6.642 Continuum Electromechanics
Fall 2008

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.642 Continuum Electromechanics

Problem Set #8 — Issued: 11/25/08
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Suggested Reading: Sections 8.17, 8.18
Final Exam: Dec. 9, 2004, 3-5PM. Open book, open notes. Focus on fluid interfacial stability.

Problem 1

Prob. 8.18.2 (Melcher, *Continuum Electromechanics*)

Corrections: Part (b) $\gamma = -\frac{\beta}{2} \pm c_{\pm}$

Part (c) $h_z \begin{pmatrix} \ell \\ 0 \end{pmatrix} = 0$

Problem 2

Prob. 8.18.3 (Melcher)

Correction: $\omega^2 = \frac{k^2 k_z^2 \left|I_4\right|^2 + I_1 I_3}{I_1 I_2}$

where, consistent with the usage in Section 8.14, E_o is the equilibrium electric field evaluated at the interface between layers.

(b) Show that the dispersion equation for the layer model, based on the results of Section 8.14, takes the normalized form

$$\frac{\underline{\omega}^2 \coth(\frac{\underline{k}}{2})}{\underline{k}}\left(2 + \frac{D\rho_m}{\rho_m}\right) = \frac{1}{2}\left[\frac{\frac{V_o}{d} Dq_e - gD\rho_m}{\frac{|V_o|}{d}|Dq_e|}\right] + S\left(\frac{1}{8\underline{k}} - \frac{1}{16}\right)$$

(c) Using $\underline{k} = 1$, $D\underline{\rho}_m = 0$, $V_o/|V_o| = 1$, $Dq_e/|Dq_e| = 1$ and $S = 1$, compare the prediction of the first eigenfrequency to the first resonance frequency predicted in the weak-gradient approximation and to the "exact" result shown in Fig. 8.18.2a. Compare the analytical expression to that for the weak-gradient imposed field approximation in the long-wave limit. Should it be expected that the layer approximation would agree with numerical results for very short wavelengths?

(d) How should the model be refined to include the second mode in the prediction?

Prob. 8.18.2 A layer of magnetizable liquid is in static equilibrium, with mass density and permeability having vertical distributions $\rho_s(x)$ and $\mu_s(x)$ (Fig. P18.8.2). The equilibrium magnetic field $H_s(x)$ is assumed to also have a weak gradient in the x direction, even though such a field is not irrotational. (For example, this gradient represents fields in the cylindrical annulus between concentric pole faces, where the poles have radii large compared to the annulus depth ℓ. The gradient in H_s is a quasi-one-dimensional model for the circular geometry.) Assume that the fluid is perfectly insulating and inviscid.

(a) Show that the perturbation equations can be reduced to

$$D(\mu_s D\hat{h}_z) - k^2\mu_s\hat{h}_z - j\frac{k_z^2}{\omega}H_s D\mu_s\hat{v}_x = 0$$

$$D(\rho_s D\hat{v}_x) - k^2\left(\rho_s - \frac{\mathcal{N}}{\omega^2}\right)\hat{v}_x + j\frac{k^2 H_s D\mu_s}{\omega}\hat{h}_z = 0$$

where $k^2 = k_y^2 + k_z^2$, $\vec{H} = H_s\vec{i}_z + \vec{h}$ and $\mathcal{N} = -g\,D\rho_s + \frac{1}{2}D\mu_s DH_s^2$

(b) As an example, assume that the profiles are $\rho_s = \rho_m \exp\beta x$, $\mu_s = \mu_m \exp\beta x$, H_s = constant. Show that solutions are a linear combination of $\exp\gamma x$, where

$$\gamma = -\frac{\beta}{2} \pm c_\pm; \quad c_\pm = \left[\left(\frac{\beta}{2}\right)^2 + k^2 + a \pm b\right]^{1/2}; \quad b = \left[\left(\frac{g\beta k^2}{2\omega^2}\right)^2 + \frac{k^2k_z^2}{\omega^2}\frac{H_s^2\mu_m\beta^2}{\rho_m}\right]^{1/2}$$

$$a = g\beta k^2/2\omega^2$$

(c) Assume that boundary conditions are $\hat{v}_x\binom{\ell}{0} = 0$, $\hat{h}_z\binom{\ell}{0} = 0$, and show that the eigenvalue equation is

$$\frac{2b}{a^2 - b^2}\sinh c_+\ell \sinh c_-\ell = 0$$

and that eigenfrequencies are

$$\omega_n^2 = \frac{k^2k_z^2H_s^2\mu_m\beta^2}{K_n^4\rho_m} - \frac{g\beta k^2}{K_n^2}; \quad K_n^2 = \left(\frac{n\pi}{\ell}\right)^2 + \left(\frac{\beta}{2}\right)^2 + k^2$$

(d) Discuss the stabilizing effect of the magnetic field on the bulk Rayleigh-Taylor instability.

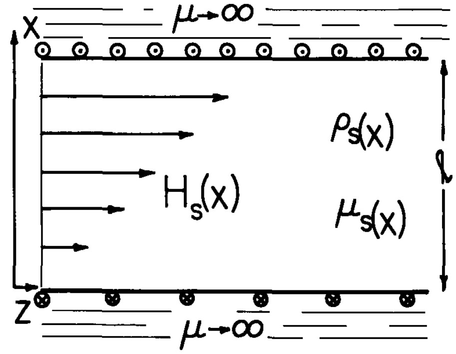

Fig. P8.18.2

Fig. 8.18.2 (continued)

(e) Discuss the analogous electric coupling with $\mu_s \rightarrow \varepsilon_s$ and $H_s \rightarrow E_s$ and describe the analogous physical configuration.

Prob. 8.18.3 As a continuation of Problem 8.18.2, prove that the principle of exchange of stabilities holds, and specifically that the eigenfrequencies are given by

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$$\omega^2 = \frac{k^2 k_z^2 |I_4|^2 + I_1 I_3}{I_1 I_2}$$

where

$$I_1 = \int_0^{\ell} (\mu_s |D\hat{h}_z|^2 + k^2 \mu_s |\hat{h}_z|^2) dx \; ; \qquad I_2 = \int_0^{\ell} (\rho_s |D\hat{v}_x|^2 + k^2 \rho_s |\hat{v}_x|^2) dx$$

$$I_3 = \int_0^{\ell} k^2 \mathcal{N} |\hat{v}_x|^2 dx \; ; \qquad I_4 = \int_0^{\ell} H_s D\mu_s \hat{v}_x^* \, \hat{h}_z dx$$