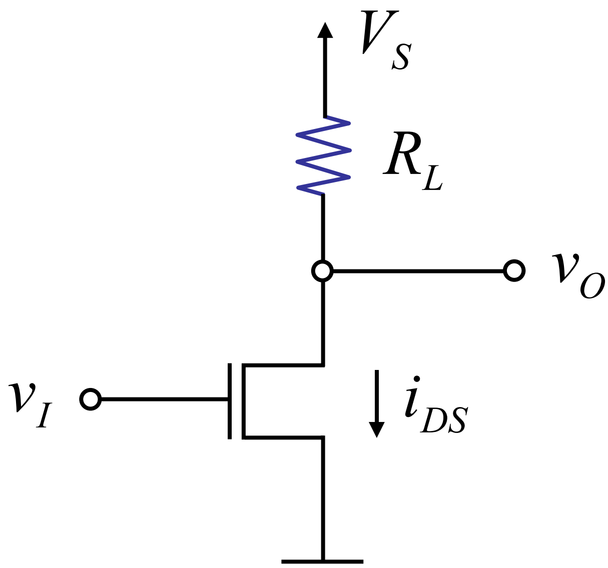
Amplifiers --
Small Signal Model

Review

- MOSFET amp



- Saturation discipline — operate MOSFET only in saturation region
- Large signal analysis
  1. Find v_O vs v_I under saturation discipline.
  2. Valid v_I, v_O ranges under saturation discipline.

Reading: Small signal model -- Chapter 8

Large Signal Review

1 $\quad v_O \ \ vs \ \ v_I$

$$v_O = V_S - \frac{K}{2}(v_I - 1)^2 R_L$$

valid for $v_I \geq V_T$

and

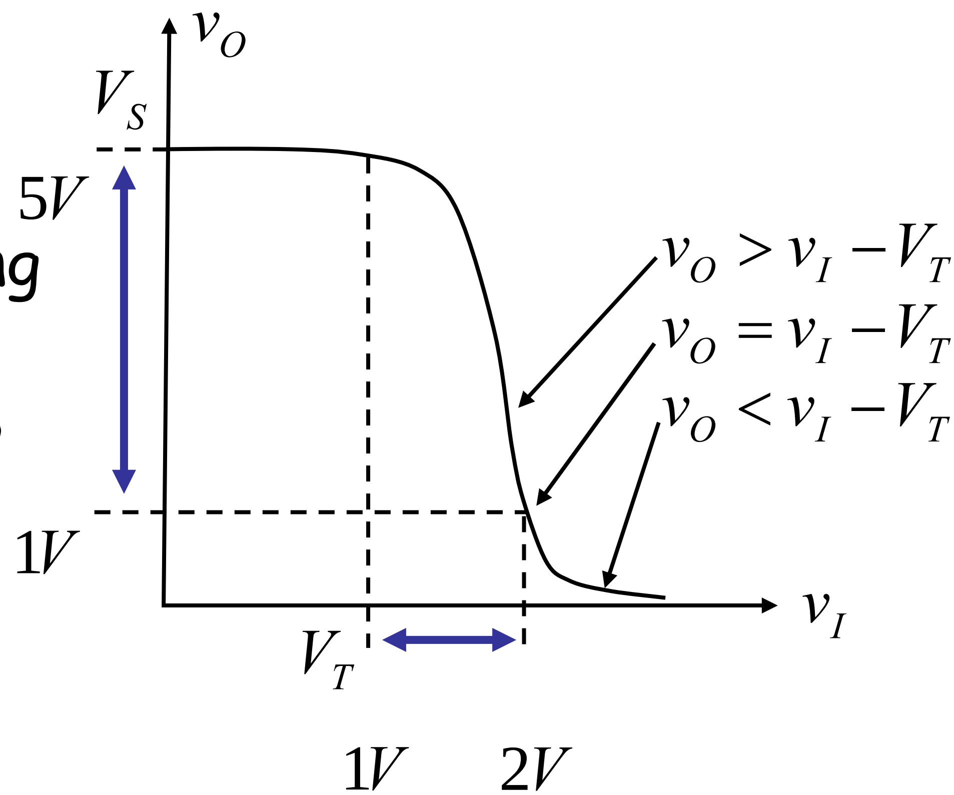
$v_O \geq v_I - V_T$

(same as $i_{DS} \leq \frac{K}{2} v_O^{\ 2}$)

