1.010 Uncertainty in Engineering
Fall 2008

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Homework Set #9
Due November 20, 2008 (in class)

1. Let X_1 and X_2 be the gains, in millions of dollars, from investing \$1million in two different stocks. The objective of investing is of course to maximize gains with a minimum of uncertainty. Suppose that X_1 and X_2 have the following second moment characteristics:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sim \left(\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \begin{bmatrix} 0.3^2 & 0 \\ 0 & 0.2^2 \end{bmatrix} \right)$$

(a) Which stock type would you prefer for investment?
(b) As an alternative, consider investing \$500k in stock 1 and \$500k in stock 2. Would you consider this strategy more attractive? Give an intuitive explanation of your result.
(c) How should you invest your \$1 million to minimize uncertainty on the returns?

2. The thickness H of a coal layer varies randomly with geographical location. Specifically, the thicknesses H_A and H_B at locations A and B have second moment characteristics:

$$\begin{bmatrix} H_A \\ H_B \end{bmatrix} \sim \left(\begin{bmatrix} m \\ m \end{bmatrix}, \sigma^2 \begin{bmatrix} 1 & 0.8^{d/10} \\ 0.8^{d/10} & 1 \end{bmatrix} \right)$$

where m = 5m, σ = 2m and d is the distance in meters between points A and B. A boring is made at location A, giving H_A =4m. Find and plot the BLUE estimate of ($H_B|H_A$=4m) and the standard deviation of the estimation error as a function of distance d, for $0 < d <$ 20m. Comment on the behavior of these functions.

3. Rivergauges A, B and C are located at three distant points on the same river stream. The daily fluxes Q_A, Q_B and Q_C (in m^3/s) at locations A, B and C, respectively, have second moment characteristics:

$$\begin{bmatrix} Q_A \\ Q_B \\ Q_C \end{bmatrix} \sim \left(\begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 1.5^2 & 2.6 & 2 \\ 2.6 & 2.5^2 & 5.9 \\ 2 & 5.9 & 3.5^2 \end{bmatrix} \right)$$

For some technical reason gauge A failed to report a measurement of today's flux. What is the best linear estimate of Q_A if Q_B = $3.5m^3/s$ and Q_C = $6.8m^3/s$? What is the variance of the estimation error?