

Massachusetts Institute of Technology  
**Department of Electrical Engineering and Computer Science**  
 6.685 Electric Machines

Problem Set 5 Solutions

October 15, 2005

**Problem 1:** 1. The fifth and seventh (space) harmonics of flux density are:

$$B_{r5} = \frac{3}{2} \frac{\mu_0 N I}{2g} \frac{4}{5\pi} \sin(\omega t + 5\theta)$$

$$B_{r7} = \frac{3}{2} \frac{\mu_0 N I}{2g} \frac{4}{7\pi} \sin(\omega t - 7\theta)$$

Now, the angle with respect to the machine rotor is  $\theta'$ , and then, if rotor angle is  $\phi = \omega t + \xi$

$$\omega t + 5\theta = 6\omega t + 5\theta' + 5\xi$$

$$\omega t - 7\theta = -6\omega t - 7\theta' - 7\xi$$

So the relative phase velocities of the fifth and seventh harmonic fields are both negative (the rotor is turning faster than they are) and have magnitude of 6/5 and 6/7 of synchronous speed (with respect to the rotor). With respect to the rotor, then, the fields are:

$$B_{r5} = \frac{3}{2} \frac{\mu_0 N I}{2g} \frac{4}{5\pi} \sin(6\omega t + 5\theta' + 5\xi)$$

$$B_{r7} = -\frac{3}{2} \frac{\mu_0 N I}{2g} \frac{4}{7\pi} \sin(6\omega t + 7\theta' + 7\xi)$$

The voltage induced by this is found by using Faraday's Law, which in this case yields:

$$E'_z = \omega' R B_r$$

and then multiplying by twice the length for the two halves of the winding. (An alternative, which gives the same answer is to integrate to find flux and then differentiating with respect to time). In either case the answer is:

$$V_6 = \frac{3}{2} \frac{4}{\pi} \frac{\mu_0 N I}{2g} 2Rl\omega \operatorname{Re} \left\{ \left( -j \frac{6}{25} e^{j5\xi} + j \frac{6}{49} e^{j7\xi} \right) e^{j6\omega t} \right\}$$

2. The maximum and minimum values of voltage are easily estimated from the above, and they are about 393 and 128 volts, peak, respectively.
3. See Figure 1 and attached script.