Large Signal Review

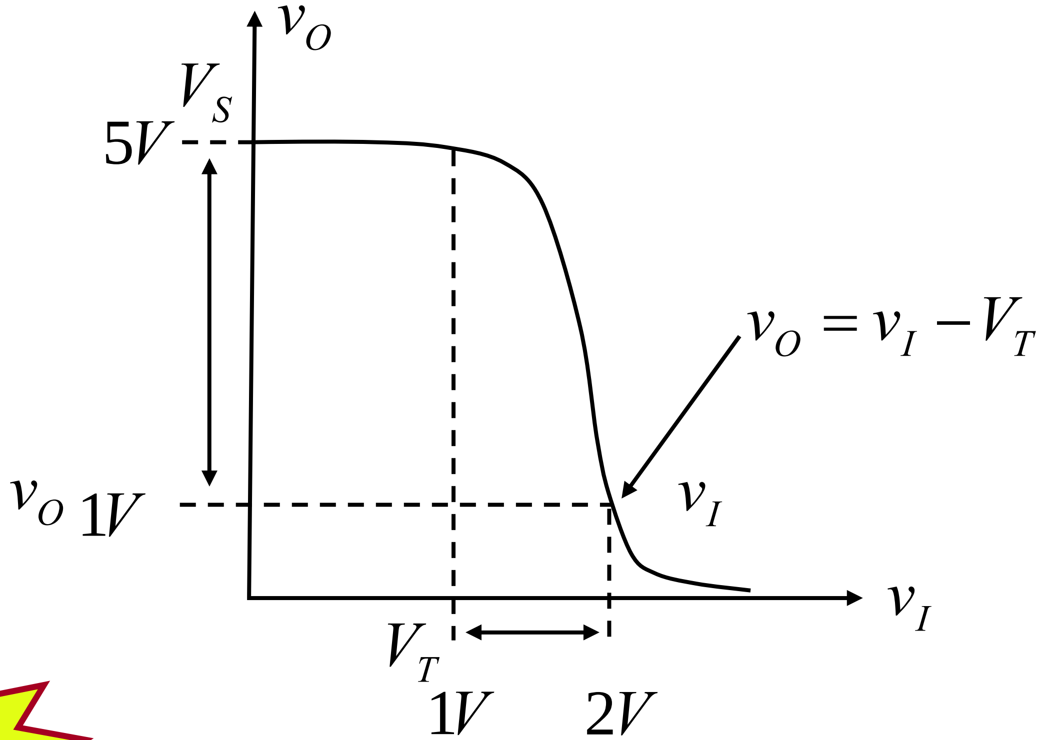
2 Valid operating ranges

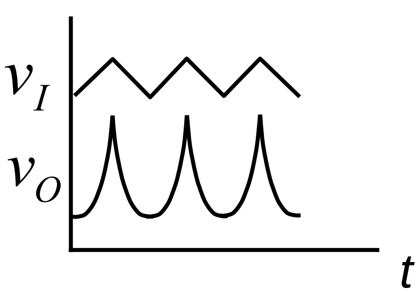
corresponding interesting region for v_O



"interesting" region for v_I . Saturation discipline satisfied.

