6.851 Advanced Data Structures (Spring'10)

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Problem 3   Sample Solutions

Ray Shooting in Simple Polygons With every step we reduce the WBBST total weight of the current subtree by at least a factor of 2. We finish once we reach a subtree with weight ω_i. Hence, we solve the recurrence $T(\omega) = 1 + T(\omega/2)$. The base case is $T(\omega_i) = 1$, so we get $T(\Omega) = O(1 + log(\Omega/\omega_i))$.

Suppose each concave chain in the balanced pseudo-triangulation is stored in a WBBST, where the weight of an edge i equals the number edges in the opposing polygon ω_i. We consider two adjacent pseudo-triangles, t_a and t_b , crossed by the ray in this algorithm. Let i be the edge the ray crosses to move from t_a into t_b . In t_b the ray homes-in on the next edge it crosses, $i + 1$, in a concave chain, which has at most ω_i edges, and so the total time spent searching the WBBST for the home-in chain in tb is $O(\log(\omega_i/\omega_{i+1}))$. The sum telescopes, and its result is the difference in the logs of two pseudo-triangle sizes, which is no larger than $O(\log n)$. The ray-shooting algorithm traverses no more than $O(\log n)$ triangles in total, giving the total runtime of $O(\log n)$.

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