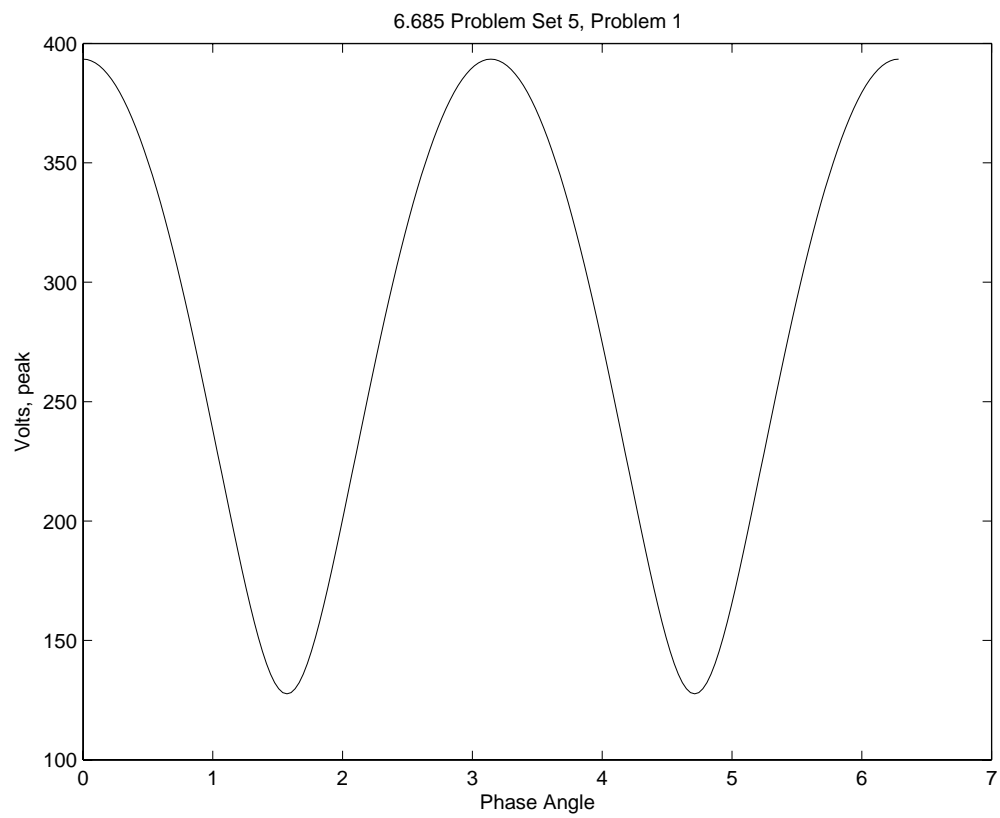


Figure 1: Induced Voltage Magnitude

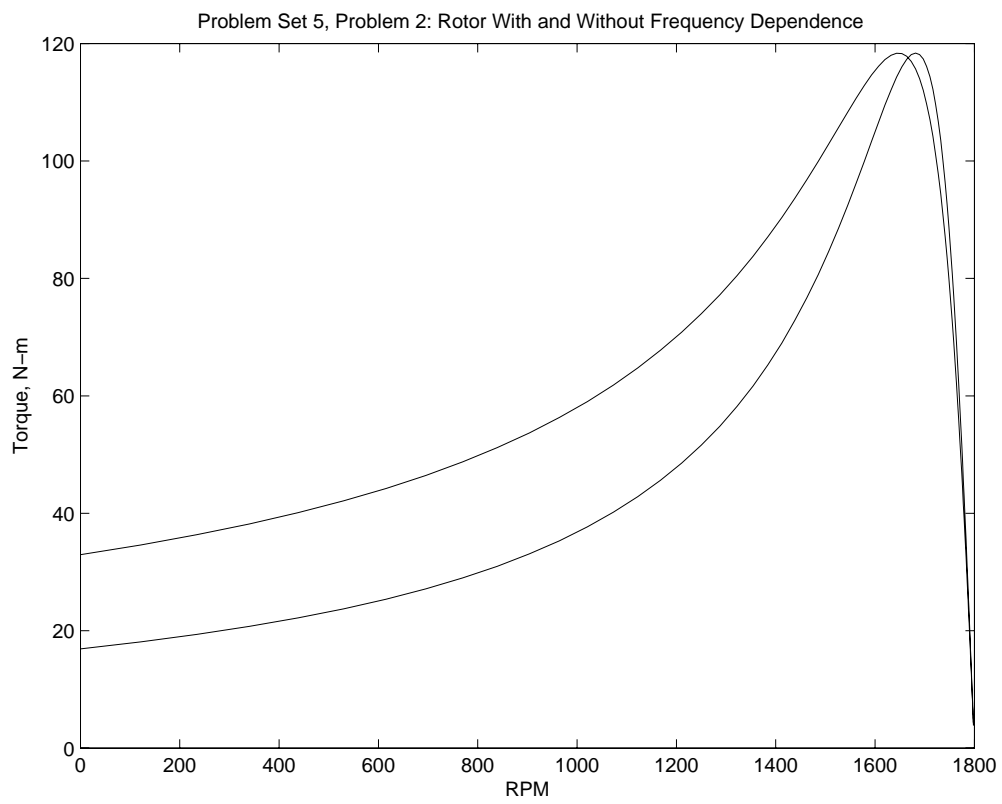


Figure 2: Comparison of Torque-Speed Curves With and Without Rotor Frequency Dependence

**Problem 2:** This is a straightforward circuit problem: the script is taken from Chapter 8 of the notes and is appended. The frequency modification of rotor resistance is also straightforward: See the script (appended).

**Problem 3:** As it turns out, this problem is worked in almost its entirety in Chapter 8 of the notes. In the section entitled 'Deep Slots' is a formula for the slot impedance per unit length:

$$Z_{\text{slot}} = \frac{1}{w_s} \frac{1+j}{\sigma\delta} \coth(1+j) \frac{h_s}{\delta}$$

where  $w_s$  and  $h_s$  are width and height of the slot,  $\sigma$  is conductivity of the slot material and  $\delta$  is 'skin depth':

$$\delta = \sqrt{\frac{2}{\omega_s \sigma \mu_0}}$$

Slot resistance includes as well the inductive impedance of the slot depression:

$$Z_{\text{tot}} = Z_{\text{slot}} + j\omega\mu_0 \frac{h_s}{w_s}$$

This is programmed in an attached script and the results are shown in Figure 3. Slot resistance is the real part and is shown in Figure 4