But…

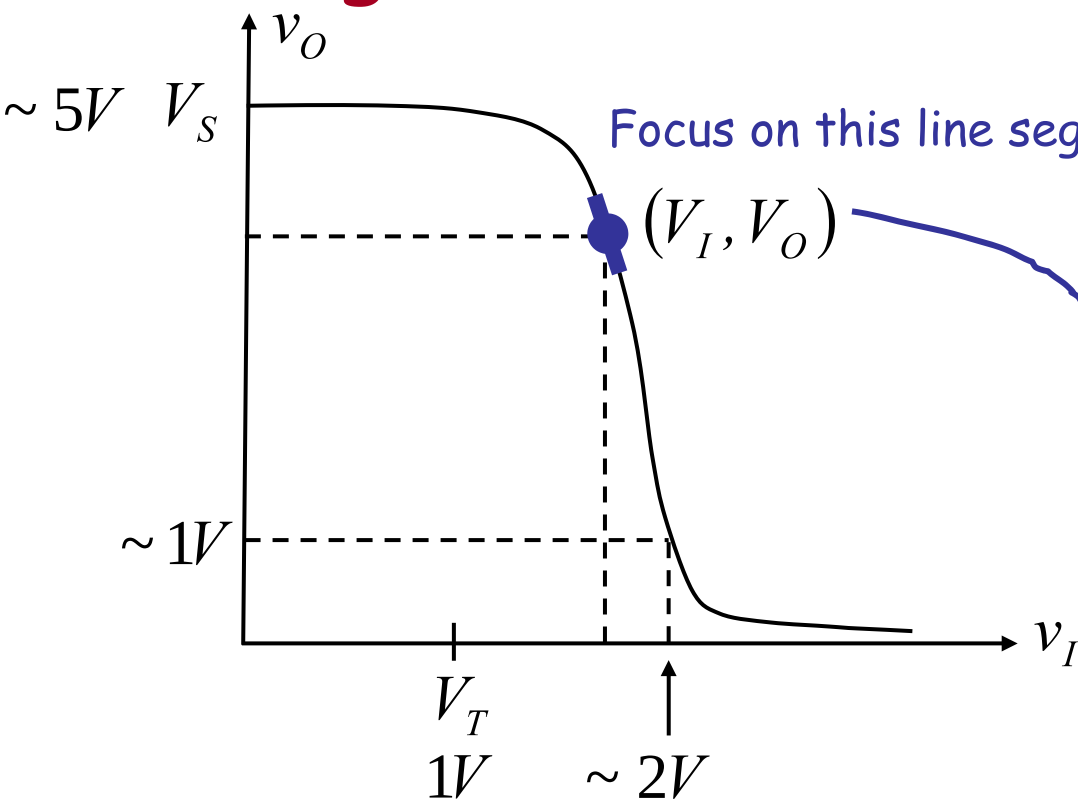




Amplifies alright,
but distorts

Amp is nonlinear … ☹

Small Signal Model



$$v_O = V_S - \frac{K(v_I - V_T)^2}{2} R_L$$

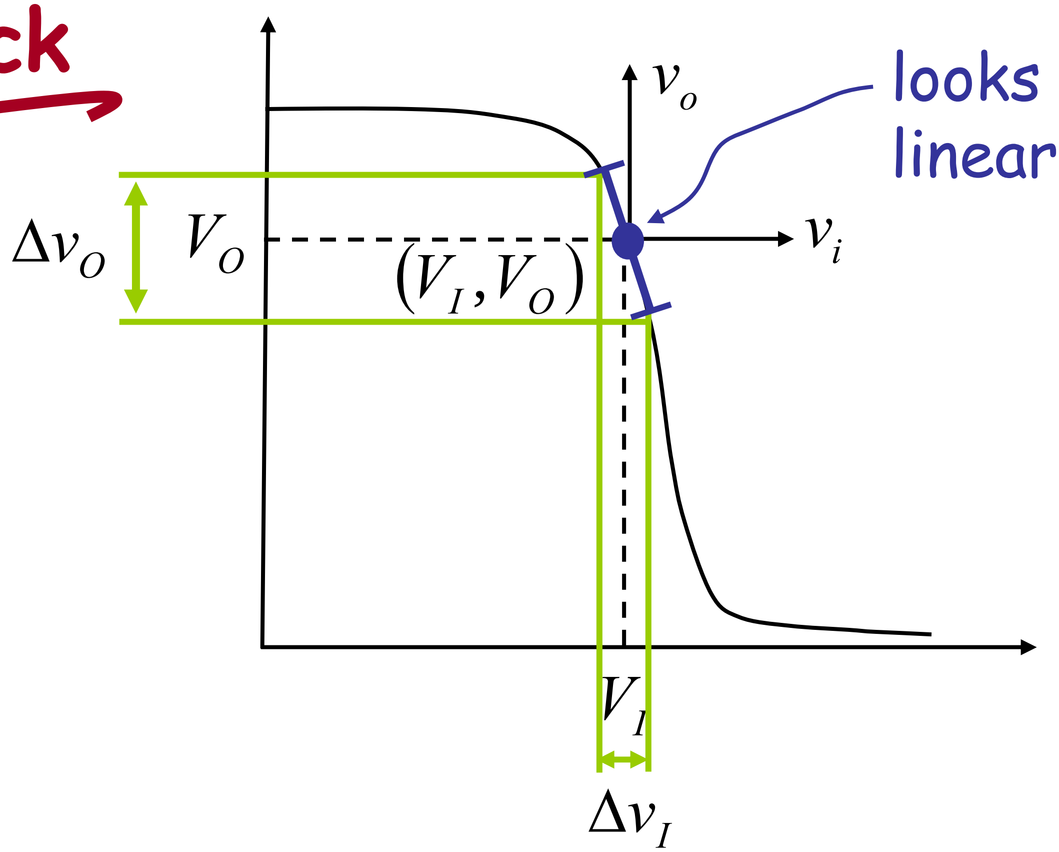
Amp all right, but nonlinear!

