\max} = \frac{E_c}{1-\nu^2}(\nu\bar{\varepsilon}_{\theta c\max} + \bar{\varepsilon}_{lc\max} + \varepsilon_{bl}) + \frac{\nu}{1-\nu}\bar{\sigma}_r$$

where

$$\varepsilon_{bl}(z) = \frac{t}{2}\frac{d^2w}{dz^2} = t\beta^2 w_p e^{-\beta z}\left[\cos\beta z - \sin\beta z\right]$$

where in turn $\beta = \left(\frac{\alpha}{4DR}\right)^{\frac{1}{4}}$

Since $\bar{\varepsilon}_{\theta c\max}$ *and* $\bar{\varepsilon}_{lc\max}$ reach the maximum value at $z \to \infty$, therefore, the strain due to bending $\varepsilon_{bl} = 0$ as $z \to \infty$.

Now, the maximum longitudinal stress shall be offset by the tendon prestress to get zero net concrete stress upon pressurization and therefore we get

$$\sigma_{lsprestree} = \frac{1-\chi_{ls}}{\chi_{ls}}\sigma_{lc\max}$$

Similarly, the maximum stress in hoop direction

$$\sigma_{\theta c\max} = \frac{E_c}{1-\nu^2}(\bar{\varepsilon}_{\theta c\max} + \nu\bar{\varepsilon}_{lc\max}) + \frac{\nu}{1-\nu}\bar{\sigma}_r$$

Then, we get

$$\sigma_{\theta sprestree} = \frac{1-\chi_{\theta s}}{\chi_{\theta s}}\sigma_{\theta c\max}$$

For the two cases where the longitudinal tendon pitch is equal to 165 mm and 200 mm, respectively, we just substitute corresponding numbers to the above equations, and obtain the final results:

165 mm:

$$\sigma_{lsprestree} = 99.5MPa$$

$$\sigma_{\theta sprestree} = 104.36MPa$$

200 mm:

$$\sigma_{lsprestree} = 123.7MPa$$

$$\sigma_{\theta sprestree} = 104.41MPa$$

According to these result, when changing the longitudinal tendon pitch from 165 mm to 200mm, the required prestress in the longitudinal direction is increased by almost 25%, however the required prestress in the hoop direction almost remains unchanged.