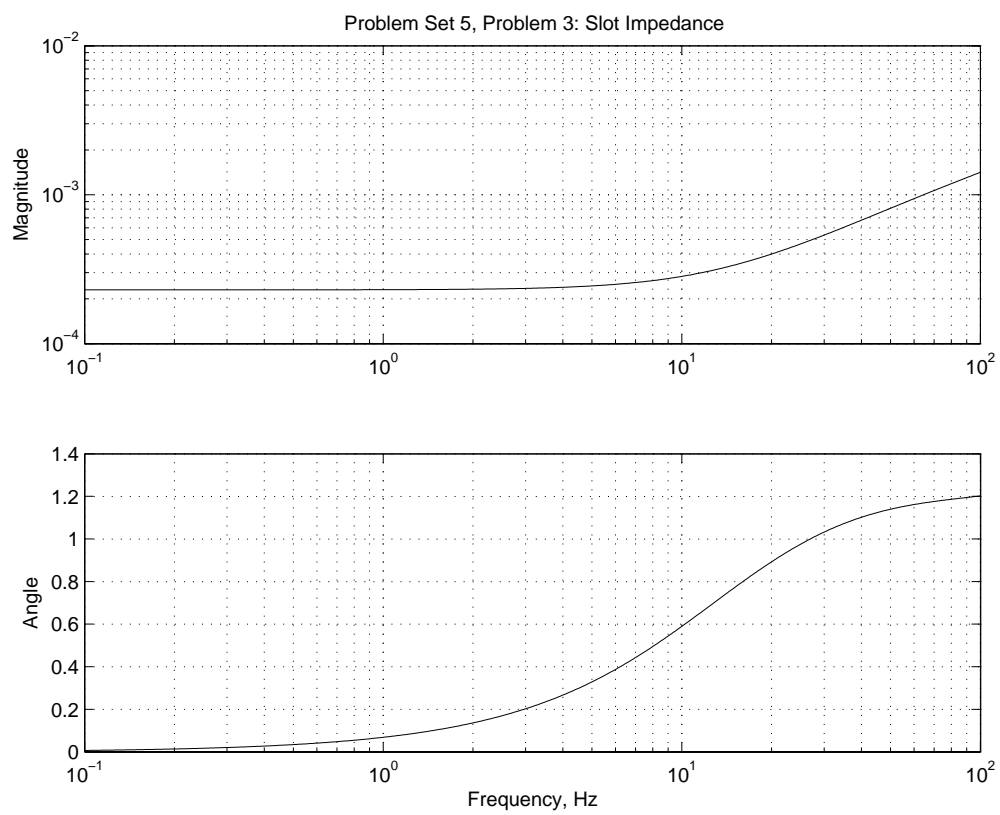


Figure 3: Rotor Slot Impedance Per Unit Length

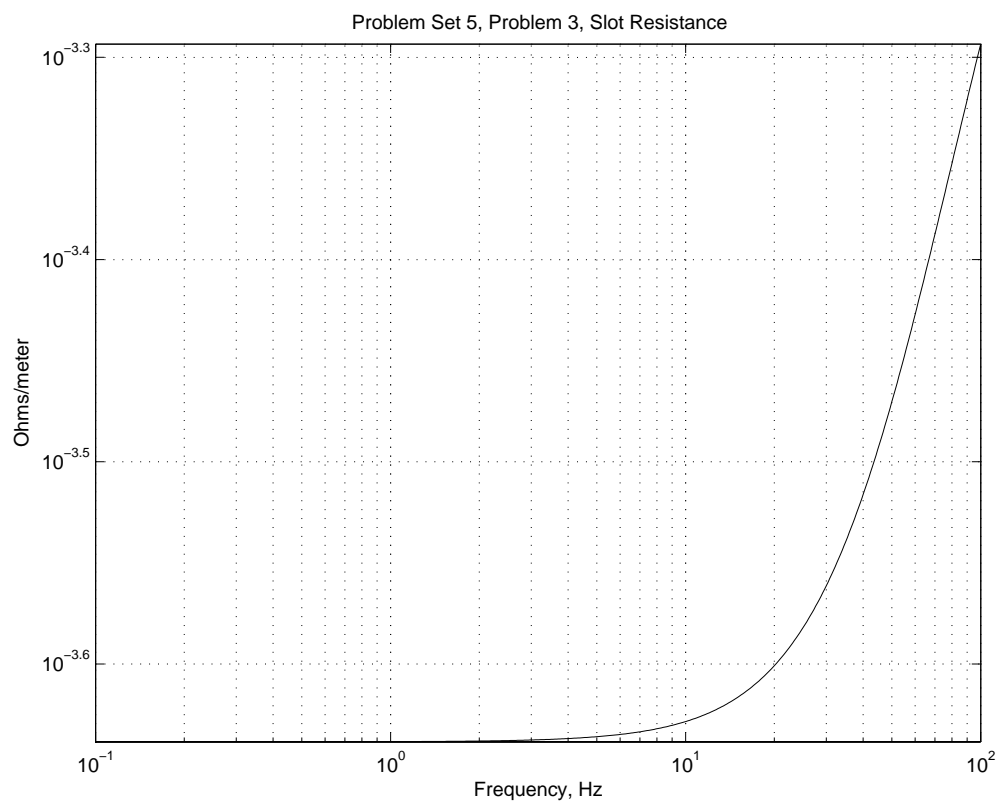


Figure 4: Rotor Slot Resistance Per Unit Length

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% Problem Set 5, Problem 1

% Parameters
N=2;
I=10000;
R = 1;
L = 3;
g = .05;
muzero = pi*4e-7;
om = 2*pi*60;

squig = 0:pi/100:2*pi;