Hmmm ... So what about our linear amplifier ???

Insight:

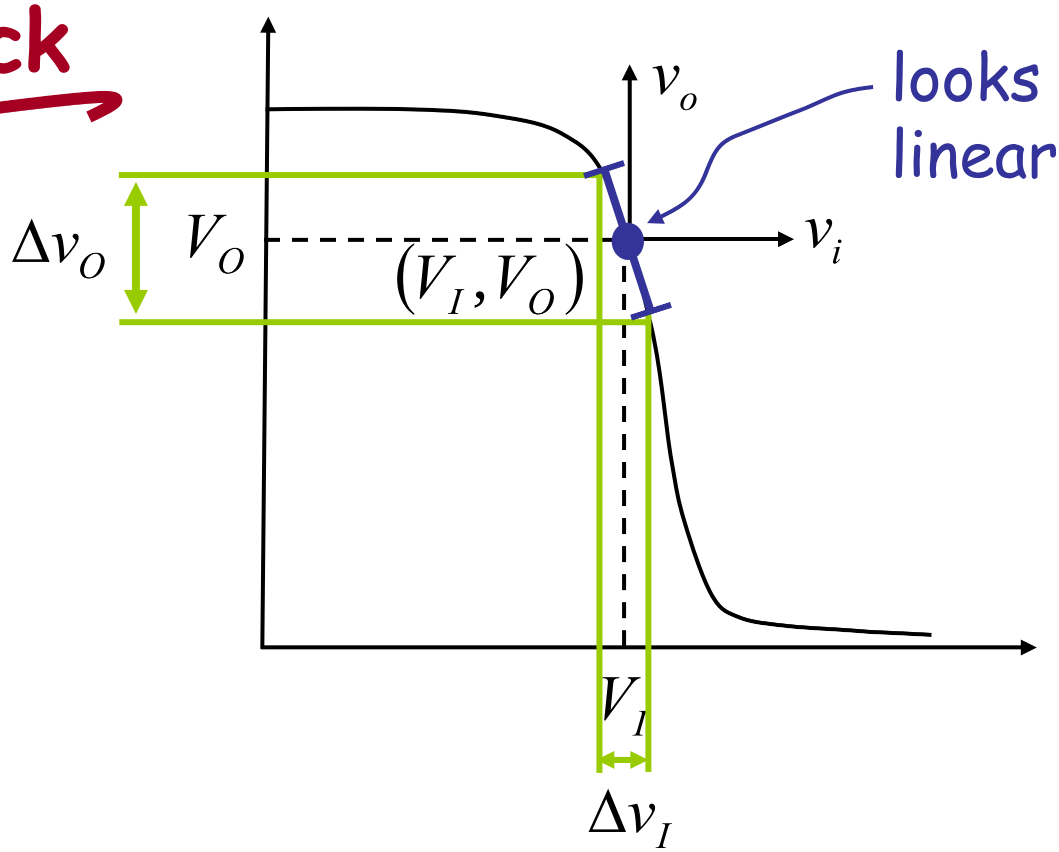
But, observe v_I vs v_O about some point (V_I, V_O) ... looks quite linear !

Trick



- Operate amp at V_I, V_O
  → DC "bias" (good choice: midpoint of input operating range)
- Superimpose small signal on top of V_I
- Response to small signal seems to be approximately linear

Trick



- Operate amp at V_I, V_O
  → DC "bias" (good choice: midpoint of input operating range)
- Superimpose small signal on top of V_I
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Let's look at this in more detail —

I graphically

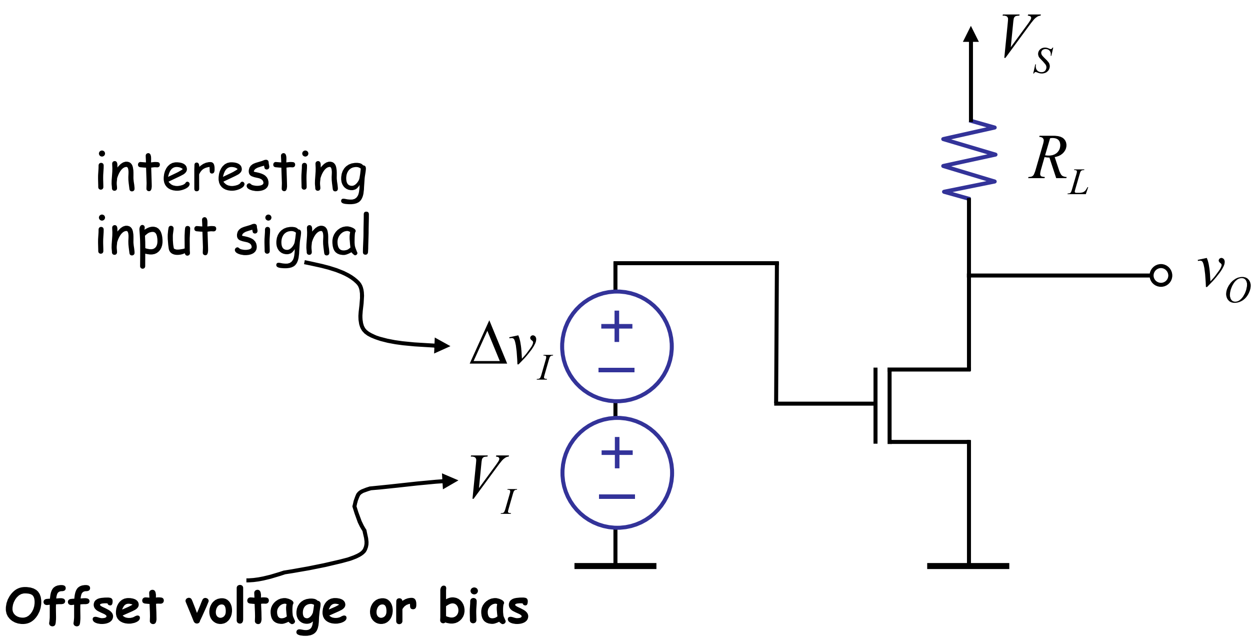
II mathematically

III from a circuit viewpoint

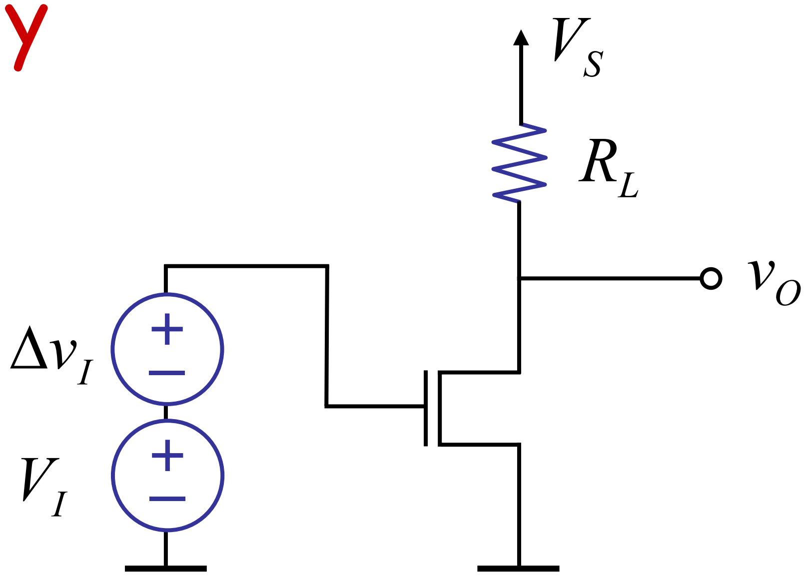
next week

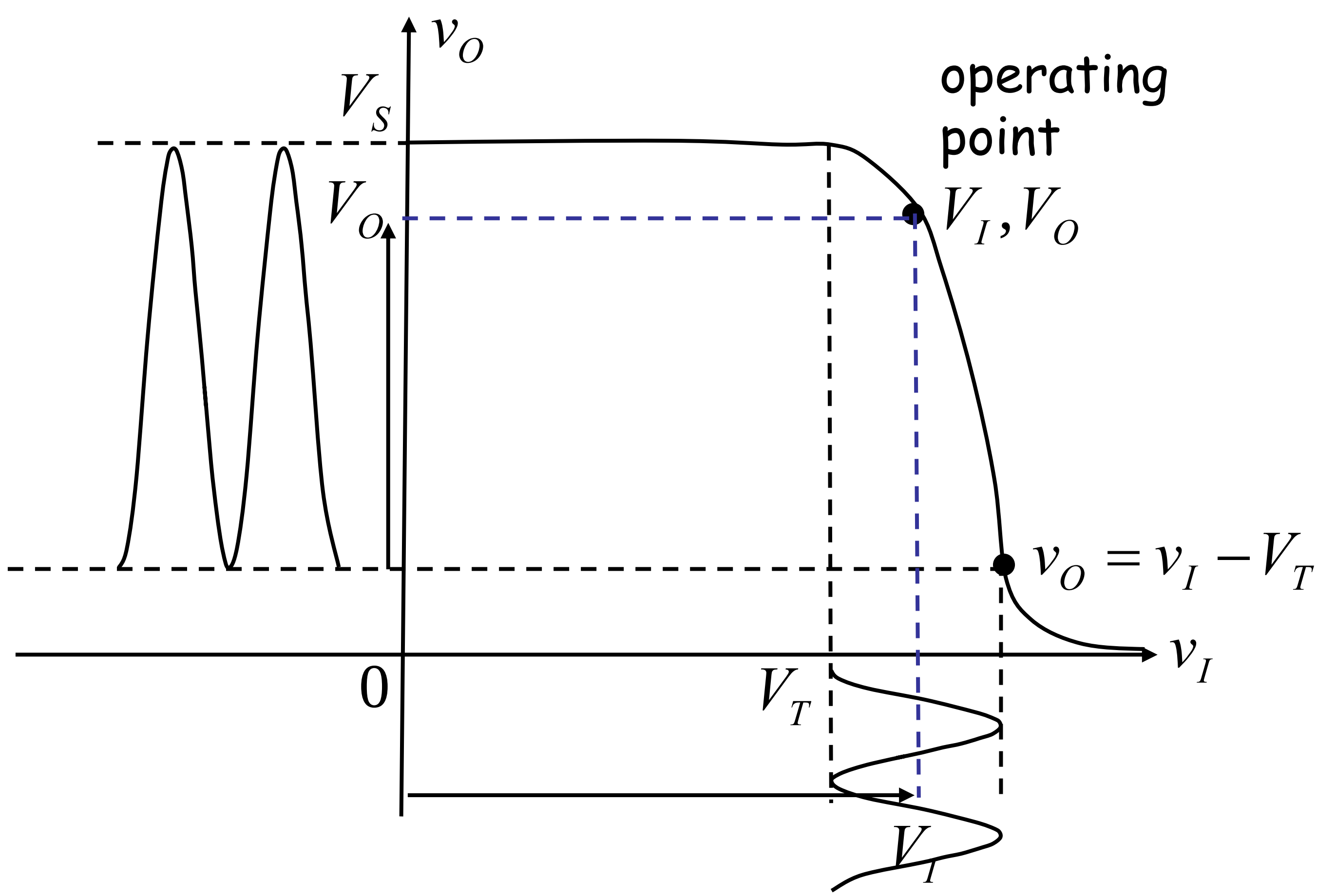
I Graphically

We use a DC bias V_I to "boost" interesting input signal above V_T, and in fact, well above V_T.