A = abs((1/25) .* exp(j*5 .* squig) + (1/49) .* exp(j*7 .* squig));
V = (6/pi)*(muzero*N*I*R*L*12*om/(2*g)) .* A;

figure(1)
plot(squig, V)
title('6.685 Problem Set 5, Problem 1')
ylabel('Volts, peak')
xlabel('Phase Angle')

Vmax = (6/pi)*(muzero*N*I*6*om*R*L/g)*(1/25+1/49)
Vmin = (6/pi)*(muzero*N*I*6*om*R*L/g)*(1/25-1/49)

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% -----
% Torque-Speed Curve for an Induction Motor
% Assumes the classical model
% This is a single-circuit model
% Required parameters are R1, X1, X2, R2, Xm, Vt, Ns
% Assumed is a three-phase motor
% This thing does a motoring, full speed range curve
% Copyright 1994, 1997 James L. Kirtley Jr.
% See 6.11s Copyright Notice for Rules of Use
% Modified to do a comparison
% -----

X1 = 1.15;
X2 = 3.4;
Xm = 116;
R1 = .5;
R2 = .3;
p = 2;
f = 60;
Vt = 277;

Ns = 60*f/p;           % Synchronous Speed, RPM
oms = 2*pi*Ns/60;     % Synchronous speed, rad/sec
s = logspace(-3, 0, 100); % get a nice vector of slips

% Part 1: Assume rotor resistance is frequency independent
N = Ns .* (1 - s);    % Speed, in RPM
Rr = R2 ./ s;         % Rotor resistance
Zr = j*X2 + Rr;       % Total rotor impedance
Za = j*Xm .* Zr ./ (j*Xm + Zr); % Air-gap impedance
Zt = R1 + j*X1 + Za;  % Terminal impedance
Ia = Vt ./ Zt;        % Terminal Current
I2 = Ia .* j*Xm ./ (j*Xm + Zr); % Rotor Current
Pag = 3 .* abs(I2) .^2 .* Rr; % Air-Gap Power
Pm = Pag .* (1 - s);  % Converted Power
Trqa = Pag ./ oms;    % Developed Torque

% Part 2: add frequency dependence

fr = f .* s;          % relative rotor frequency
Rr = R2 .* (1 + sqrt(fr ./ f)) ./ s; % Rotor resistance
Zr = j*X2 + Rr;       % Total rotor impedance
Za = j*Xm .* Zr ./ (j*Xm + Zr); % Air-gap impedance
Zt = R1 + j*X1 + Za;  % Terminal impedance
Ia = Vt ./ Zt;        % Terminal Current

```



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I2 = Ia .* j*Xm ./ (j*Xm + Zr); % Rotor Current
Pag = 3 .* abs(I2) .^2 .* Rr; % Air-Gap Power
Pm = Pag .* (1 - s); % Converted Power
Trqb = Pag ./ oms; % Developed Torque

figure(1)
plot(N, Trqa, N, Trqb)
title('Problem Set 5, Problem 2: Rotor With and Without Frequency Dependence');
ylabel('Torque, N-m');
xlabel('RPM');

```

```

% 6.685 Problem Set 5, Problem 3
% Complex Slot Impedance
% Parameters:
hd = .001;           % slot depression depth
wd = .001;           % slot depression width
ws = .005;           % slot width
hs = .015;           % slot height
sig = 5.8e7;         % conductivity
muzero = pi*4e-7;   % muzero

% frequency uses a log space
f = logspace(-1, 2, 500);
om = 2*pi .* f;      % do this in radians/second
delt = sqrt((2/(muzero*sig)) ./om);

% now here is slot impedance
zslot = (j*muzero*hd/wd) .* om + ((1+j)/(sig*ws)) .* coth((1+j)*hs ./ delt) ./ delt;

figure(1)
subplot 211
loglog(f, abs(zslot))
title('Problem Set 5, Problem 3: Slot Impedance')
ylabel('Magnitude')
grid on
subplot 212
semilogx(f, angle(zslot))
ylabel('Angle')
xlabel('Frequency, Hz')
grid on

rslot = real(zslot);
figure(2)
loglog(f, rslot)
title('Problem Set 5, Problem 3, Slot Resistance')
ylabel('Ohms/meter')
xlabel('Frequency, Hz')

```