Graphically





Good choice for operating point:
midpoint of input operating range

Small Signal Model

aka incremental model
aka linearized model

Notation —

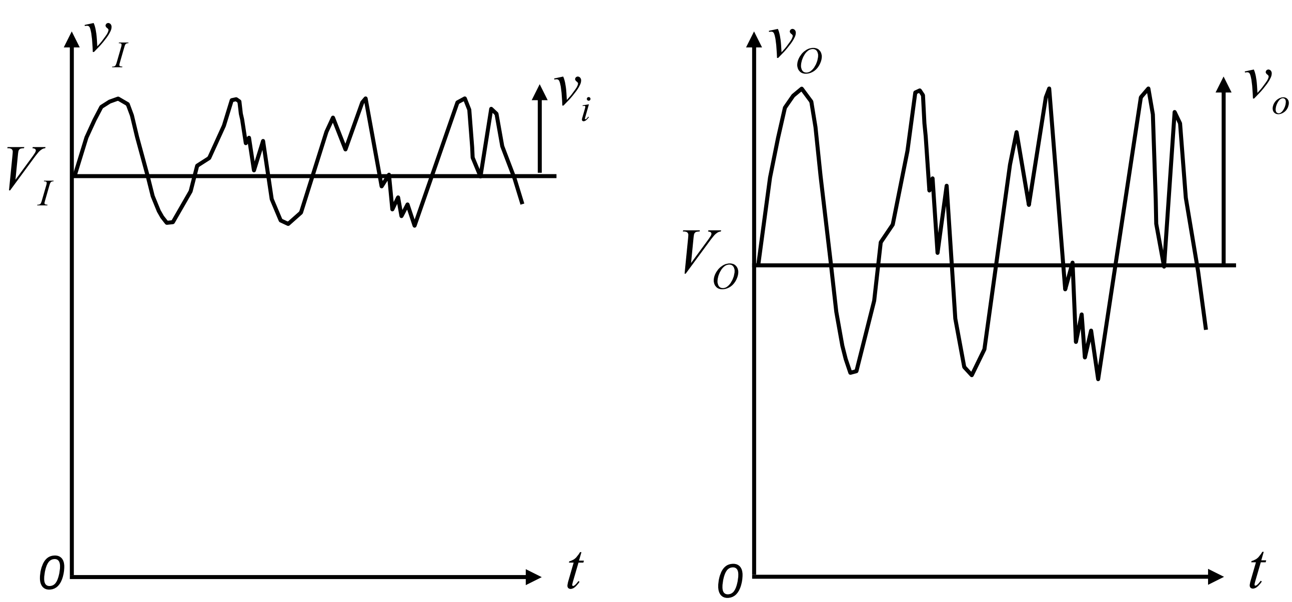
Input: $v_I = V_I + v_i$

total variable → v_I; DC bias → V_I; small signal (like Δv_I) → v_i

bias voltage aka operating point voltage

Output: $v_O = V_O + v_o$

Graphically,



II Mathematically

(... watch my fingers)

$$v_O = V_S - \frac{R_L K}{2}\left(v_I - V_T\right)^2 \qquad \Bigg| \qquad V_O = V_S - \frac{R_L K}{2}\left(V_I - V_T\right)^2 \; ✸$$

substituting $v_I = V_I + v_i$ $\qquad v_i << V_I$

$$v_O = V_S - \frac{R_L K}{2}\left(\left[V_I + v_i\right] - v_T\right)^2$$

$$= V_S - \frac{R_L K}{2}\left(\left[V_I - V_T\right] + v_i\right)^2$$

$$= V_S - \frac{R_L K}{2}\left(\left[V_I - V_T\right]^2 + 2\left[V_I - v_T\right]v_i + \cancel{v_i^2}\right)$$

$$\cancel{V_O} + v_o = \cancel{V_S - \frac{R_L K}{2}\left(V_I - V_T\right)^2} - R_L K\left(V_I - V_T\right)v_i$$

From ✸,

$$v_o = -R_L \underbrace{K\left(V_I - V_T\right)}_{g_m} v_i$$

related to V_I

Mathematically

$$v_o = -R_L \underbrace{K\left(V_I - V_T\right)}_{g_m} v_i$$

g_m related to V_I

$$v_o = -g_m R_L \; v_i$$

For a given DC operating point voltage V_I, $V_I - V_T$ is constant. So,

$$v_o = -\underbrace{A}_{\text{constant w.r.t. } v_i} v_i$$

constant w.r.t. v_i

In other words, our circuit behaves like a linear amplifier for small signals

Another way

$$v_O = V_S - \frac{R_L K}{2}\left(v_I - V_T\right)^2$$

$$\underbrace{v_o = \frac{d}{dv_I}\left.\left[V_S - \frac{R_L K}{2}\left(v_I - V_T\right)^2\right]\right|_{v_I = V_I}}_{\text{slope at } V_I} \cdot v_i$$

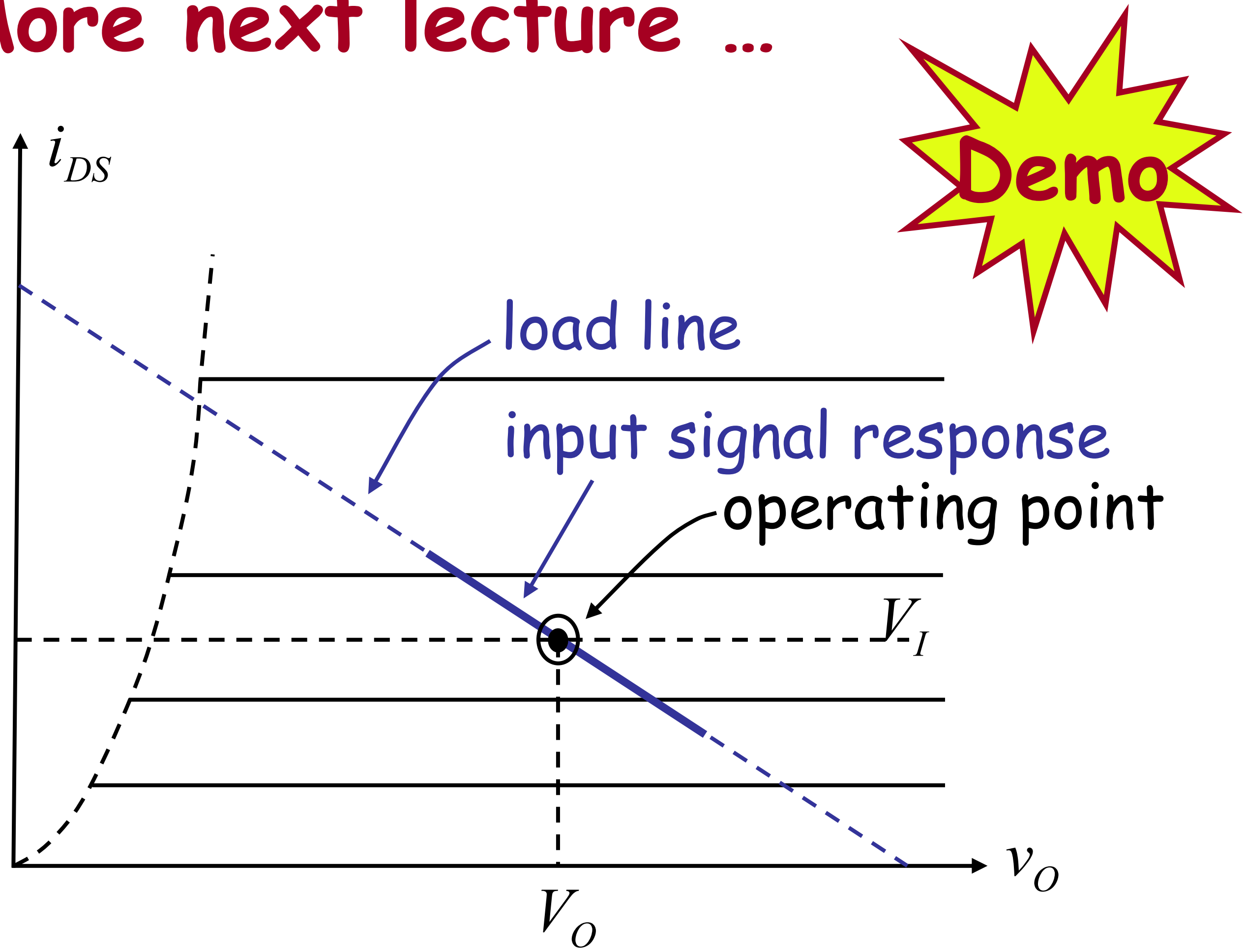
$$v_o = -R_L K\left(V_I - V_T\right) \cdot v_i$$

$$g_m = K\left(V_I - V_T\right)$$

$$A = -g_m R_L \qquad \text{amp gain}$$

Also, see Figure 8.9 in the course notes for a graphical interpretation of this result

More next lecture …



How to choose the bias point:

1. Gain component $g_m \propto V_I$
2. v_i gets big → distortion.
   So bias carefully
3. Input valid operating range.
   Bias at midpoint of input operating range for maximum swing.